Chapter 249

# Exponentially Weighted Moving Average (EWMA) Charts

## Introduction

This procedure generates exponentially weighted moving average (EWMA) control charts for variables. Charts for the mean and for the variability can be produced. The format of the control charts is fully customizable. The data for the subgroups can be in a single column or in multiple columns. This procedure permits the defining of stages. The target value can be entered directly or estimated from the data, or a subset of the data. Sigma may be estimated from the data, or a standard sigma value may be entered. Means and ranges/standard deviations can be stored to the spreadsheet.

## Exponentially Weighted Moving Average Control Charts

Similar to the CUSUM chart, the EWMA chart is useful in detecting small shifts in the process mean. These charts are used to monitor the mean of a process based on samples taken from the process at given times (hours, shifts, days, weeks, months, etc.). The measurements of the samples at a given time constitute a subgroup.

The EWMA chart relies on the specification of a target value and a known or reliable estimate of the standard deviation. For this reason, the moving average chart is better used after process control has been established.
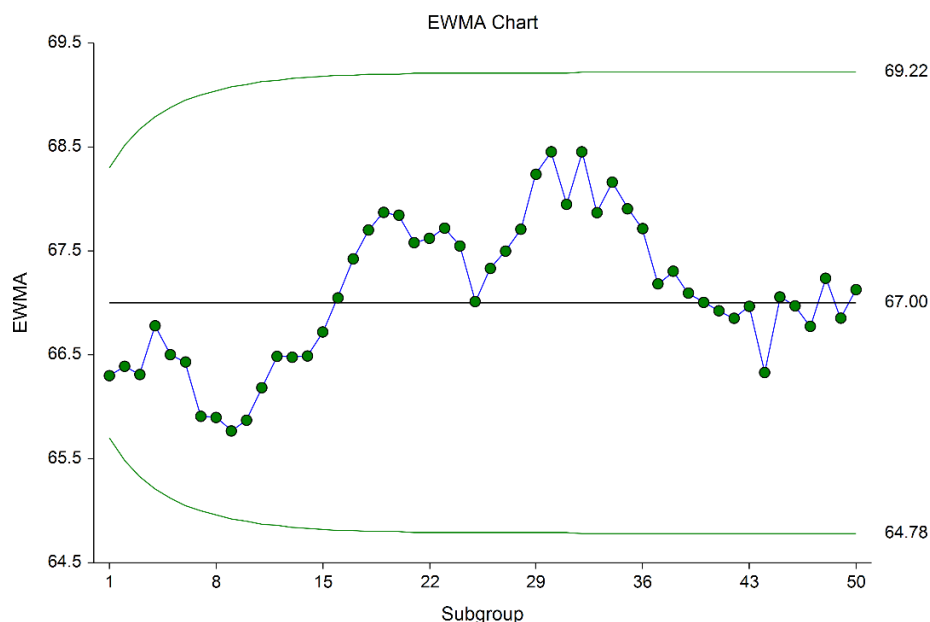
# Other Control Charts for the Mean and Variation of a Process

Common control charts for detecting large process changes, or for establishing initial control of the process are the X-bar and R charts (or X-bar and s charts).

Once a reliable estimate of the mean and standard deviation is available, the EWMA and CUSUM charts are useful in detecting smaller shifts in the process mean.

The EWMA chart may also be used when only a single response is available at each time point. Another option for single responses are the individuals and moving range (I-MR) control charts.

# Control Chart Formulas

Suppose we have $k$ subgroups, each of size $n$. Let $x_{ij}$ represent the measurement in the $j^{th}$ sample of the $i^{th}$ subgroup.

## Formulas for the Points on the Chart

The $i^{th}$ subgroup mean is calculated using

$$\bar{x}_i = \frac{\sum_{j=1}^{n} x_{ij}}{n}$$

The points of the chart are obtained from the $\bar{x}_i$'s by the following exponential smoothing operation

$$z_i = \lambda \bar{x}_i + (1 - \lambda) z_{i-1}$$

The value of $z_0$ is set to the target mean. The value of $\lambda$ is specified by the user.

Values for the points of the EWMA variation chart are given by substituting $\bar{x}_i$ with the range or standard deviation.

## Estimating the EWMA Chart Center Line (Grand Mean)

In the EWMA Charts procedure, the target mean may be input directly, or it may be estimated from a series of subgroups. If it is estimated from the subgroups the formula for the grand average is

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij}}{\sum_{i=1}^{k} n_i}.$$

If the subgroups are of equal size, the above equation for the grand mean reduces to

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{k} \bar{x}_i}{k} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k}.$$

## Estimating Sigma – Sample Ranges

Either the range or the standard deviation of the subgroups may be used to estimate sigma, or a known (standard) sigma value may be entered directly. If the standard deviation (sigma) is to be estimated from the ranges, it is estimated as

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

where

$$\bar{R} = \frac{\sum_{i=1}^{k} R_i}{k}$$

$$d_2 = \frac{E(R)}{\sigma} = \frac{\mu_R}{\sigma}$$

The calculation of $E(R)$ requires the knowledge of the underlying distribution of the $x_{ij}$'s. Making the assumption that the $x_{ij}$'s follow the normal distribution with constant mean and variance, the values for $d_2$ are derived through the use of numerical integration. It is important to note that the normality assumption is used and that the accuracy of this estimate requires that this assumption be valid.

When $n$ is one, we cannot calculate $R_i$ since it requires at least two measurements. The procedure in this case is to use the ranges of successive pairs of observations. Hence, the range of the first and second observation is computed, the range of the second and third is computed, and so on. The average of these approximate ranges is used to estimate $\sigma$.

## Estimating Sigma – Sample Standard Deviations

If the standard deviation (sigma) is to be estimated from the standard deviations, it is estimated as

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

where

$$\bar{s} = \frac{\sum_{i=1}^{k} s_i}{k}$$

$$c_4 = \frac{E(s)}{\sigma} = \frac{\mu_s}{\sigma}$$

The calculation of $E(s)$ requires the knowledge of the underlying distribution of the $x_{ij}$'s. Assuming that the $x_{ij}$'s follow the normal distribution with constant mean and variance, the values for $c_4$ are obtained from

$$c_4 = \sqrt{\frac{2}{n-1}} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$$

## Estimating Sigma – Weighted Approach

When the sample size is variable across subgroups, a weighted approach is recommended for estimating sigma (Montgomery, 2013):

$$\hat{\sigma} = \bar{s} = \left[\frac{\sum_{i=1}^{k}(n_i - 1)s_i^2}{\sum_{i=1}^{k} n_i - k}\right]^{1/2}$$

## EWMA Chart Limits

The lower and upper control limits for the EWMA chart are calculated using the formulas

$$LCL_i = \mu_0 - m\left(\frac{\hat{\sigma}}{\sqrt{n}}\right)\sqrt{\frac{\lambda}{2 - \lambda}[1 - (1 - \lambda)^{2i}]}$$

$$UCL_i = \mu_0 + m\left(\frac{\hat{\sigma}}{\sqrt{n}}\right)\sqrt{\frac{\lambda}{2 - \lambda}[1 - (1 - \lambda)^{2i}]}$$

where $m$ is a multiplier (often set to 2.7 or 3), and $\lambda$ is the exponential smoothing constant.

Note that the values of limits change with each successive subgroup but tend to level off at around the tenth subgroup.

Values for the limits of the EWMA variation chart are given by substituting $\mu_0$ with the range or standard deviation center line value (such as R-bar or s-bar).

# Data Structure

In this procedure, the data may be in either of two formats. The first data structure option is to have the data in several columns, with one subgroup per row.

**Example Dataset**

| S1 | S2 | S3 | S4 | S5 |
|----|----|----|----|----|
| 2  | 6  | 3  | 8  | 5  |
| 8  | 8  | 7  | 7  | 9  |
| 6  | 2  | 2  | 4  | 3  |
| 5  | 6  | 7  | 6  | 10 |
| 48 | 2  | 6  | 5  | 0  |
| .  | .  | .  | .  | .  |
| .  | .  | .  | .  | .  |
| .  | .  | .  | .  | .  |

The second data structure option uses one column for the response data, and either a subgroup size or a second column defining the subgroups.

**Alternative Example Dataset**

| Response | Subgroup |
|----------|----------|
| 2 | 1 |
| 6 | 1 |
| 3 | 1 |
| 8 | 1 |
| 5 | 1 |
| 8 | 2 |
| 8 | 2 |
| 7 | 2 |
| 7 | 2 |
| 9 | 2 |
| 6 | 3 |
| 2 | 3 |
| . | . |
| . | . |
| . | . |

In the alternative example dataset, the Subgroup column is not needed if every subgroup is of size 5 and the user specifies 5 as the subgroup size. If there are missing values, the Subgroup column should be used, or the structure of the first example dataset.

# Quality Control Chart Format Window Options

This section describes a few of the specific options available on the first tab of the control chart format window, which is displayed when a quality control chart format button is pressed. Common options, such as axes, labels, legends, and titles are documented in the Graphics Components chapter.
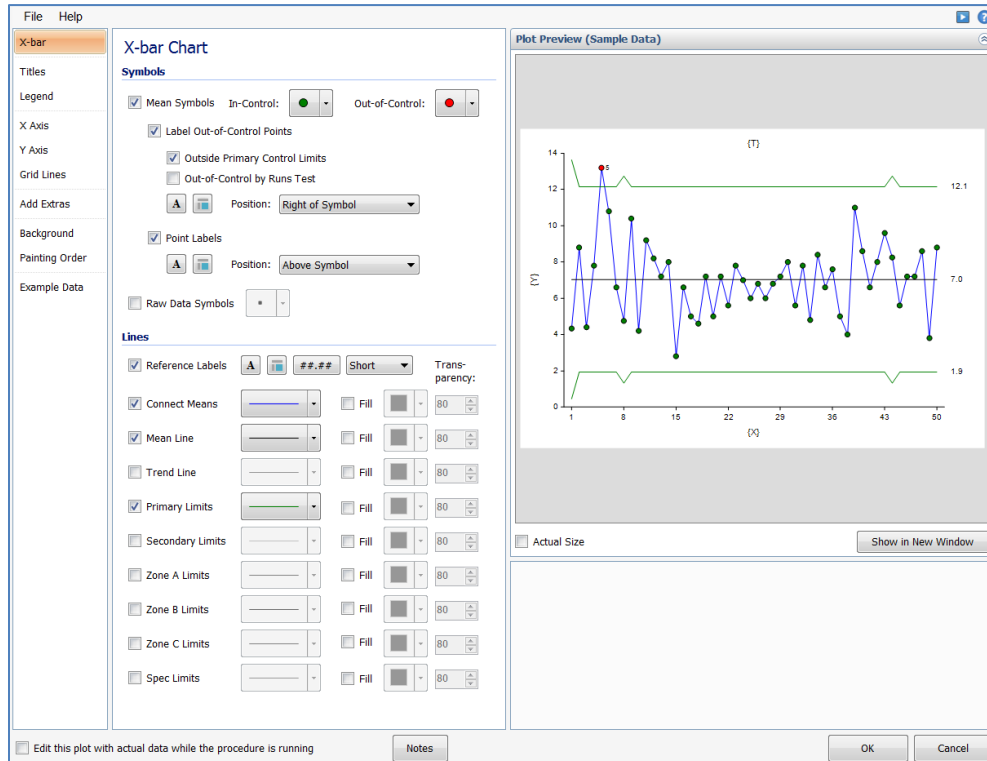


## [Xbar] / [Range] Chart Tab

### Symbols Section

You can modify the attributes of the symbols using the options in this section.
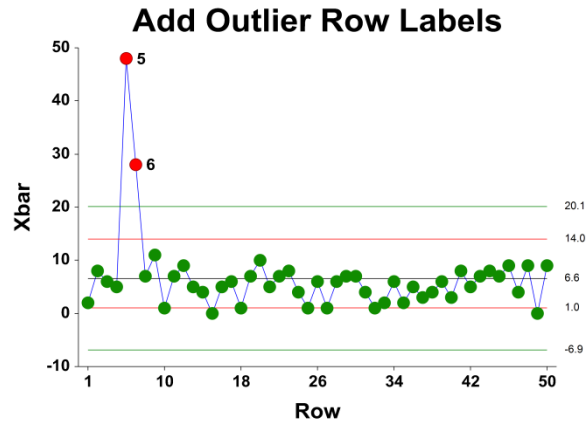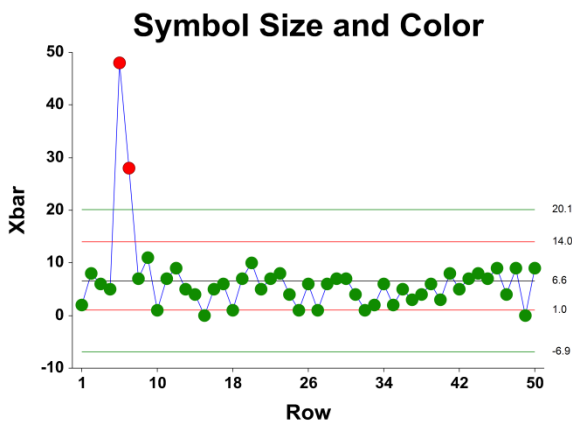
A wide variety of sizes, shapes, and colors are available for the symbols. The symbols for in-control and out-of-control points are specified independently. There are additional options to label out-of-control points. The label for points outside the primary control limits is the subgroup number. The label for points that are out-of-control based on the runs test is the number of the first runs test that is signaled by this point.

The user may also specify a column of point labels on the procedure variables tab, to be used to label all or some of the points of the chart. The raw data may also be shown, based on customizable raw data symbols.

## Lines Section

You can specify the format of the various lines using the options in this section. Note that when shading is desired, the fill will be to the bottom for single lines (such as the mean line), and between the lines for pairs of lines (such as primary limits).
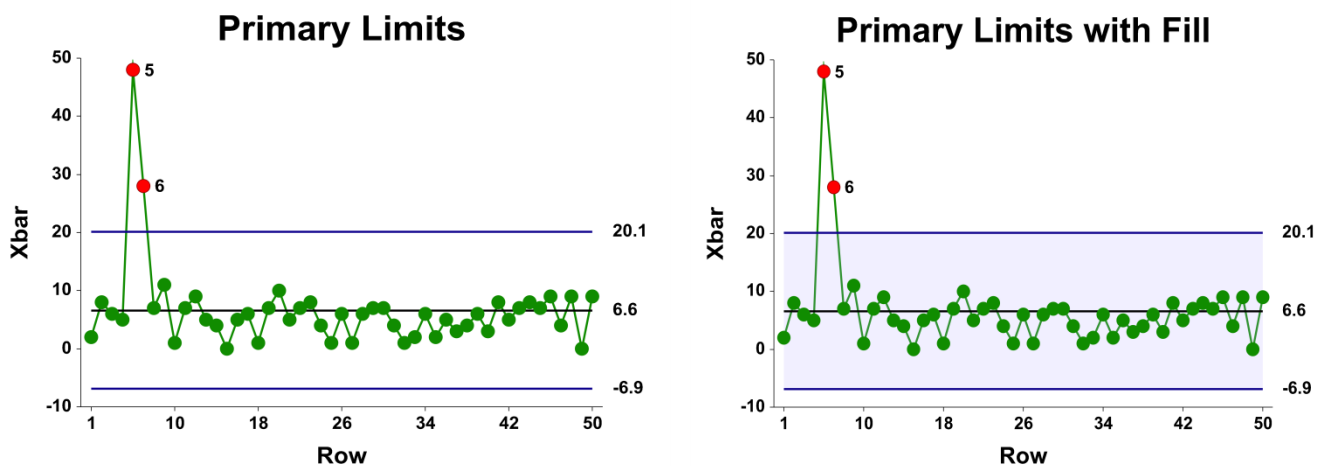


Lines for the zones, secondary limits, and specification limits are also specified here.

# Titles, Legend, Numeric Axis, Group Axis, Grid Lines, and Background Tabs

Details on setting the options in these tabs are given in the Graphics Components chapter. The legend does not show by default but can easily be included by going t1o the Legend tab and clicking the Show Legend checkbox.

# Example 1 – EWMA Chart

This section presents an example of how to produce an EWMA chart. The data represent 50 subgroups of size 5. The data used are in the QC dataset. We will analyze the variables D1 through D5 of this dataset.

## Setup

To run this example, complete the following steps:

**1    Open the QC example dataset**
- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **QC** and click **OK**.

**2    Specify the Exponentially Weighted Moving Average (EWMA) Charts procedure options**
- Find and open the **Exponentially Weighted Moving Average (EWMA) Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

  Variables Tab

  Data Variables .................................................**D1-D5**

  Target & Sigma Tab

  Target Value Specification...............................**Enter Target Value(s)**
  Target Value(s) ...............................................**67**
  Sigma Specification ........................................**Enter Standard Sigma Value(s)**
  Sigma Value(s) ...............................................**8**

**3    Run the procedure**
- Click the **Run** button to perform the calculations and generate the output.

## Target Value Section

**Target Value ($\mu_0$) Section for Subgroups 1 to 50**

Number of Subgroups:    50

| Target Value Specification | Value |
|---|---|
| User-Specified Target Value | 67 |

This section displays the target value that is used in the EWMA chart.

## Target Value Specification

This is the method by which the target value is obtained.

## Target Value

This is value of the target value used to create the moving average chart.

# Sigma Specification Section

**Sigma Estimation Section for Subgroups 1 to 50**

| Sigma Specification | User-Specified Sigma Value |
|---|---|
| User-Specified | 8 |

This section shows the specification of the standard deviation used in the EWMA chart.
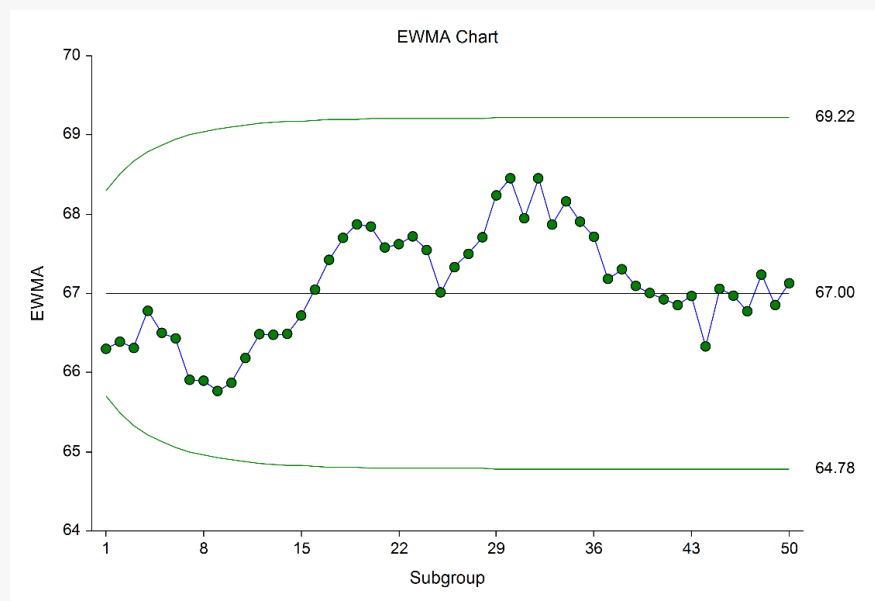
## Sigma Specification

This is the method by which the sigma value is obtained.

## User-Specified Sigma Value

This is value of the sigma entered by the user, and is the value used to create the moving average chart.

# EWMA Chart

**Chart Section for Subgroups 1 to 50**



This EWMA plot shows the progress of the weighted moving averages across the 50 subgroups. There does not appear to be an indication of a change in the process mean.

# Example 2 – EWMA Chart with Additional Formatting

This example uses the same setup as Example 1, except that a variety of improvements are made in the plot format. These improvements are made by clicking the EWMA Chart format button on the EWMA Chart tab.

The settings for this example are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

## EWMA Chart

**Chart Section for Subgroups 1 to 50**
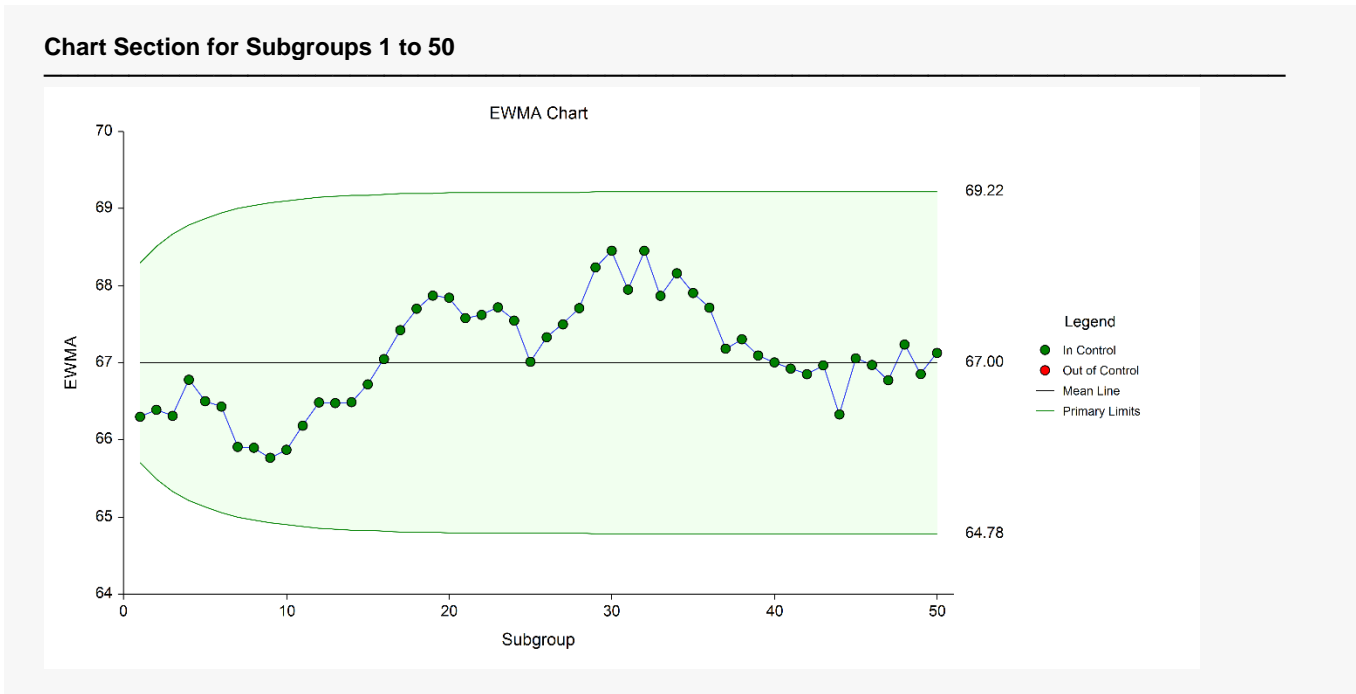


As shown here, a variety of enhancements can be made to the formatting of the control charts to make the chart as easy to read as possible.