Chapter 239

Operating Characteristic Curves for Acceptance Sampling for Attributes

Introduction

The *Operating Characteristic Curves for Acceptance Sampling for Attributes* procedure is a companion procedure to the procedure *Acceptance Sampling for Attributes*. This procedure is used view graphically the probability of lot acceptance versus the lot proportion defective for a given sample size and acceptance number. Plots with multiple curves for multiple sample sizes or multiple acceptance numbers may be generated using this procedure.

In this procedure, the lot size can be assumed to be infinite (or continuous) and use the binomial distribution for calculations, or the lot can have a fixed size, whereupon the calculations are based on the hypergeometric distribution.
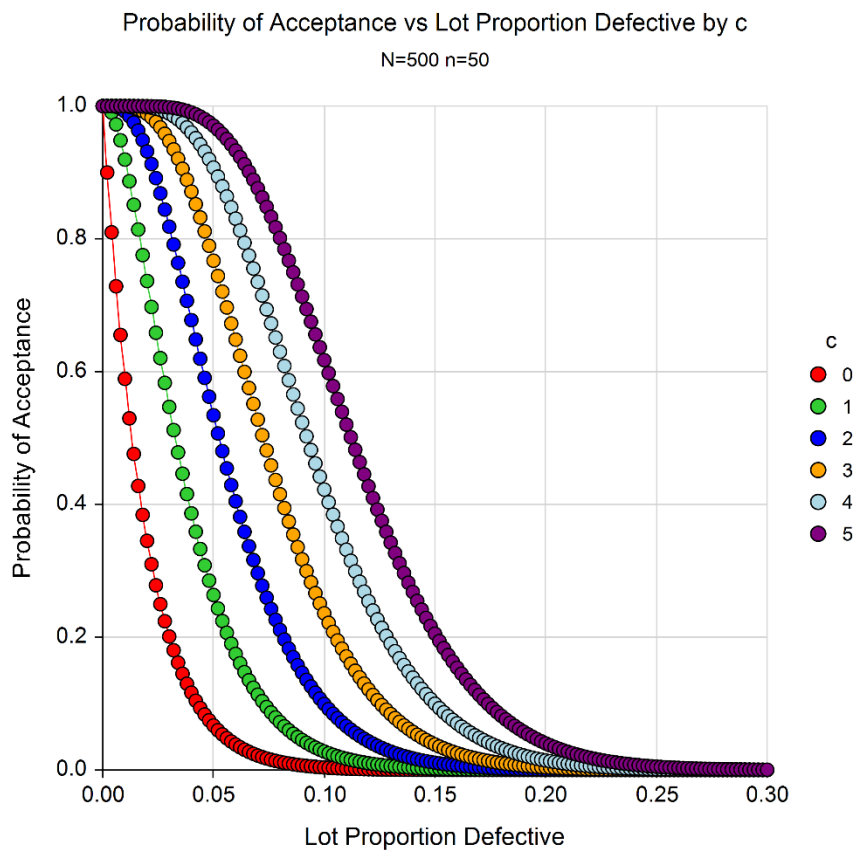
Technical details

Define *N* to be the lot size (possibly infinite), *n* as the size of the sample to be drawn, and *c* to be the acceptance number (the highest number of nonconforming units for which the lot will still be accepted). Let *X* denote the number of nonconforming units in the sample. If the lot size is assumed to be infinite, the probability of acceptance, P_a, for a given lot proportion defective, *p*, is given by

$$P_a = \sum_{X=0}^{c} \frac{n!}{X!\,(n-X)!} p^X (1-p)^{n-X}$$

If the lot size is finite, the hypergeometric distribution is used, where, for each *X*,

$$h(X; N, M, n) = \frac{\binom{M}{X}\binom{N-M}{n-X}}{\binom{N}{n}}$$

where *M* = [N*p*].

The probability of acceptance is given by

$$P_a = \sum_{X=0}^{c} h(X; N, M, n)$$

Example 1 – Generating an Operating Characteristic Curve

Suppose a quality engineer is to receive a lot of 500 units. The engineer is considering sampling 50 units and would like to examine the operating characteristic curve for an acceptance number of 3. To begin, the engineer will examine the full range of lot proportion defectives (0 to 1).

Setup

To run this example, complete the following steps:

1   Specify the Operating Characteristic Curves for Acceptance Sampling for Attributes procedure options

- Find and open the **Operating Characteristic Curves for Acceptance Sampling for Attributes** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1(a or b)** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Lot Size (N)...**500**
Sample Size (n) **50**
Acceptance Number (c) **3**
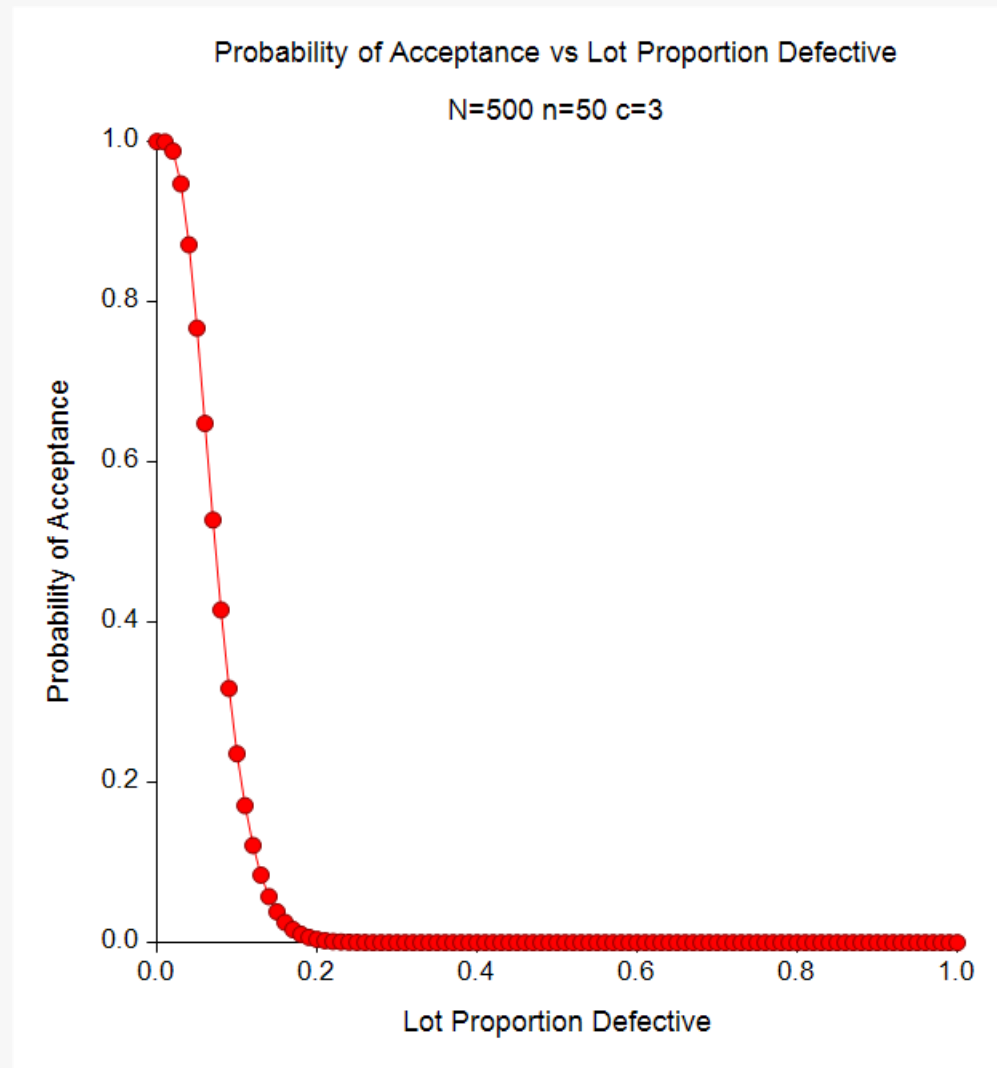Proportion Defective Range.............................**0 to 1 by 0.01**

2   Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Output

Click the Calculate button to perform the calculations and generate the following output.

Chart Section

Probability of Acceptance vs Lot Proportion Defective

N=500 n=50 c=3

Numeric Results
_____

Lot Size (N) = 500
_____

Sample Size (n)	Acceptance Number (c)	Proportion Defective	Probability of Acceptance
50	3	0.0000	1.00000
50	3	0.0100	0.99959
50	3	0.0200	0.98814
50	3	0.0300	0.94723
50	3	0.0400	0.87112
50	3	0.0500	0.76708
50	3	0.0600	0.64824
50	3	0.0700	0.52769
50	3	0.0800	0.41537
50	3	0.0900	0.31727
.	.	.	.
.	.	.	.
.	.	.	.

_____

|	
Sample Size (n)	The size of the sample drawn from the lot.
c	Acceptance Number. The largest number of nonconforming items from the sample of size n for which the lot should still be accepted.
Proportion Defective	The proportion of the items of the lot that is assumed to be defective.
Probability of Acceptance	The probability that the lot will be accepted, given the corresponding sample size, acceptance number, and assumed proportion defective.

References
_____

Montgomery, D. C. 2013. Introduction to Statistical Quality Control, 7th Edition. John Wiley & Sons.
Kenett, Ron S. and Zacks, Shelemyahu 2014. Modern Industrial Statistics, 2nd Edition. John Wiley & Sons. West Sussex, United Kingdom.
_____

Each line gives the probability of acceptance based on the corresponding proportion defective. These reports also give some text references, as well as the definitions for each column.

Since the probability of acceptance is close to 0 when the lot proportion defective reaches 0.2, the Proportion Defective Range is changed to 0 to 0.2 by 0.002, and the procedure is run again.

Chart Section
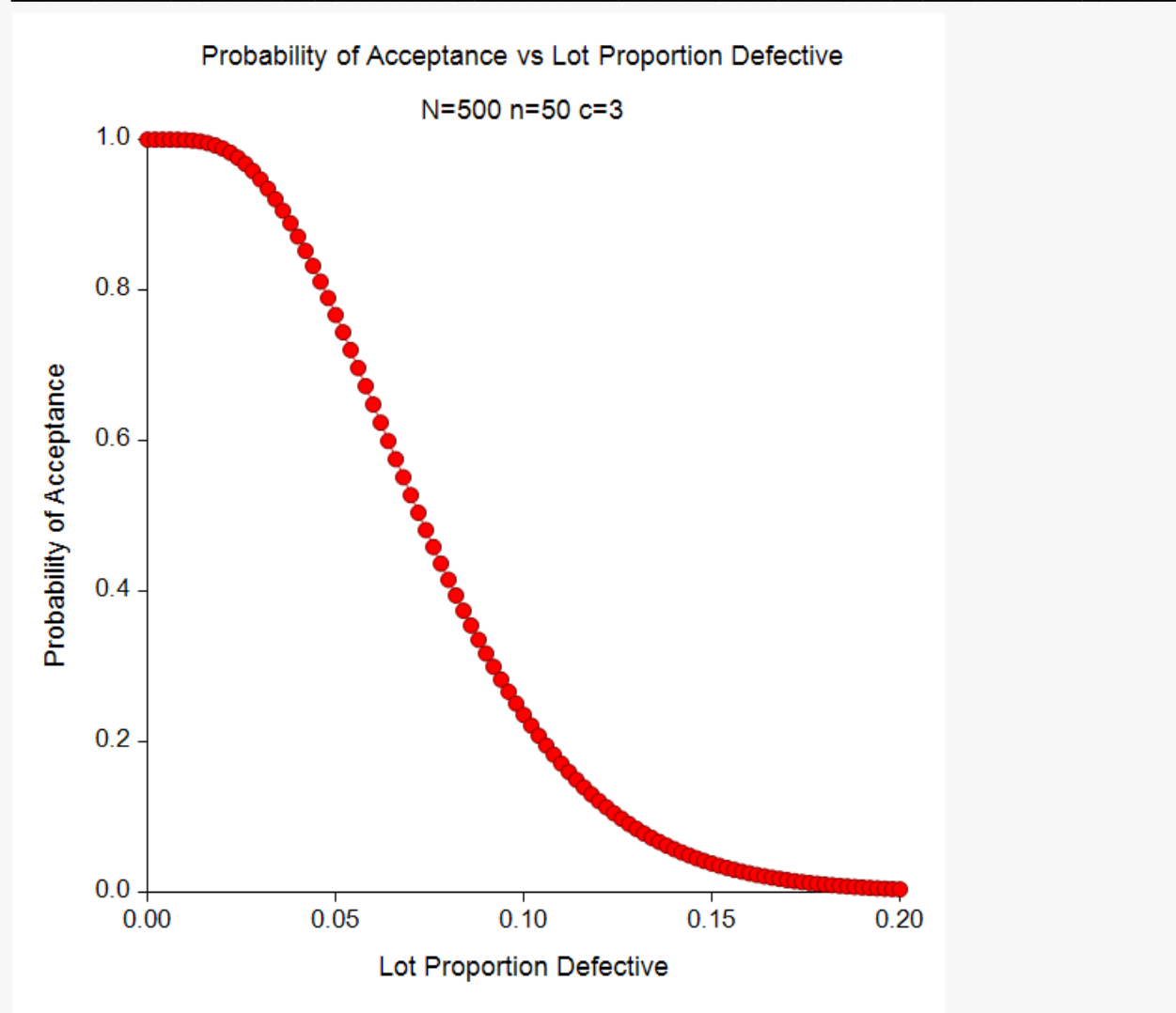
This second chart gives a nice range for the Lot Proportion Defective relative to the Probability of Acceptance.

Example 2 – OC Curves for Various Acceptance Numbers

Continuing with Example 1, suppose the quality engineer would like to see various curves for different values of the acceptance number. In particular, the engineer would like to examine acceptance numbers of 0, 1, 2, 3, 4, 5.

Setup

To run this example, complete the following steps:

1   Specify the Operating Characteristic Curves for Acceptance Sampling for Attributes procedure options

- Find and open the **Operating Characteristic Curves for Acceptance Sampling for Attributes** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.
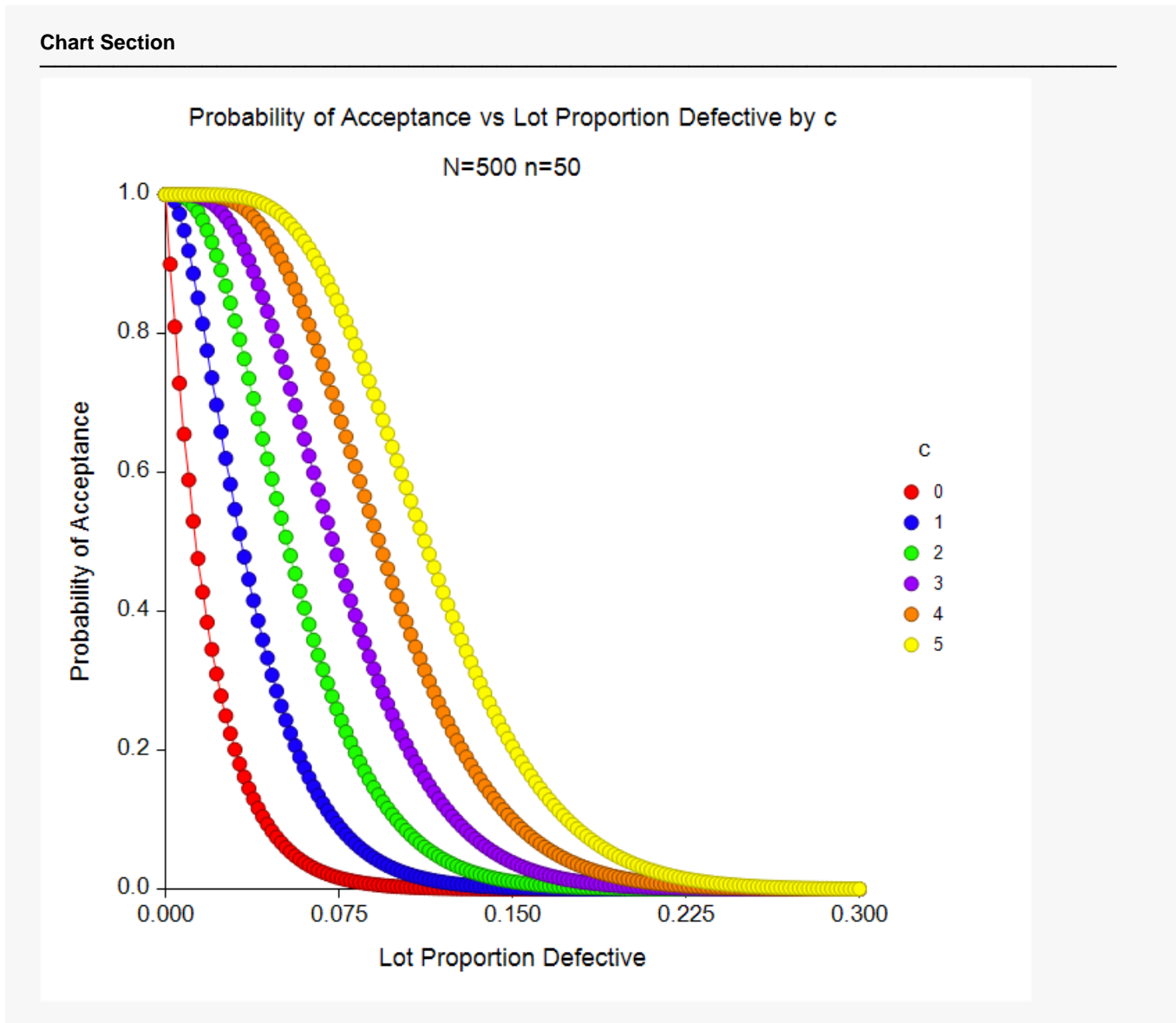
    Variables Tab
    _____

    Lot Size (N)..**500**
    Sample Size (n) **50**
    Acceptance Number (c) **0 1 2 3 4 5**
    Proportion Defective Range............................**0 to 0.3 by 0.002**

2   Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Output

Click the Calculate button to perform the calculations and generate the following output.

Chart Section

This plot shows the operating characteristic curve for each acceptance number.

Example 3 – Validation using Kenett and Zacks (2014)

Kenett and Zacks (2014), pp. 260-261, give an operating characteristic curve example in which the sample size is 50, and the acceptance number is 1.

Setup

To run this example, complete the following steps:

1   Specify the Operating Characteristic Curves for Acceptance Sampling for Attributes procedure options

- Find and open the **Operating Characteristic Curves for Acceptance Sampling for Attributes** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Lot Size (N)..**Infinite**
Sample Size (n) **50**
Acceptance Number (c) **1**
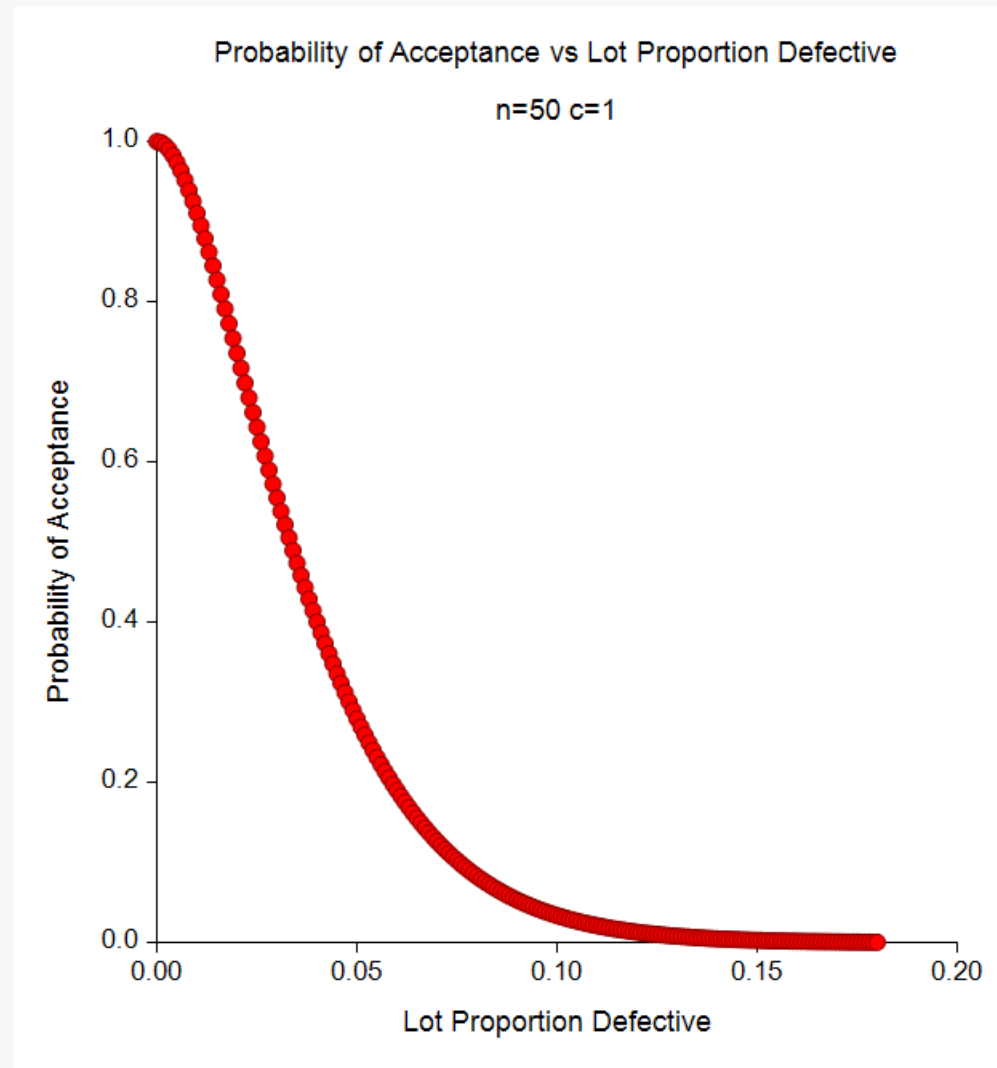Proportion Defective Range............................**0 to 0.18 by 0.001**

2   Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Output

Click the Calculate button to perform the calculations and generate the following output.

Chart Section

This plot matches the plot shown in Kenett and Zacks (2014).

Example 4 – Validation using Montgomery (2013)

Montgomery (2013), p. 657, gives an operating characteristic curve example in which the sample size is 89, and the acceptance numbers are 0, 1, and 2.

Setup

To run this example, complete the following steps:

1 Specify the Operating Characteristic Curves for Acceptance Sampling for Attributes procedure options

- Find and open the **Operating Characteristic Curves for Acceptance Sampling for Attributes** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.
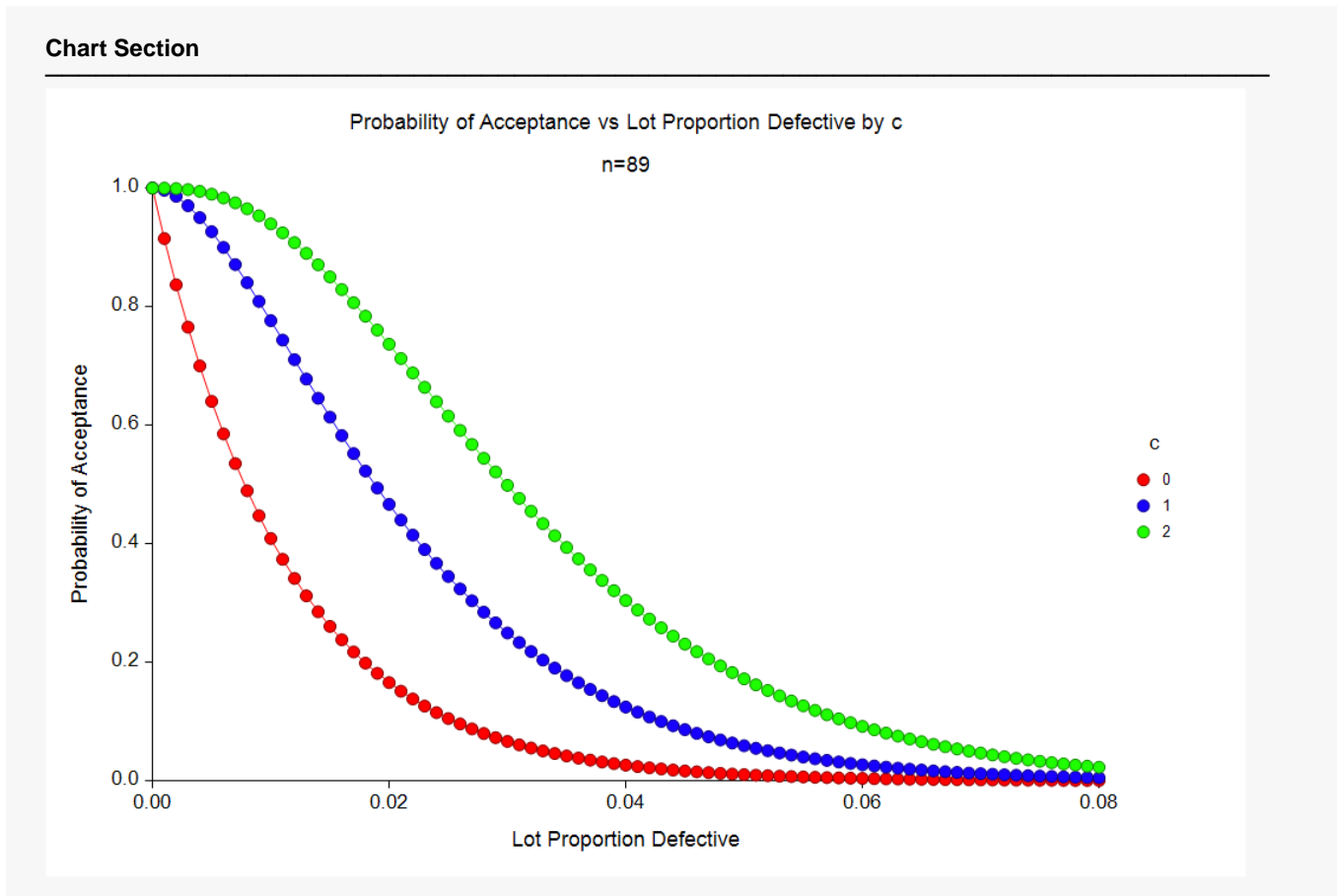
Variables Tab

Lot Size (N)...**Infinite**
Sample Size (n) **89**
Acceptance Number (c) **0 1 2**
Proportion Defective Range.............................**0 to 0.08 by 0.001**

2 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Output

Click the Calculate button to perform the calculations and generate the following output.

Chart Section

This plot matches the plot shown in Montgomery (2013).