Chapter 585

Tolerance Intervals

Introduction

This procedure calculates one-, and two-, sided tolerance intervals based on either a distribution-free (nonparametric) method or a method based on a normality assumption (parametric). A two-sided *tolerance interval* consists of two limits between which a given proportion β of the population falls with a given confidence level $1-\alpha$. A one-sided tolerance interval is similar but consists of a single upper or lower limit.

Technical Details

Let $X_1, X_2, \cdots, X_n$ be a random sample for a population with distribution function $F(X)$. A $(\beta, 1-\alpha)$ two-sided β-content tolerance interval (T_L, T_U) is defined by

$$\Pr[F(T_U) - F(T_L) \geq \beta] \geq 1-\alpha$$

A $(\beta, 1-\alpha)$ lower, one-sided β-content tolerance bound T_L is defined by

$$\Pr[1 - F(T_L) \geq \beta] \geq 1-\alpha$$

A $(\beta, 1-\alpha)$ upper, one-sided β-content tolerance bound T_U is defined by

$$\Pr[F(T_U) \geq \beta] \geq 1-\alpha$$

Note that a one-sided tolerance limit is the same as the one-sided confidence limit of the quantile of *F*.

Distribution-Free Tolerance Intervals

The definition of two-sided distribution-free tolerance intervals is found in many places. We use the formulation given by Bury (1999). The only distributional assumption made about *F* is that it is a continuous, non-decreasing, probability distribution. That is, these intervals should not be used with discrete data. Given this, the tolerance limits are

$$T_L = X_{(r)}, \quad T_U = X_{(s)}$$

where *r* and *s* are two order indices. The values of *r* and *s* are determined using the formula

$$\sum_{i=0}^{n-2c} \binom{n}{i} \beta^i (1-\beta)^{n-i} \geq 1-\alpha$$

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where

$$r = c$$

$$s = n - c + 1$$

The value of *c* is found as the largest value for which the above inequality is true.

A lower, one-sided tolerance bound is $X_{(r)}$ where *r* is the largest value for with the following inequality is true.

$$\sum_{i=0}^{n-r} \binom{n}{i} \beta^i (1-\beta)^{n-i} \geq 1 - \alpha$$

An upper, one-sided tolerance bound is $X_{(s)}$ where *s* is the largest value for with the following inequality is true.

$$\sum_{i=0}^{s-1} \binom{n}{i} \beta^i (1-\beta)^{n-i} \geq 1 - \alpha$$

Normal-Distribution Tolerance Interval

The limits discussed in this section are based on the assumption that *F* is the normal distribution.

Two-Sided Limits

In this case, the two-sided tolerance interval is defined by the interval

$$T_L = \bar{x} - ks, \quad T_U = \bar{x} + ks$$

The construction reduces to the determination of the constant *k*. Howe (1969) provides the following approximation which is 'nearly' exact for all values of *n* greater than one

$$k = uvw$$

where

$$u = z_{\frac{1+\beta}{2}} \sqrt{1 + \frac{1}{n}}$$

$$v = \sqrt{\frac{n-1}{\chi^2_{n-1,\alpha}}}$$

$$w = \sqrt{1 + \frac{n - 3 - \chi^2_{n-1,\alpha}}{2(n+1)^2}}$$

Note that originally, Howe (1969) used *n* – 2 in the above definition of *w*. But Guenther (1977) gives the corrected version using *n* – 3 shown above.

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One-Sided Bound

A one-sided tolerance bound ('bound' is used instead of 'limit' in the one-sided case) is given by

$$T_U = \bar{x} + ks$$

Here *k* is selected so that

$$Pr\left(t'_{n-1,\delta} = k\sqrt{n}\right) = 1 - \alpha$$

where $t'_{f,\delta}$ represents a noncentral *t* distribution with *f* degrees of freedom and noncentrality

$$\delta = z_\beta \sqrt{n}\,.$$

Data Structure

The data are contained in a single column.

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Example 1 – Generating Tolerance Intervals

This section presents a detailed example of how to generate tolerance intervals for the *Height* variable in the Height dataset.

Setup

To run this example, complete the following steps:

1 Open the Height example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Height** and click **OK**.

2 Specify the Tolerance Intervals procedure options

- Find and open the **Tolerance Intervals** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Variables..**Height**
Population Percentages for Tolerances **50 75 80 90 95 99**

Reports Tab

Values Decimal Places...................................**3**
Means Decimal Places **3**
Probabilities Decimal Places...........................**2**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Descriptive Statistics

Descriptive Statistics of Height

Count	Mean	Standard Deviation	Standard Error	Minimum	Maximum	Range
20	62.100	8.441	1.887	51.000	79.000	28.000

This report was defined and discussed in the Descriptive Statistics procedure chapter. We refer you to the Summary Section of that chapter for details.

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Two-Sided Tolerance Intervals

Two-Sided 95% Tolerance Intervals of Height

Percent of Population Between Limits	Parametric Lower Tolerance Limit	Parametric Upper Tolerance Limit	Nonparametric Lower Tolerance Limit	Nonparametric Upper Tolerance Limit
50.00	54.074	70.126	52.000	73.000
75.00	48.411	75.789	51.000	79.000
80.00	46.850	77.350		
90.00	42.527	81.673		
95.00	38.777	85.423		
99.00	31.449	92.751		

Notes:
The parametric (normal-based) limits assume that the data follow the normal distribution.
The nonparametric (distribution-free) limits make no special distributional assumption.

This section gives the parametric and nonparametric two-sided tolerance intervals.

Percent of Population Between Limits

This is the percentage of population values that are contained in the tolerance interval.

Parametric Lower (Upper) Tolerance Limits

These are the values of the limits of a tolerance interval based on the assumption that the population is normally distributed.

Nonparametric Lower (Upper) Tolerance Limits

These are the values of the limits of a distribution-free tolerance interval. These intervals make no distributional assumption.

Lower One-Sided Tolerance Bounds

Lower One-Sided 95% Tolerance Bounds of Height

Percent of Population Greater Than Bound	Parametric Lower Tolerance Bound	Nonparametric Lower Tolerance Bound
50.00	60.264	56.000
75.00	52.254	52.000
80.00	50.524	51.000
90.00	45.842	
95.00	41.875	
99.00	34.285	

Notes:
The parametric (normal-based) limit assumes that the data follow the normal distribution.
The nonparametric (distribution-free) limit makes no special distributional assumption.

This section gives the parametric and nonparametric one-sided tolerance bounds.

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Percent of Population Greater Than Bound

This is the percentage of population values that are above the tolerance bound.

Parametric Lower Tolerance Bound

This is the lower parametric (normal distribution) tolerance bound.

Nonparametric Lower (Upper) Tolerance Limits

This is the lower nonparametric (distribution-free) tolerance bound. Note that some values are missing because of the small sample size in this example.

Upper One-Sided Tolerance Bounds

Upper One-Sided 95% Tolerance Bounds of Height

Percent of Population Greater Than Bound	Parametric Upper Tolerance Bound	Nonparametric Upper Tolerance Bound
50.00	63.936	65.000
75.00	71.946	73.000
80.00	73.676	76.000
90.00	78.358	79.000
95.00	82.325	79.000
99.00	89.915	79.000

Notes:
The parametric (normal-based) limit assumes that the data follow the normal distribution.
The nonparametric (distribution-free) limit makes no special distributional assumption.

This section gives the parametric and nonparametric one-sided tolerance bounds.

Percent of Population Less Than Bound

This is the percentage of population values that are below the tolerance bound.

Parametric Lower Tolerance Bound

This is the upper parametric (normal distribution) tolerance bound.

Nonparametric Lower (Upper) Tolerance Limits

This is the upper nonparametric (distribution-free) tolerance bound.

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Normality Test Section

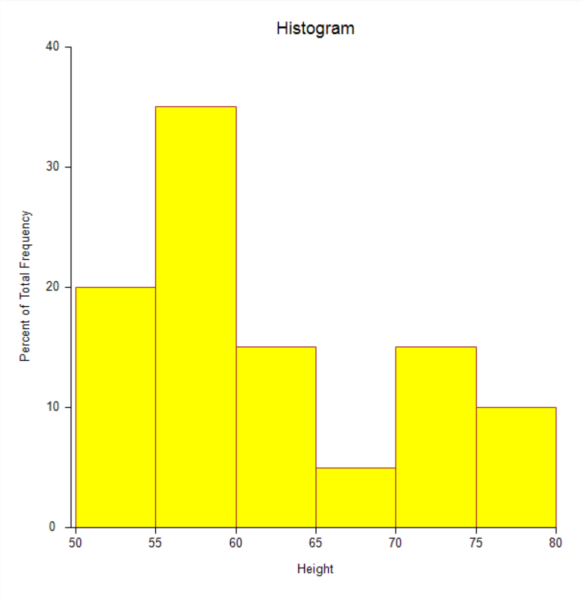
Normality Test Section of Height

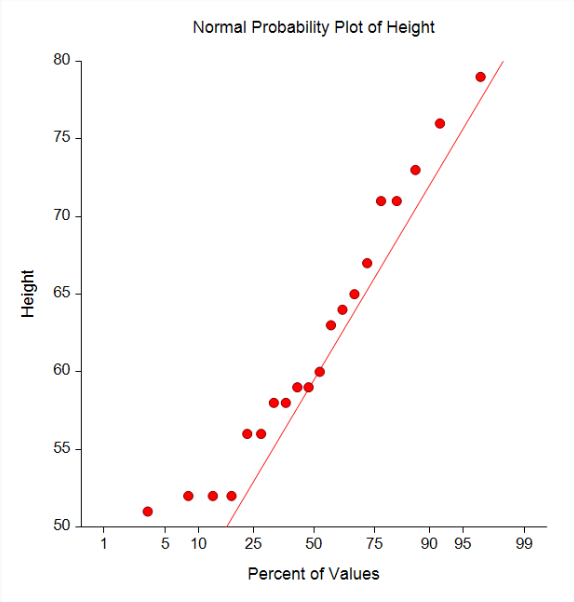
Test Name	Test Value	Prob Level	10% Critical Value	5% Critical Value	Decision (5%)
Shapiro-Wilk W	0.937	0.21			Can't reject normality
Anderson-Darling	0.443	0.29			Can't reject normality
Kolmogorov-Smirnov	0.148		0.176	0.192	Can't reject normality
D'Agostino Skewness	1.037	0.30	1.645	1.960	Can't reject normality
D'Agostino Kurtosis	-0.786	0.43	1.645	1.960	Can't reject normality
D'Agostino Omnibus	1.692	0.43	4.605	5.991	Can't reject normality

This report was defined and discussed in the Descriptive Statistics procedure chapter. We refer you to the Normality Test Section of that chapter for details.

Plots Section

Plots Section of Height





The plots section displays a histogram and a probability plot to allow you to assess the accuracy of the normality assumption.

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