Chapter 302

Point-Biserial and Biserial Correlations

Introduction

This procedure calculates estimates, confidence intervals, and hypothesis tests for both the point-biserial and the biserial correlations.

The *point-biserial correlation* is a special case of the product-moment correlation in which one variable is continuous and the other variable is binary (dichotomous). The categories of the binary variable do not have a natural ordering. For example, the binary variable gender does not have a natural ordering. That is, it does not matter whether the males are coded as a zero or a one. Such variables are often referred to as nominal binary variables. It is assumed that the continuous data within each group created by the binary variable are normally distributed with equal variances and possibly different means.

The *biserial correlation* has a different interpretation which may be explained with an example. Suppose you have a set of bivariate data from the bivariate normal distribution. The two variables have a correlation sometimes called the product-moment correlation coefficient. Now suppose one of the variables is dichotomized by creating a binary variable that is zero if the original variable is less than a certain variable and one otherwise. The biserial correlation is an estimate of the original product-moment correlation constructed from the point-biserial correlation. For example, you may want to calculate the correlation between IQ and the score on a certain test, but the only measurement available with whether the test was passed or failed. You could then use the biserial correlation to estimate the more meaningful product-moment correlation.

The formulas used are found in Tate (1954, 1955), Sheskin (2011), and an article by Kraemer (2006).

Technical Details

Point-Biserial Correlation

Suppose you want to find the correlation between a continuous random variable Y and a binary random variable X which takes the values zero and one. Assume that n paired observations (Y_k, X_k), k = 1, 2, ..., n are available. If the common product-moment correlation r is calculated from these data, the resulting correlation is called the *point-biserial correlation*.

Sheskin (2011) gives the formula for the point-biserial correlation coefficient as

$$r_{pb} = \left(\frac{\bar{Y}_1 - \bar{Y}_0}{s_Y}\right)\sqrt{\frac{np_0(1-p_0)}{n-1}}$$

where

$$s_Y = \sqrt{\frac{\sum_{k=1}^{n}(Y_k - \bar{Y})^2}{n-1}}$$

$$\bar{Y} = \frac{\sum_{k=1}^{n} Y_k}{n}$$

$$p_1 = \frac{\sum_{k=1}^{n} X_k}{n}$$

$$p_0 = 1 - p_1$$

Tate (1954) shows that, for large samples, the distribution of r_{pb} is normal with mean ρ and variance

$$\sigma_r^2 = \frac{(1-\rho^2)^2}{n}\left[1 + \rho^2\left(\frac{1 - 6p_0(1-p_0)}{4p_0(1-p_0)}\right)\right]$$

This population variance can be estimated by substituting the sample value r_{pb} for ρ. An approximate confidence interval based on the normal distribution can be calculated from these quantities using

$$r_{pb} \pm z_{\alpha/2}\sqrt{\frac{\left(1-r_{pb}^2\right)^2}{n}\left[1 + r_{pb}^2\left(\frac{1 - 6p_0(1-p_0)}{4p_0(1-p_0)}\right)\right]}$$

The hypothesis that $\rho = 0$ can be tested using the following test which is equivalent to the two-sample t-test.

$$t_{pb} = \frac{r_{pb}\sqrt{n-2}}{\sqrt{1-r_{pb}^2}}$$

This test statistic follows Student's t distribution with $n - 2$ degrees of freedom.

Biserial Correlation

Suppose you want to find the correlation between a pair of bivariate normal random variables when one has been dichotomized. Sheskin (2011) states that the biserial correlation can be calculated from the point-biserial correlation r_{pb} using the formula

$$r_b = \left(\frac{r_{pb}}{h}\right)\sqrt{p_0(1-p_0)}$$

where

$$h = \frac{e^{-u^2/2}}{\sqrt{2\pi}}$$

$$Pr[Z \geq u | Z \sim N(0,1)] = p_1$$

Kraemer (2006) gives a method for constructing a large sample confidence interval for ρ_b which is described as follows. Let $g(x)$ be Fisher's z-transformation

$$g(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

then

$$g\left(\frac{2r_b}{\sqrt{5}}\right) \sim N\left[g\left(\frac{2\rho_b}{\sqrt{5}}\right), \frac{5}{4n}\right]$$

It follows that a $(1 - \alpha)$% confidence interval for g, denote G_1 and G_2, can be calculated using

$$G_1 = g\left(\frac{2r_b}{\sqrt{5}}\right) - \left|z_{\alpha/2}\right|\sqrt{\frac{5}{4n}}$$

$$G_2 = g\left(\frac{2r_b}{\sqrt{5}}\right) + \left|z_{\alpha/2}\right|\sqrt{\frac{5}{4n}}$$

These limits can then be inverted to obtain corresponding confidence limits for ρ_b. The result is

$$CL_1 = \frac{\sqrt{5}}{2}\left(\frac{e^{2G_1} - 1}{e^{2G_1} + 1}\right)$$

$$CL_2 = \frac{\sqrt{5}}{2}\left(\frac{e^{2G_2} - 1}{e^{2G_2} + 1}\right)$$

A large sample z-test of ρ_b =0 based on g(x) can be constructed as follows

$$z = \frac{g\left(\frac{2r_b}{\sqrt{5}}\right)}{\sqrt{\frac{5}{4n}}}$$

Example 1 – Correlating Test Result with IQ

This example correlates the IQ scores of 100 subjects with their result on a pass-fail test. The researcher will quantify the correlation using the point-biserial correlation coefficient. These data are contained on the *IQ Test* dataset.

Setup

To run this example, complete the following steps:

1 Open the IQTest example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **IQTest** and click **OK**.

2 Specify the Point-Biserial and Biserial Correlations procedure options

- Find and open the **Point-Biserial and Biserial Correlations** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Input Type.....**One or More Continuous Variables and a Binary Variable**
Continuous Variable(s) .....**IQ**
Binary Variable .....**Test**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Point-Biserial and Biserial Correlations

Point-Biserial and Biserial Correlations

Continuous Variable = IQ, Binary Variable = Test

Type	Correlation r	Lower 95.0% C.L. of ρ	Upper 95.0% C.L. of ρ	Std Dev of ρ	r^2	Count N	N0/N P	Test for ρ = 0	Prob Level
Pt-Biserial	0.7435	0.6690	0.8181	0.0380	0.5529	100	0.5000	11.008	0.0000
Biserial	0.9319	0.8421	0.9943		0.8684	100	0.5000	10.729	0.0000

This report shows the point-biserial correlation and associated confidence interval and hypothesis test on the first row. It shows the biserial correlation and associated confidence interval and hypothesis test on the second row.

Type

The type of correlation coefficient shown on this row. Note that, although the names point-biserial and biserial sound similar, these are two different correlations that come from different models.

Correlation

The computed values of the point-biserial correlation and biserial correlation. Note that since the assignment of the zero and one to the two binary variable categories is arbitrary, the sign of the point-biserial correlation can be ignored. This is not true of the biserial correlation.

Lower and Upper 95% C.L. of ρ

These are the lower and upper limits of a two-sided, 95% confidence interval for the corresponding correlation.

Std Dev of ρ

This is the standard deviation of the estimate of the point-biserial correlation. This value is not available for the biserial correlation.

r^2

This is the r-squared value for the correlation presented on this row. R-squared is a measure of the strength of the relationship.

Count N

This is the total sample size.

N0/N P

This is the proportion of the sample that is in the group defined by the binary variable being 0. It is the value of p_0 in the formulas presented earlier in the chapter.

Test for ρ = 0

This is value of the test statistic used to test the hypothesis that the correlation is zero. For the point-biserial correlation, this is the value of the t-test with N – 2 degrees of freedom. It is identical to the two-sample t-test for testing whether the means are different.

For the biserial correlation, this is the value of the z-test which is based on the standard normal distribution.

Prob Level

This is the p-value of the hypothesis test mentioned above. If it is less than 0.05 (or whatever value you choose), then the test is 'significant' and the null hypothesis that the correlation is zero is rejected.

Means, Standard Deviations, and Confidence Intervals of Means

Means, Standard Deviations, and Confidence Intervals of Means

Continuous Variable = IQ, Binary Variable = Test

Name	Count	Mean	Standard Deviation	Lower 95.0% C.L.	Upper 95.0% C.L.
Test = 0	50	100.24	5.227713	98.7543	101.7257
Test = 1	50	111.22	4.734976	109.8743	112.5657
Combined	100	105.73	7.420767	104.2576	107.2024
Difference	100	10.98	4.987433	9.000521	12.95948

This report shows the descriptive statistics of the two individual groups, the combination of both groups, and the difference between the two groups.

Tests of Normality and Equal Variance

Tests of the Normality and Equal Variance Assumptions

Continuous Variable = IQ, Binary Variable = Test

Assumption	Test Name	Test Value	Prob Level	Conclusion (α = 0.050)
Normality of Test = 0	Shapiro-Wilk	0.977	0.4359	Cannot reject normality
Normality of Test = 1	Shapiro-Wilk	0.972	0.2894	Cannot reject normality
Equal Variances	Brown-Forsythe	0.182	0.6710	Cannot reject equal variances

This report presents the results of the Shapiro-Wilk normality test of each group as well as the Brown-Forsythe Equal Variance test (sometimes called the Modified-Levene test).

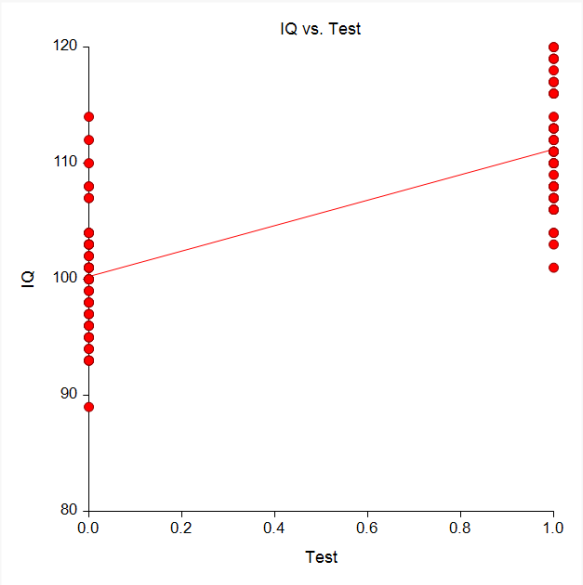
Note that the point-biserial correlation demands that the variances are equal but is robust to mild non-normality. On the other hand, the biserial correlation is robust to unequal variances, but demands that the data are normal.

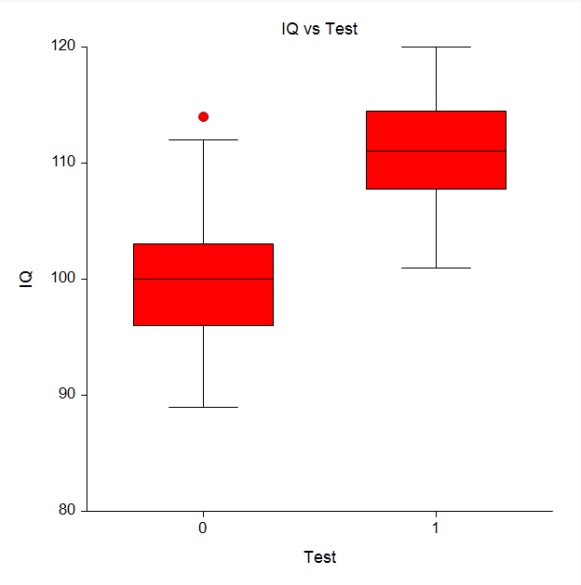
This report presents the usual descriptive statistics.

This report displays a brief summary of a linear regression of Y on X.

Plots to Evaluate Correlation

Plots to Evaluate Correlation

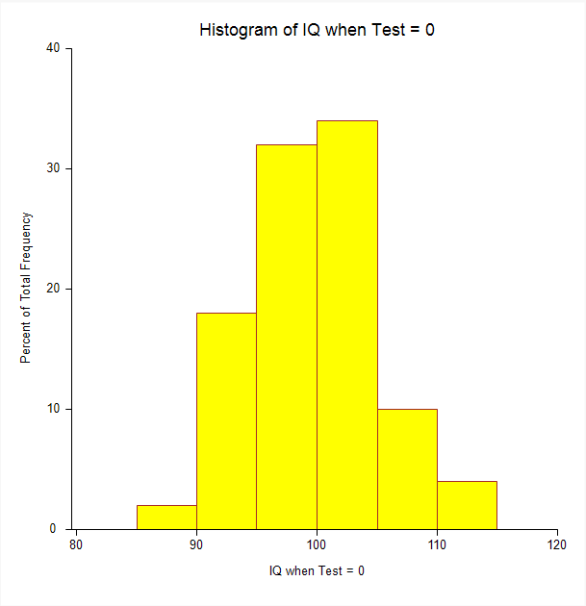


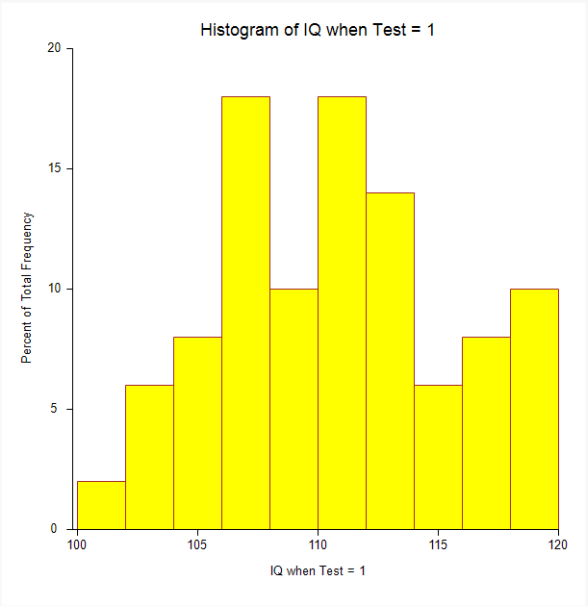


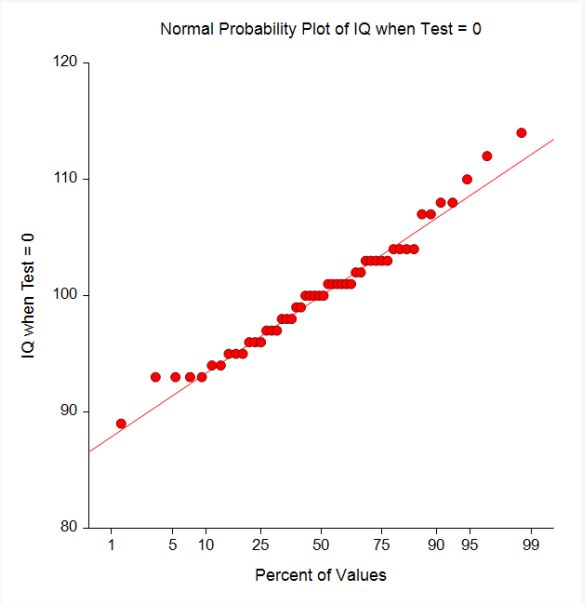
These plots let you investigate the relationship between the two variables more closely. The box plot is especially useful for comparing the variances of the two groups.

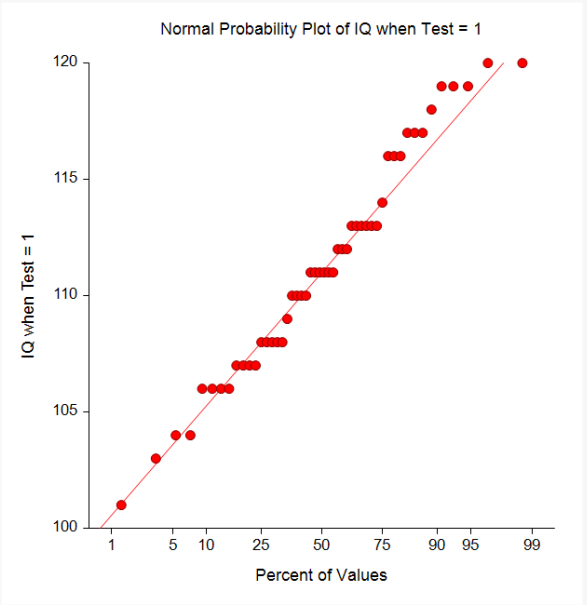
Plots to Evaluate Normality

Plots to Evaluate Normality









The histograms and normal probability plots help you assess the viability of the assumption of normality within each group.