

## Chapter 203

# Grubbs' Outlier Test

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## Introduction

It is well known that outliers (extreme points) often distort the results of an analysis. Because of this, every analysis should begin with either a graphical or statistical check about the possibility of outliers. This procedure computes Grubbs' test (1950) for detecting outliers in normal populations. It also computes Rosner's (2011) test for many outliers. We also recommend Barnett and Lewis (1994) for many more outlier tests.

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## Deleting Outliers

Once outliers have been detected, a decision has to be made about what to do with them. Most would agree that at the minimum, these values should be investigated for input errors. If the outlier appears to be valid, then what? Some would recommend adopting a technique that performs well when outliers are present such as robust or nonparametric techniques. A pragmatic approach is to run the analysis with and without the outliers and watch whether the fundamental conclusions change.

Most statisticians would agree that outliers should not be removed automatically. They should be carefully studied. One question that must be asked is will more outliers likely occur? If so, they must be dealt with.

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## Technical Details

This section provides the technical details of this test. We follow the presentation of Rosner (2011).

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### Grubbs' Test for a Single Outlier

Grubbs' (1950) procedure tests the hypothesis that the value that is the furthest from the sample mean is an outlier. Suppose you have a sample of  $n$  observations, labelled  $X_1$  to  $X_n$ , that are assumed to follow the normal distribution. To test the null hypothesis that no outlier is present versus the alternative hypothesis that a single outlier is present, Grubbs proposed that the quantity *ESD* (extreme studentized deviate) be used where

$$ESD = \max_{i=1, \dots, n} \frac{|X_i - \bar{X}|}{s}$$

## Grubbs' Outlier Test

The significance level of ESD can be determined using

$$ESD = \frac{n-1}{\sqrt{n}} \sqrt{\frac{t^2}{n-2+t^2}}$$

where  $t$  is short for  $t_{n-2,p}$  and  $p = 1 - \alpha/(2n)$ . Grubbs' tabulated this result, but we have solved this so that a  $p$ -value can be calculated. The above is for two-sided tests. For one-sided tests, substitute  $\alpha/(n)$  for  $\alpha/(2n)$ .

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## Rosner's ESD Many-Outlier Procedure

You might be tempted to apply Grubbs' test to the maximum ESD, then removing it and recalculating Grubbs' test on the reduced sample, and so on until a non-significant result is found. However, because of features of multiply outliers called swamping and masking, this procedure will not find all of the outliers. However, Rosner (2011) presents a procedure that will identify a block of outliers, assuming that  $n$  is greater than 20. His procedure is as follows.

1. Determine a value  $k$  that is a few larger than the anticipated number of outliers. This may be a number or a percentage of the sample size.
2. Compute the mean, standard deviation, and  $ESD_{(n)}$  for the sample of  $n$  value.
3. Calculate the significance level of  $ESD_{(n)}$ . Call it  $P_{ESD(n)}$ .
4. Remove the value from value corresponding to  $ESD_{(n)}$ .
5. Repeat steps 2, 3, and 4 on each successive sample after removing value corresponding to ESD at each step.
6. Consider the  $k$  values:  $P_{ESD(n)}$ ,  $P_{ESD(n-1)}$ ,  $P_{ESD(n-2)}$ , ...,  $P_{ESD(n-k+1)}$ .
7. Beginning with  $P_{ESD(n-k+1)}$ , look for the first value that is less than a preset significance level, say 0.05.
8. Designate this value as an outlier and all values before it also as outliers, whether their individual values of  $P_{ESD}$  were significant or not. For example, suppose  $n$  is 50 and  $k$  is 5, and the values of  $P_{ESD}$  beginning with  $P_{ESD(50)}$  are:  $P_{ESD(50)} = 0.107$ ,  $P_{ESD(49)} = 0.033$ ,  $P_{ESD(48)} = 0.0482$ ,  $P_{ESD(47)} = 0.203$ , and  $P_{ESD(46)} = 0.428$ . If  $\alpha$  were 0.05, we would design the observation corresponding to the first 3 values of ESD as outliers, even though the  $P_{ESD(50)}$  by itself was not significant according to Grubbs' test.

## Example 1 – Search for Outliers in the Grubbs Dataset

This section presents an example of how to check a variable for outliers. A test score was calculated on each of 100 subjects. These scores were stored in the first column of the Grubbs dataset. The researchers wish to check whether there are any outliers in the data.

### Setup

To run this example, complete the following steps:

#### 1 Open the Grubbs example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Grubbs** and click **OK**.

#### 2 Specify the Grubbs' Outlier Test procedure options

- Find and open the **Grubbs' Outlier Test** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Data Input Type ..... **One Response Variable**

Response Variable ..... **Score**

#### 3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

### Descriptive Statistics

#### Descriptive Statistics for Score

Parameter	Value
Count	100
Mean	50.1268
Standard Deviation	14.1729

This report provides the basic descriptive statistics of the variable.

## Grubbs' Single Outlier Test and Rosner's ESD Many-Outliers Test

### Grubbs' Single-Outlier Test and Rosner's ESD Many-Outliers Test for Score

Alternative Hypothesis: Two-Sided

Value of Possible Outlier	ESD  Z	Grubbs' Single-Outlier Test Prob Level	Conclude Outlier by Rosner's Procedure?
1.2345	3.4497	0.0381	Yes
98.3123	3.5718	0.0223	Yes
4.1234	3.6787	0.0137	Yes
81.1439	2.6205	0.7519	No
78.8369	2.5302	0.9820	No

The report provides the results of Grubbs' and Rosner's outlier tests.

### Value of Possible Outlier

These are the data values whose outlier test results are shown. The number of values displayed here is controlled by the 'Maximum Number of Possible Outliers' settings. The values that are the most extreme from the sample mean are shown.

### ESD |Z|

These are the ESD values corresponding to the data values shown to the left. The first value, 3.4497, is calculated from all the data. The second value, 3.5718, is calculated from the 99 values that remain after the first value, 1.2345, is omitted. The third value, 3.6787, is calculated from the 98 values that remain after both 1.2345 and 98.3123 are omitted. And so on.

### Grubbs' Single-Outlier Test Prob Level

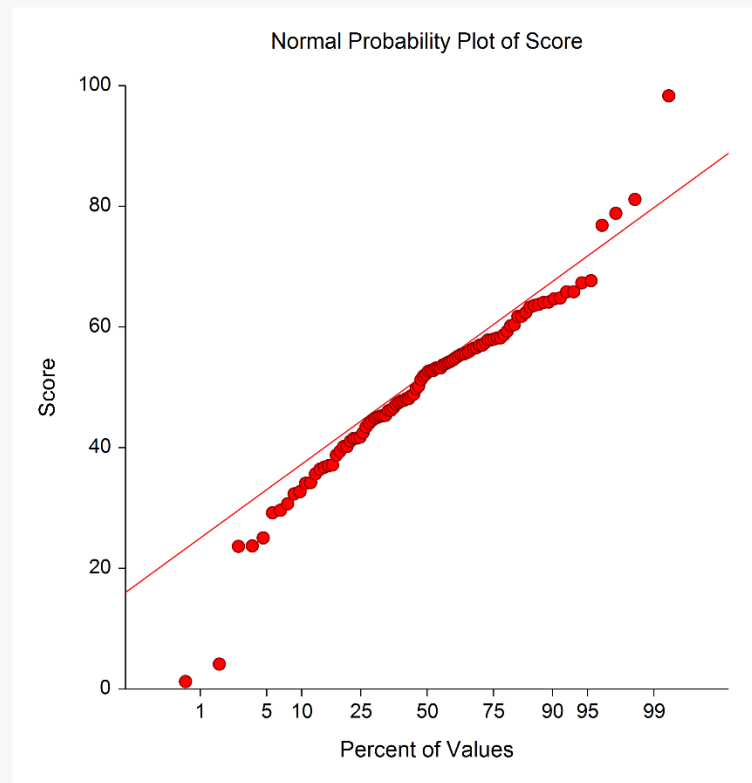
These are the p-values corresponding to the ESD values to the left. It is possible for the calculation to yield a number greater than one. When this occurs, the p-value is reset to 1.0.

### Conclude Outlier by Rosner's Procedure?

Rosner's test is conducted by proceeding up the Grubbs Prob Level from the bottom. When a value is found that is less than alpha (e.g., 0.05), that row and all above it are concluded to be statistically significant. In the example, the top three rows. Note that they are concluded to be outliers *even if their individual p-values are greater than 0.05*.

## Normal Probability Plot

### Normal Probability Plot



If the observations fall along a straight line, this indicates the data follow a normal distribution. Outliers are shown at either end of the trend line.