

## Chapter 259

# U Charts

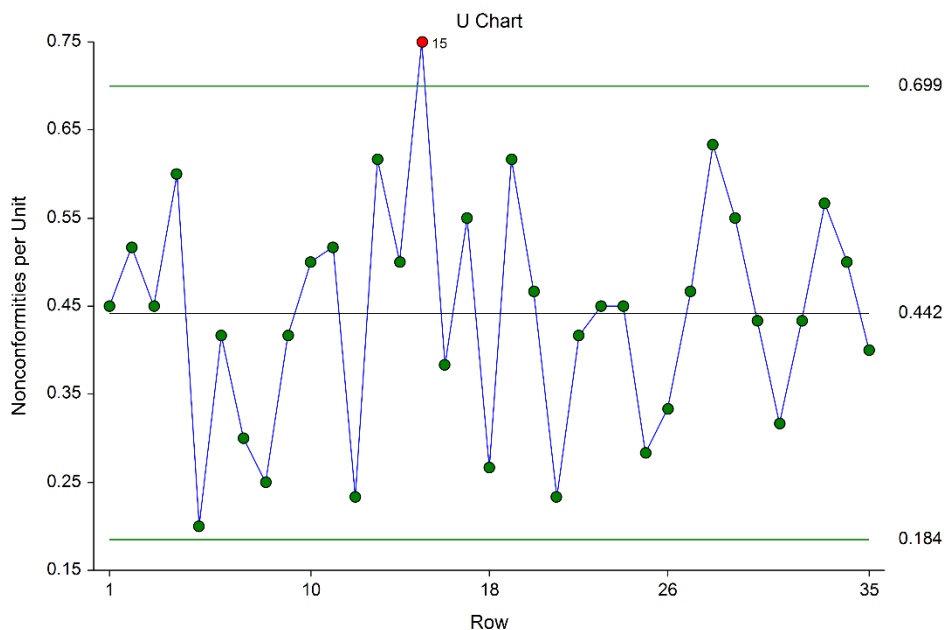
## Introduction

This procedure generates the U control chart for the number of nonconformities per unit, based on the number of nonconformities per sample. The format of the control charts is fully customizable. This procedure permits the defining of stages. For the U chart, the value for U can be entered directly or U can be estimated from the data, or a sub-set of the data. A list of out-of-control points can be produced in the output, if desired, and the per unit number of nonconformities values may be stored to the spreadsheet.

## U Control Charts

C charts are used to monitor the number of nonconformities on a unit of a process based on samples of units taken from the process at given times (hours, shifts, days, weeks, months, etc.). Typically, an initial series of samples is used to estimate the average number of nonconformities per unit. The estimated value is then used to produce control limits for the per unit number of nonconformities. During this initial phase, the process should be in control. If points are out-of-control during the initial (estimation) phase, the assignable cause should be determined and the sample should be removed from estimation.

Once the control limits have been established for the U chart, these limits may be used to monitor the per unit number of nonconformities (defects) of the process going forward. When a point is outside these established control limits it indicates that the per unit number of nonconformities of the process is out-of-control. An assignable cause is suspected whenever the control chart indicates an out-of-control process.



## The U Chart versus the C Chart

The C chart is used when a single unit will be examined for nonconformities at each time point. The U chart is used when a number of units will be sampled at each time point, and a per unit average number of nonconformities will be obtained.

## U Chart Formulas

Suppose we have  $k$  samples, each of size  $n_i$ . Let  $D_i$  represent the total number of nonconformities in the  $i^{\text{th}}$  sample.

### Formulas for the Points on the Chart

The  $i^{\text{th}}$  per unit number of nonconformities  $u_i$  is calculated using

$$u_i = \frac{D_i}{n_i},$$

### Estimating the U Chart Center Line

In the U Charts procedure, the center line proportion may be input directly, or it may be estimated from a series of samples. If it is estimated from the samples the formula for the centerline proportion is

$$\bar{u} = \frac{\sum_{i=1}^k D_i}{\sum_{i=1}^k n_i}$$

If all the samples are the same size, this formula reduces to

$$\bar{u} = \frac{\sum_{i=1}^k D_i}{kn} = \frac{\sum_{i=1}^k u_i}{k}$$

## U Chart Limits

The lower and upper control limits for the U chart are calculated using the formulas

$$LCL = \bar{u} - m \sqrt{\frac{\bar{u}}{n_i}}$$

$$UCL = \bar{u} + m \sqrt{\frac{\bar{u}}{n_i}}$$

where  $m$  is a multiplier (usually set to 3) chosen to control the likelihood of false alarms (out-of-control signals when the process is in control).

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## Runs Tests

The strength of control charts comes from their ability to detect sudden changes in a process that result from the presence of assignable causes. Unfortunately, the U chart is poor at detecting drifts (gradual trends) or small shifts in the process. For example, there might be a positive trend in the last ten samples, but until a per unit number of nonconforming units goes above the upper control limit, the chart gives no indication that a change has taken place in the process.

Runs tests can be used to check control charts for unnatural patterns that are most likely caused by assignable causes. Runs tests are sometimes called “pattern tests”, “out-of-control” tests, or “zones rules”.

While runs tests may be helpful in identifying patterns or smaller shifts in the proportion, they also increase the likelihood of false positive indications. The rate of false positives is typically measured using the average run length (the average length of a run before a false positive is indicated). When several runs tests are used the average run length of the control chart becomes very short.

In order to perform the runs tests, the control chart is divided into six equal zones (three on each side of the center line). Since the control limit is three sigma limits (three standard deviations of the proportion) in width, each zone is one sigma wide and is labeled A, B, or C, with the C zone being the closest to the center line. There is a lower zone A and an upper zone A. The same is true for B and C. The runs tests look at the pattern in which points fall in these zones.

The runs tests used in this procedure are described below.

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### Test 1: Any Single Point Beyond Zone A

This runs test simply indicates a single point is beyond one of the two three-sigma limits.

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### Test 2: Two of Three Successive Points in Zone A or Beyond

This usually indicates a shift in the process average. Note that the two points have to be in the same Zone A, upper or lower. They cannot be on both sides of the center line. The third point can be anywhere.

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### Test 3: Four of Five Successive Points in Zone B or Beyond

This usually indicates a shift in the process average. Note that the odd point can be anywhere.

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### Test 4: Eight Successive Points in Zone C or Beyond

All eight points must be on one side of the center line. This is another indication of a shift in the process average.

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## Test 5: Fifteen Successive Points Fall in Zone C on Either Side of the Center Line

Although this pattern might make you think that the variation in your process has suddenly decreased, this is usually not the case. It is usually an indication of stratification in the sample. This happens when the samples come from two distinct distributions having different means. Perhaps there are two machines that are set differently. Try to isolate the two processes and check each one separately.

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## Test 6: Eight of Eight Successive Points Outside of Zone C

This usually indicates a mixture of processes. This can happen when two supposedly identical production lines feed a single production or assembly process. You must separate the processes to find and correct the assignable cause.

There are, of course, many other sets of runs tests that have been developed. You should watch your data for trends, zig-zags, and other nonrandom patterns. Any of these conditions could be an indication of an assignable cause and would warrant further investigation.

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## Issues in Using Control Charts

There are several additional considerations surrounding the use of control charts that will not be addressed here. Some important questions are presented below without discussion. For a full treatment of these issues, you should consider a statistical quality control text such as Ryan (2011) or Montgomery (2013).

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### Sample Size

How many items should be sampled for each sample? How does the sample size affect my use of control charts? What about unequal sample sizes?

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### Dealing with Out-of-Control Points

How do you deal with out-of-control points once they have been detected? Should they be included or excluded in the process estimate?

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### Control Limit Multiplier

Three-sigma limits are very common. When should one consider a value other than three?

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### Startup Time

How many samples should be used to establish control for my process?

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## Data Structure

In this procedure, the data are entered in two columns. One column contains the sample size for each sample, the other column contains the total number of nonconformities of the sample.

### Example Dataset

N	Nonconf
50	2
50	8
50	6
50	3
50	4
50	2
50	7
50	1
50	9
50	7
50	3
50	5
.	.
.	.
.	.

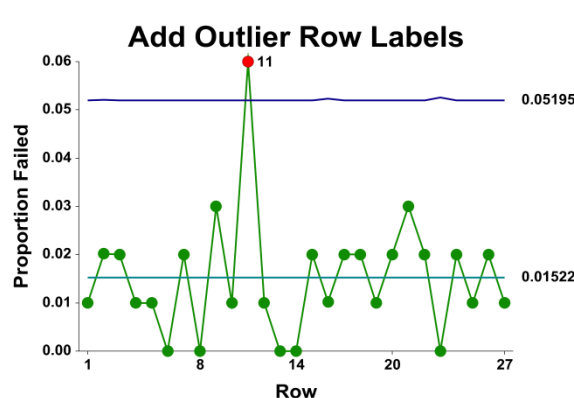
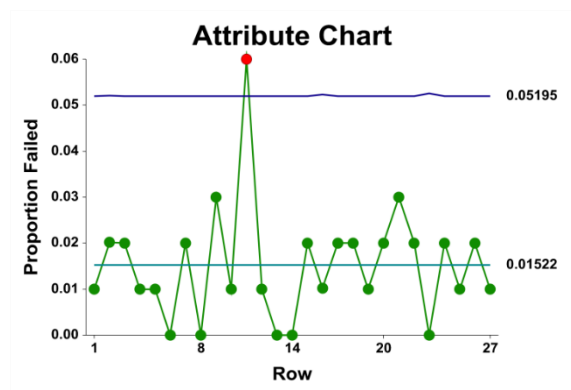
## Attribute Chart Format Window Options

This section describes the specific options available on the Attribute Chart Format window, which is displayed when the Attribute Chart Format button is clicked. Common options, such as axes, labels, legends, and titles are documented in the Graphics Components chapter.

### Attribute Chart Tab

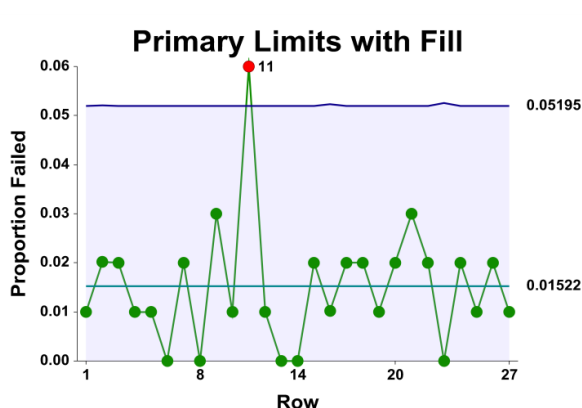
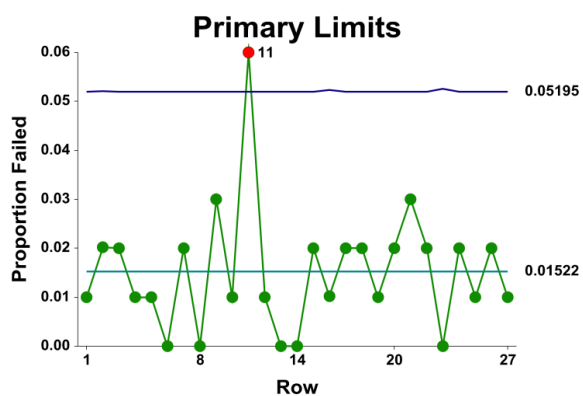
#### Symbols Section

You can modify the attributes of the symbols using the options in this section.



#### Lines Section

You can specify the format of the various lines using the options in this section. Note that when shading is desired, the fill will be to the bottom for single lines (such as the mean line), and between the lines for pairs of lines (such as primary limits).



### Titles, Legend, Numeric Axis, Group Axis, Grid Lines, and Background Tabs

Details on setting the options in these tabs are given in the Graphics Components chapter. The legend does not show by default but can easily be included by going to the Legend tab and clicking the Show Legend checkbox.

## Example 1 – U Chart Analysis (Phase I)

This section presents an example of how to run an initial U Chart analysis to establish control limits. In this example, investigators are examining a production line for a newly developed tire. Sixty tires are to be inspected for nonconformities each day for 35 days to establish control of the process. The daily total number of nonconformities for the sixty tires is in the Tire Nonconformities dataset.

### Setup

To run this example, complete the following steps:

**1 Open the Tire Nonconformities example dataset**

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select Tire Nonconformities and click OK.

**2 Specify the U Charts procedure options**

- Find and open the **U Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Sample Size Variable .....Count  
 Number of Nonconformities Variable .....NC

**3 Run the procedure**

- Click the **Run** button to perform the calculations and generate the output.

### Estimation and Limit Section

**Estimation and Limit Section for Samples 1 to 35**

Number of Samples: 35  
 Sample Size Variable: Count  
 Number of Nonconformities Variable: NC

Average Sample Size	Average # of Nonconf./Sample	Total Sample Size	Total # Nonconf.	Average # of Nonconf./Unit	Nonconformities/Unit Limits	
					Lower Limit	Upper Limit
60.00	26.514286	2100	928	0.4419048	0.184445	0.699365

This section displays the calculation values and limits that are to be used in the U chart. The formulas for each calculation are described in the U chart formula section toward the beginning of this chapter. These results show that the estimated number of nonconformities is about 0.44 per tire.

U Charts

**Average Sample Size**

This is the average of all the sample sizes.

**Average Number of Nonconformities per Sample**

This is the average number of nonconforming units in a sample (of 60 tires).

**Total Sample Size**

This is the sum of all the sample sizes.

**Total Number of Nonconformities**

This is the sum of all the numbers of nonconforming units across all samples.

**Average Number of Nonconformities per Unit**

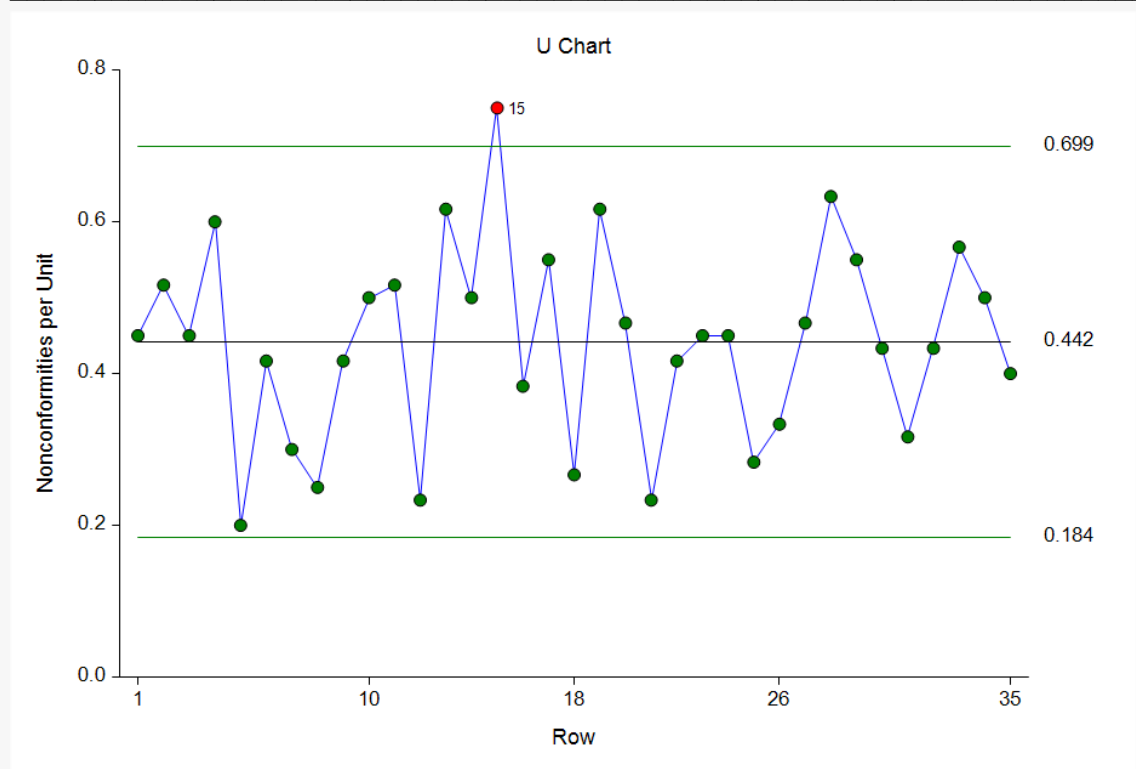
This is the total number of nonconformities divided by the total sample size. It is the center line estimate.

**Number of Nonconformities per Unit Limits**

These are the lower and upper control limits for the U chart.

**U Chart**

Chart Section for Samples 1 to 35





## U Charts

The U chart shows the sequence of per unit nonconformities. The chart shows that sample 15 merits further investigation for an assignable cause. After reviewing the records, the investigators found that the 15<sup>th</sup> sample was produced on a day when one of the curing molds partially failed. The investigators determine that this was the assignable cause and that sample 15 should be removed from process estimation (see Example 2).

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## Out-of-Control List

**Out-of-Control List for Samples 1 to 35**

Row	Number of Nonconform./Unit	Row Label	Reason
15	0.7500	15	beyond control limits

This report provides a list of the samples that failed one of the runs tests (including points outside the control limits). The report shows that sample 15 is beyond the control limits.

## Example 2 – U Chart Revised (Phase I)

This section presents a continuation of the previous example. In this example the limits are revised to exclude the data from sample 15, due to assignable cause.

### Setup

To run this example, complete the following steps:

**1 Open the Tire Nonconformities example dataset**

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select Tire Nonconformities and click OK.

**2 Specify the U Charts procedure options**

- Find and open the **U Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

```

Sample Size Variable .....Count
Number of Nonconformities Variable .....NC
Specification Method .....Remove Rows Variable
Use all rows except rows where variable .....Remove
equals .....1
    
```

**3 Run the procedure**

- Click the **Run** button to perform the calculations and generate the output.

### Estimation and Limit Section

**Estimation and Limit Section for Samples 1 to 35 where Remove is not equal to 1**

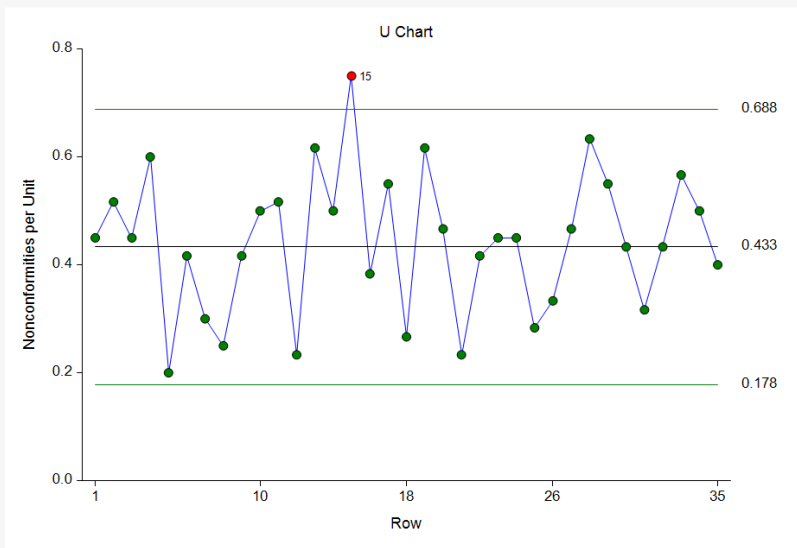
```

Number of Samples:          34
Sample Size Variable:      Count
Number of Nonconformities Variable:  NC
    
```

Average Sample Size	Average # of Nonconf./Sample	Total Sample Size	Total # Nonconf.	Average # of Nonconf./Unit	Nonconformities/Unit Limits	
					Lower Limit	Upper Limit
60.00	25.970588	2040	883	0.4328431	0.178036	0.687650

U Charts

Chart Section for Samples 1 to 35



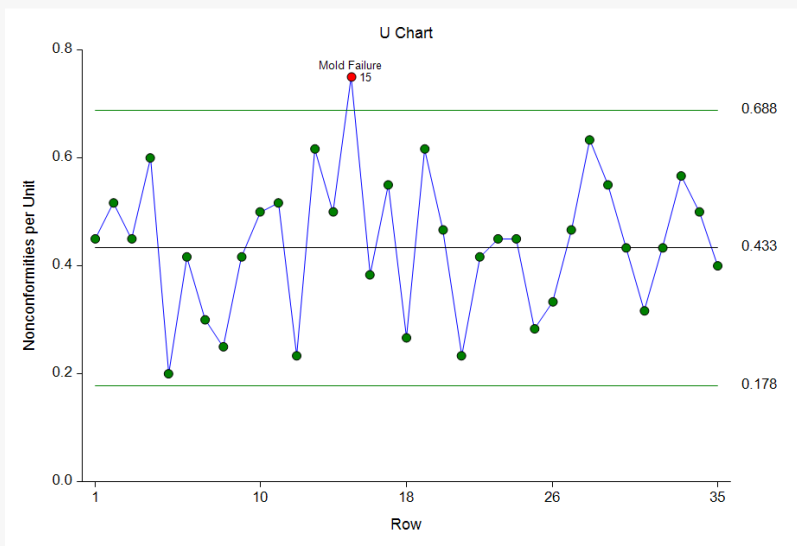
Out-of-Control List for Samples 1 to 35

Row	Number of Nonconform./Unit	Row Label	Reason
15	0.7500	15	beyond control limits

The limits in this report and chart are now based on all samples except sample 15. The value for sample 15 is included in the chart, but it is not included in the calculations. The removal of sample 15 causes a slight change in the center line and limits.

The U chart can be additionally enhanced by labeling the point that was not included in the calculations. To do this, set the Point Label Variable to Label.

Chart Section for Samples 1 to 35



## Example 3 – U Chart Analysis (Phase II)

Continuing with Examples 1 and 2, the investigators obtain samples for an additional 25 days. They wish to use the limits based on the first 35 samples, excluding sample 15.

### Setup

To run this example, complete the following steps:

**1 Open the Tire Nonconformities example dataset**

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select Tire Nonconformities and click OK.

**2 Specify the U Charts procedure options**

- Find and open the **U Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

```

Variables Tab
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Sample Size Variable .....CountCont
Number of Nonconformities Variable .....NCCont
Specification Method .....Keep Rows Variable
Use all rows where variable.....Keep
equals .....1
Point Label Variable.....Label
    
```

**3 Run the procedure**

- Click the **Run** button to perform the calculations and generate the output.

### U Chart Analysis Output

**Estimation and Limit Section for Samples 1 to 60 where Keep equals 1**

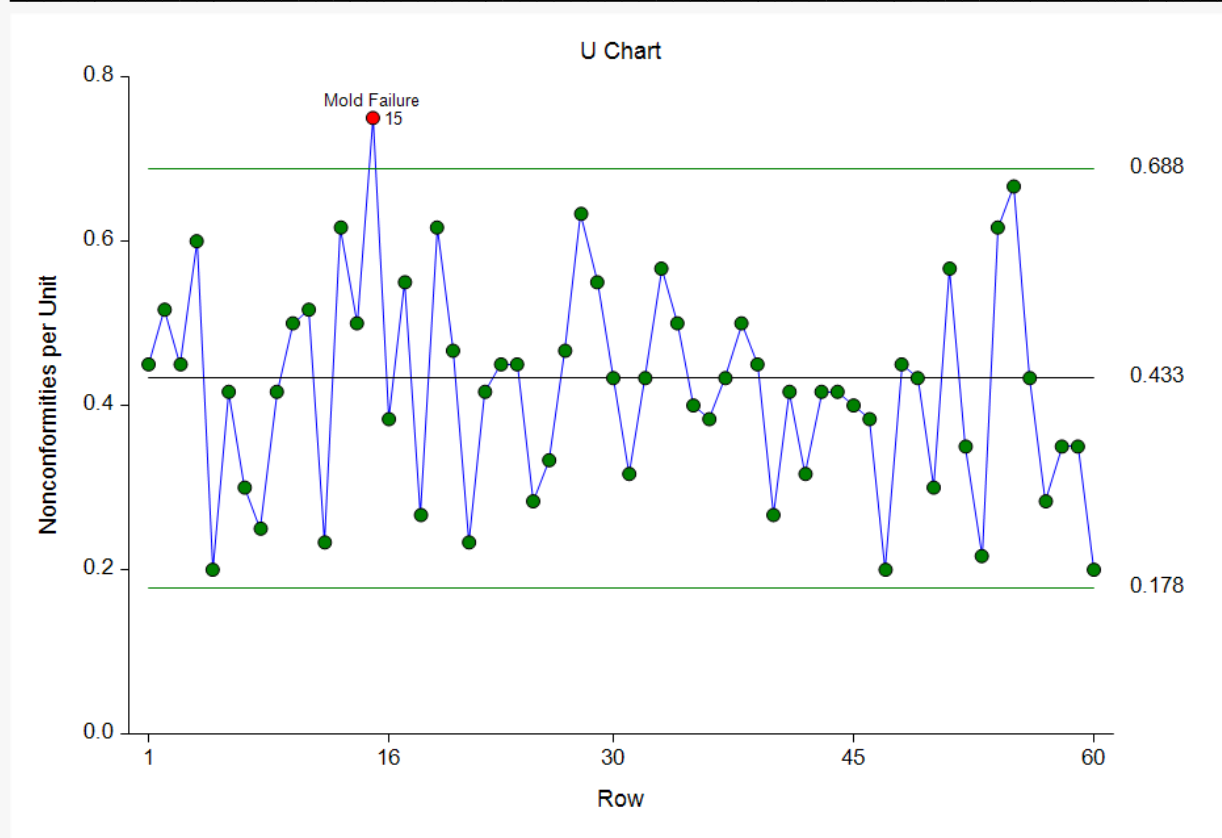
```

Number of Samples:          34
Sample Size Variable:      CountCont
Number of Nonconformities Variable:  NCCont
    
```

Average Sample Size	Average # of Nonconf./Sample	Total Sample Size	Total # Nonconf.	Average # of Nonconf./Unit	Nonconformities/Unit Limits	
					Lower Limit	Upper Limit
60.00	25.970588	2040	883	0.4328431	0.178036	0.687650

U Charts

Chart Section for Samples 1 to 60



Out-of-Control List for Samples 1 to 60

Row	Number of Nonconform./Unit	Row Label	Reason
15	0.7500	15	beyond control limits
47	0.2000	47	8 in zone C or beyond
55	0.6667	55	2 of 3 in zone A
56	0.4333	56	2 of 3 in zone A

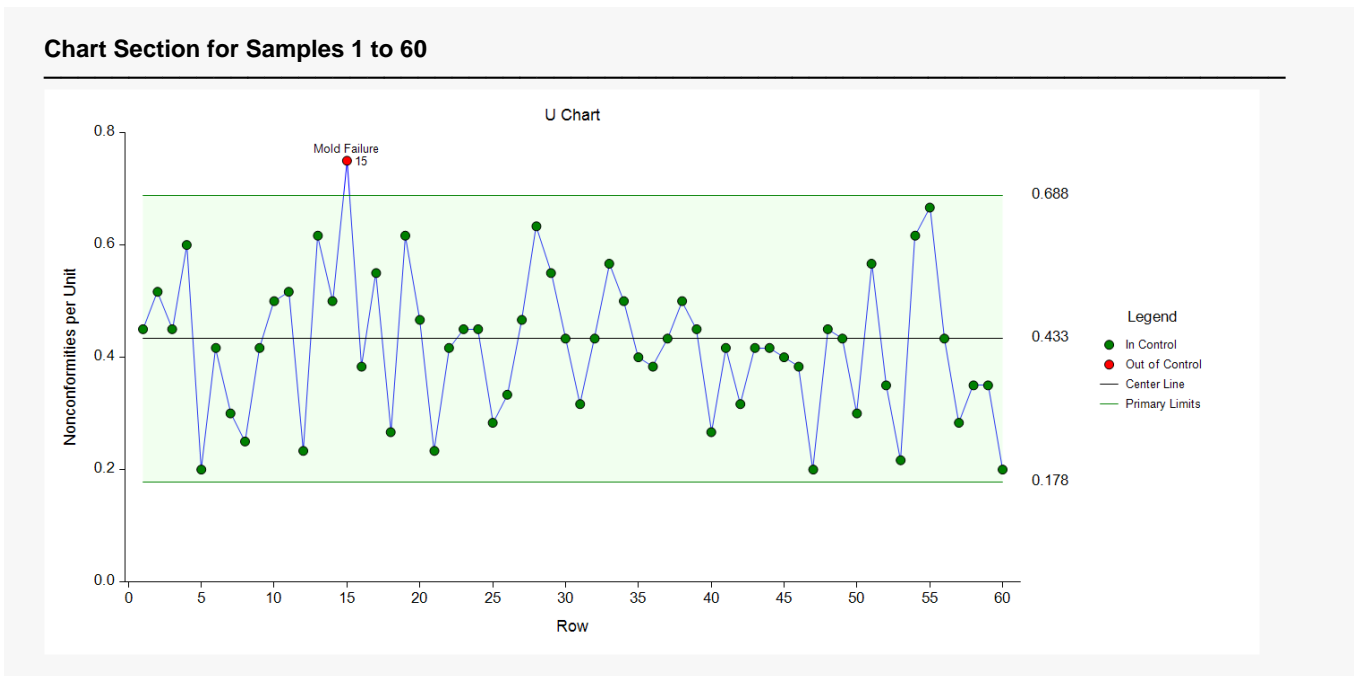
The estimation and limits section shows the same results as Example 2, since the calculations are again based only on the first 35 samples, excluding 15. The plot shows a stable continuation of the process from samples 36 to 60. Some of the additional samples give an out-of-control signal based on some of the runs tests. The investigators concluded that none of these were due to assignable causes.

## Example 4 – U Chart with Additional Formatting

This example uses the same setup as Example 3, except that a variety of improvements are made in the plot format. These improvements are made by clicking the U Chart format button on the U Chart tab.

The settings for this example are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

### U Chart



As shown here, a variety of enhancements can be made to the formatting of the control chart to make the chart as easy to read as possible.

## Example 5 – U Chart with Stages

This section presents a continuation of Examples 1, 2, and 3. After the first 60 days, a new production line process is implemented to reduce the number of tire nonconformities. The investigators obtained samples of the process for 70 additional days, using the first 30 samples of the additional 70 days (61 to 90) to determine the in-control limits of the updated process. The investigators wish to view the entire progress of the process, beginning with day 1. This example shows the use of stages to monitor a process with a change in center line and limit calculations.

### Setup

To run this example, complete the following steps:

**1 Open the Tire Nonconformities example dataset**

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select Tire Nonconformities and click OK.

**2 Specify the U Charts procedure options**

- Find and open the **U Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 5** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab	
Sample Size Variable .....	<b>CountStage</b>
Number of Nonconformities Variable .....	<b>NCStage</b>
Number of Stages.....	<b>Multiple Stages</b>
Stage Variable .....	<b>Stage</b>
Specification Method .....	<b>Keep Rows Variable</b>
Use all rows where variable.....	<b>KeepStage</b>
equals .....	<b>1</b>
Point Label Variable.....	<b>Label</b>

**3 Run the procedure**

- Click the **Run** button to perform the calculations and generate the output.

## Estimation and Limit Section

### Estimation and Limit Section for Samples 1 to 60 where KeepStage equals 1

Number of Samples: 34  
 Sample Size Variable: CountStage  
 Number of Nonconformities Variable: NCStage

Average Sample Size	Average # of Nonconf./Sample	Total Sample Size	Total # Nonconf.	Average # of Nonconf./Unit	Nonconformities/Unit Limits	
					Lower Limit	Upper Limit
60.00	25.970588	2040	883	0.4328431	0.178036	0.687650

### Estimation and Limit Section for Samples 61 to 130 where KeepStage equals 1

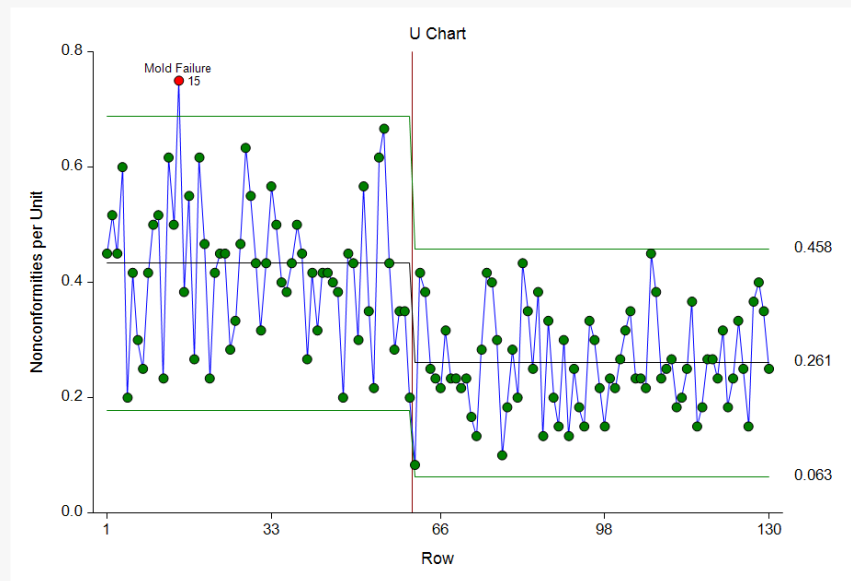
Number of Samples: 30  
 Sample Size Variable: CountStage  
 Number of Nonconformities Variable: NCStage

Average Sample Size	Average # of Nonconf./Sample	Total Sample Size	Total # Nonconf.	Average # of Nonconf./Unit	Nonconformities/Unit Limits	
					Lower Limit	Upper Limit
60.00	15.633333	1800	469	0.2605556	0.062861	0.458251

This section displays the estimation for each stage separately. The average number nonconforming is decreased by about 40%.

## U Chart

### Chart Section for Samples 1 to 130



The U chart shows the substantial decrease in the number of tire nonconformities in the second stage. The center line and limits of the second stage are estimated from samples 61 to 90.



## Out-of-Control List

### Out-of-Control List for Samples 1 to 60

Row	Number of Nonconform./Unit	Row Label	Reason
15	0.7500	15	beyond control limits
47	0.2000	47	8 in zone C or beyond
55	0.6667	55	2 of 3 in zone A
56	0.4333	56	2 of 3 in zone A

### Out-of-Control List for Samples 61 to 130

Row	Number of Nonconform./Unit	Row Label	Reason
76	0.4000	76	2 of 3 in zone A
77	0.3000	77	2 of 3 in zone A

The out-of-control list for the first stage is the same as that of the previous examples. In the second stage a couple of additional points are flagged due to two points in zone A.