Chapter 219

# Two-Sample T-Test for Superiority by a Margin

## Introduction

This procedure provides reports for making inference about the superiority of a treatment mean compared to a control mean from data taken from independent groups. The question of interest is whether the treatment mean is better than the control mean by some superiority margin. Another way of saying this is that the treatment is better than the control by some value called the *margin*.

Three different test statistics may be used: two-sample t-test, the Aspin-Welch unequal-variance t-test, and the nonparametric Mann-Whitney U (or Wilcoxon Rank-Sum) test.

## Technical Details

Suppose you want to evaluate the superiority of a continuous random variable $X_T$ as compared to a second random variable $X_C$ using data on each variable taken on the different subjects. Assume that $n_T$ observations ($X_{Tk}$), $k$ = 1, 2, …, $n_T$ are available from the treatment group and that $n_C$ observations ($X_{Ck}$), $k$ = 1, 2, …, $n_C$ are available from the control group.

### Superiority by a Margin Test

This discussion is based on the book by Rothmann, Wiens, and Chan (2012) which discusses the two-independent sample case. Assume that higher values are better, that $\mu_T$ and $\mu_C$ represent the means of the two variables, and that $M$ is the positive *superiority margin.* The null and alternative hypotheses when the **higher values are better** are

$$\text{H0: } (\mu_T - \mu_C) \leq M$$

$$\text{H1: } (\mu_T - \mu_C) > M$$

or

$$\text{H0: } \mu_T \leq \mu_C + M$$

$$\text{H1: } \mu_T > \mu_C + M$$

If, on the other hand, we assume that **higher values are worse**, then null and alternative hypotheses are

$$\text{H0: } (\mu_T - \mu_C) \geq -M$$

$$\text{H1: } (\mu_T - \mu_C) < -M$$

or

$$H0: \mu_T \geq \mu_C - M$$
$$H1: \mu_T < \mu_C - M$$

The two-sample t-test is usually employed to test that the mean difference is zero. The superiority by a margin test is a one-sided two-sample t-test that compares the difference to a non-zero quantity, *M*. One-sided editions of the Aspin-Welch unequal-variance t-test, and the Mann-Whitney U (or Wilcoxon Rank-Sum) nonparametric test are also optionally available.

# Data Structure

The data may be entered in two formats, as shown in the two examples below. The examples give the yield of corn for two types of fertilizer. The first format, shown in the first table, is the case in which the responses for each group are entered in separate columns. That is, each variable contains all responses for a single group. In the second format the data are arranged so that all responses are entered in a single column. A second column, referred to as the *grouping variable,* contains an index that gives the group (A or B) to which the row of data belongs.

In most cases, the second format is more flexible. Unless there is some special reason to use the first format, we recommend that you use the second.

**Two Response Variables**

| Yield A | Yield B |
|---|---|
| 452 | 546 |
| 874 | 547 |
| 554 | 774 |
| 447 | 465 |
| 356 | 459 |
| 754 | 665 |
| 558 | 467 |
| 574 | 365 |
| 664 | 589 |
| 682 | 534 |
| 547 | 456 |
| 435 | 651 |
| 245 | 654 |
| | 665 |
| | 546 |
| | 537 |

**Grouping and Response Variables**

| Fertilizer | Yield |
|---|---|
| A | 452 |
| A | 874 |
| A | 554 |
| A | 447 |
| A | 356 |
| . | . |
| . | . |
| . | . |
| B | 546 |
| B | 547 |
| B | 774 |
| B | 465 |
| B | 459 |
| . | . |
| . | . |
| . | . |

# Example 1 – Superiority by a Margin Test for Two Independent Samples

This section presents an example of how to test superiority by a margin. Suppose that a new fertilizer has been developed with a number of desired improvements. The researchers of the new fertilizer want to show that the new fertilizer (YldB) is better than the current fertilizer (YldA) by some margin. Further suppose that the average corn yield of the current fertilizer is about 550. The researchers want to show that the yield of the new fertilizer is more than 10% better than the current type. That is, the superiority margin is 10% of 550 which is 55.

## Setup

To run this example, complete the following steps:

**1 Open the Corn Yield example dataset**

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Corn Yield** and click **OK**.

**2 Specify the Two-Sample T-Test for Superiority by a Margin procedure options**

- Find and open the **Two-Sample T-Test for Superiority by a Margin** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Data Input Type ............................................. **Two Variables with Response Data in each Variable**
Treatment Variable ......................................... **YldB**
Control Variable ............................................. **YldA**
Higher Values Are........................................... **Better**
Superiority Margin........................................... **55**

Report Options (*in the Toolbar*)

Variable Labels ............................................... **Column Names**

**3 Run the procedure**

- Click the **Run** button to perform the calculations and generate the output.

## Descriptive Statistics and Confidence Intervals for the Group Means (μ's)

**Descriptive Statistics and Confidence Intervals for the Group Means (μ's)**

| Variable | N | Mean | Standard Deviation of the Data | Standard Error of the Mean | T* | 95% Confidence Interval Limits for the Mean (μ) Lower | Upper |
|---|---|---|---|---|---|---|---|
| YldB | 16 | 557.5 | 104.6219 | 26.15546 | 2.1314 | 501.7509 | 613.249 |
| YldA | 13 | 549.3846 | 168.7629 | 46.80641 | 2.1788 | 447.4022 | 651.367 |

This report provides basic descriptive statistics and confidence intervals for the two variables.

### Variable

These are the names of the variables or groups.

### N

This gives the number of non-missing values. This value is often referred to as the group sample size or count.

### Mean

This is the average for each group.

### Standard Deviation of the Data

The sample standard deviation is the square root of the sample variance. It is a measure of spread.

### Standard Error of the Mean

This is the estimated standard deviation for the distribution of sample means for an infinite population. It is the sample standard deviation divided by the square root of sample size.

### T*

This is the t-value used to construct the confidence interval. If you were constructing the interval manually, you would obtain this value from a table of the Student's t distribution with $n$ - 1 degrees of freedom.

### 95% Confidence Interval Limits for the Mean (μ) (Lower and Upper)

These are the lower and upper limits of an interval estimate of the mean based on a Student's t distribution with $n$ - 1 degrees of freedom. This interval estimate assumes that the population standard deviation is not known and that the data are normally distributed.

## Descriptive Statistics and Confidence Intervals for the Mean Difference (μ1 - μ2)

**Descriptive Statistics and Confidence Intervals for the Mean Difference (μ1 - μ2)**

| Variance Assumption | DF | Mean Difference | Standard Error | T* | 95% Confidence Interval Limits for the Mean Difference (μ1 - μ2) Lower | Upper |
|---|---|---|---|---|---|---|
| Equal | 27 | 8.115385 | 51.11428 | 2.0518 | -96.76247 | 112.9932 |
| Unequal | 19.17 | 8.115385 | 53.61855 | 2.0918 | -104.0426 | 120.2734 |

Given that the assumptions of independent samples and normality are valid, this section provides an interval estimate (confidence limits) of the difference between the two means. Results are given for both the equal and unequal variance cases.

### DF

The degrees of freedom are used to determine the T distribution from which T* is generated.

For the equal variance case:

$$df = n_T + n_C - 2$$

For the unequal variance case:

$$df = \frac{\left(\frac{s_T^2}{n_T} + \frac{s_C^2}{n_C}\right)^2}{\frac{\left(\frac{s_T^2}{n_T}\right)^2}{n_T - 1} + \frac{\left(\frac{s_C^2}{n_C}\right)^2}{n_C - 1}}$$

### Mean Difference

This is the difference between the sample means, $\overline{X}_T - \overline{X}_C$.

### Standard Error

This is the estimated standard deviation of the distribution of differences between independent sample means.

For the equal variance case:

$$SE_{\overline{X}_T - \overline{X}_C} = \sqrt{\left(\frac{(n_T - 1)s_T^2 + (n_C - 1)s_C^2}{n_T + n_C - 2}\right)\left(\frac{1}{n_T} + \frac{1}{n_C}\right)}$$

For the unequal variance case:

$$SE_{\overline{X}_T-\overline{X}_C} = \sqrt{\frac{s_T^2}{n_T} + \frac{s_C^2}{n_C}}$$

### T*

This is the t-value used to construct the confidence limits. It is based on the degrees of freedom and the confidence level.

### 95% Confidence Interval Limits for the Mean Difference (Lower and Upper)

These are the confidence limits of the confidence interval for $\mu_T - \mu_C$. The confidence interval formula is

$$\overline{X}_T - \overline{X}_C \pm T_{df}^* SE_{\overline{X}_T-\overline{X}_C}$$

The equal-variance and unequal-variance assumption formulas differ by the values of T* and the standard error.

## Descriptive Statistics and Confidence Intervals for the Group Medians

**Descriptive Statistics and Confidence Intervals for the Group Medians**

| | | | 95% Confidence Interval Limits for the Median | |
|---|---|---|---|---|
| Variable | N | Median | Lower | Upper |
| YldB | 16 | 546 | 465 | 651 |
| YldA | 13 | 554 | 435 | 682 |

This report provides the medians and corresponding confidence intervals for the medians of each group.

### Variable

These are the names of the variables or groups.

### N

This gives the number of non-missing values. This value is often referred to as the group sample size or count.

### Median

The median is the 50th percentile of the group data, using the AveXp(n+1) method. The details of this method are described in the Descriptive Statistics chapter under Percentile Type.

### 95% Confidence Interval Limits for the Median (Lower and Upper)

These are the lower and upper confidence limits for the median. These limits are exact and make no distributional assumptions other than a continuous distribution. No limits are reported if the algorithm for this interval is not able to find a solution. This may occur if the number of unique values is small.

## Equal-Variance T-Test for Superiority by a Margin

**Equal-Variance T-Test for Superiority by a Margin**

| | |
|---|---|
| Higher Values Are: | Better |
| Null Hypothesis (H0): | (YldB) ≤ (YldA) + 55 |
| Superiority Hypothesis (H1): | (YldB) > (YldA) + 55 |

| Alternative Hypothesis | Mean Difference | Standard Error | T-Statistic | DF | P-Value | Reject H0 and Conclude Superiority at α = 0.05? |
|---|---|---|---|---|---|---|
| μT > μC + 55 | 8.115385 | 51.11428 | -0.9173 | 27 | 0.81643 | No |

This report shows the superiority by a margin test for the equal-variance assumption. Since the Prob Level is greater than the designated value of alpha (0.05), the null hypothesis cannot be rejected.

## Aspin-Welch Unequal-Variance T-Test for Superiority by a Margin

**Aspin-Welch Unequal-Variance T-Test for Superiority by a Margin**

| | |
|---|---|
| Higher Values Are: | Better |
| Null Hypothesis (H0): | (YldB) ≤ (YldA) + 55 |
| Superiority Hypothesis (H1): | (YldB) > (YldA) + 55 |

| Alternative Hypothesis | Mean Difference | Standard Error | T-Statistic | DF | P-Value | Reject H0 and Conclude Superiority at α = 0.05? |
|---|---|---|---|---|---|---|
| μT > μC + 55 | 8.115385 | 53.61855 | -0.8744 | 19.17 | 0.80364 | No |

This report shows the superiority by a margin test for the unequal-variance assumption. Since the Prob Level is greater than the designated value of alpha (0.05), the null hypothesis cannot be rejected.

## Mann-Whitney U or Wilcoxon Rank-Sum Location Difference Test for Superiority by a Margin

**Mann-Whitney U or Wilcoxon Rank-Sum Location Difference Test for Superiority by a Margin**

| | |
|---|---|
| Higher Values Are: | Better |
| Null Hypothesis (H0): | (YldB) ≤ (YldA) + 55 |
| Superiority Hypothesis (H1): | (YldB) > (YldA) + 55 |

**Variable Details**

| Variable | Mann-Whitney U | Sum of Ranks (W) | Mean of W | Standard Deviation of W |
|---|---|---|---|---|
| YldB | 86 | 222 | 240 | 22.79789 |
| YldA | 122 | 213 | 195 | 22.79789 |

Number of Sets of Ties = 2, Multiplicity Factor = 12

**Test Results**

| Test Type | Alternative Hypothesis† | Z-Value | P-Value | Reject H0 and Conclude Superiority at α = 0.05? |
|---|---|---|---|---|
| Exact* | LocT > LocC + 55 | | | |
| Normal Approximation | LocT > LocC + 55 | 0.7895 | 0.78510 | No |
| Normal Approx. with C.C. | LocT > LocC + 55 | 0.8115 | 0.79145 | No |

† "LocT" and "LocC" refer to the location parameters of the treatment and control distributions, respectively.
* The Exact Test is provided only when there are no ties and the sample size is ≤ 20 in both groups.

This report shows the superiority by a margin test based on the Mann-Whitney U statistic. This test is documented in the Two-Sample T-Test chapter.

## Tests of Assumptions

**Tests of the Normality Assumption for YldB**

| Normality Test | Test Statistic | P-Value | Reject the Assumption of Normality at α = 0.05? |
|---|---|---|---|
| Shapiro-Wilk | 0.9593 | 0.64856 | No |
| Skewness | 0.4587 | 0.64644 | No |
| Kurtosis | 0.1291 | 0.89726 | No |
| Omnibus (Skewness or Kurtosis) | 0.2271 | 0.89267 | No |

**Tests of the Normality Assumption for YldA**

| Normality Test | Test Statistic | P-Value | Reject the Assumption of Normality at α = 0.05? |
|---|---|---|---|
| Shapiro-Wilk | 0.9843 | 0.99420 | No |
| Skewness | 0.2691 | 0.78785 | No |
| Kurtosis | 0.3081 | 0.75803 | No |
| Omnibus (Skewness or Kurtosis) | 0.1673 | 0.91974 | No |

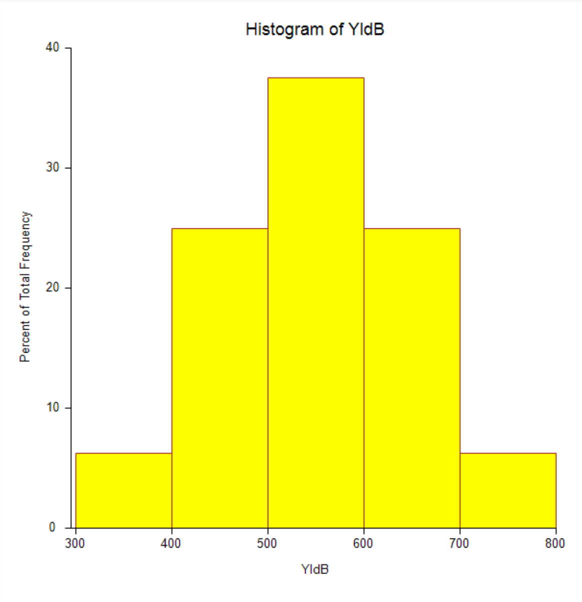
**Tests of the Equal-Variance Assumption**

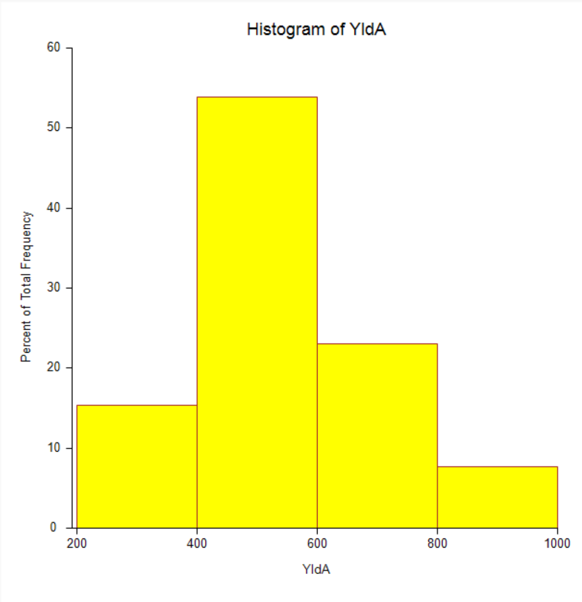
| Equal-Variance Test | Test Statistic | P-Value | Reject the Assumption of Equal Variances at α = 0.05? |
|---|---|---|---|
| Variance-Ratio | 2.6020 | 0.08315 | No |
| Modified-Levene | 1.9940 | 0.16935 | No |

This section reports the results of diagnostic tests to determine if the data are normal and the variances are close to being equal. The details of these tests are given in the Descriptive Statistics chapter.

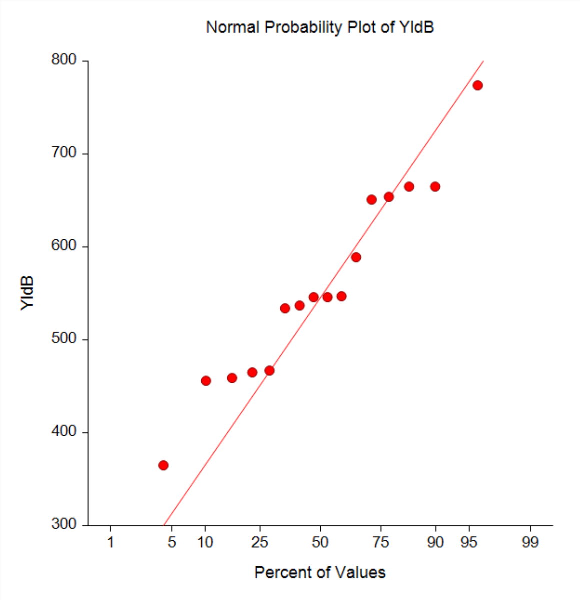
## Evaluation of Assumptions Plots

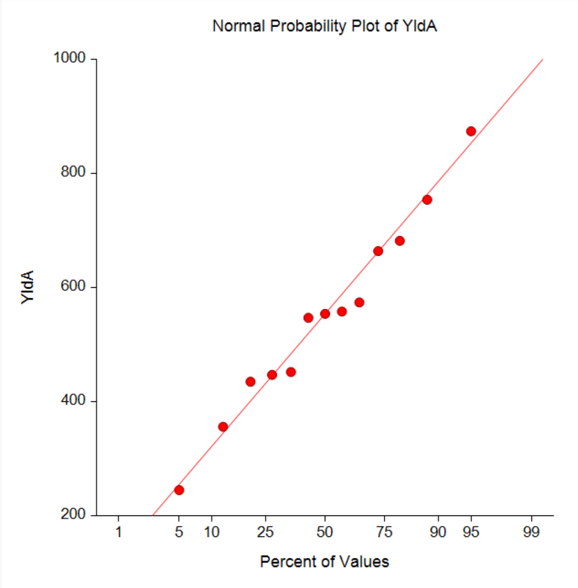
**Histograms**



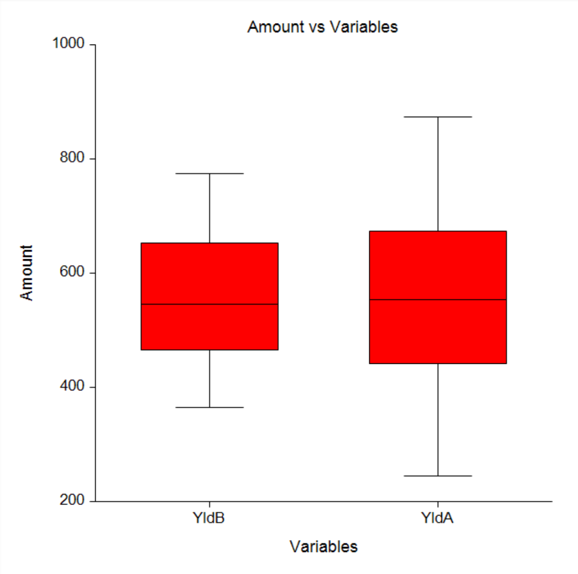



## Probability Plots




## Box Plots




These plots let you visually evaluate the assumptions of normality and equal variance. The probability plots also let you see if outliers are present in the data.