Chapter 136

Survival Parameter Conversion Tool

Introduction

This procedure can be used to generate any of the following survival parameters from the others: hazard rate, proportion surviving past a given time, mortality, and median survival time. Each of these parameters is functionally related to the others as described in the following section. This procedure may also be used to convert rates and proportions to different time units.

Functional Relationship of Survival Parameters

The parameter conversions in this tool assume an exponential survival distribution. Using the hazard rate equations below, any of the four survival parameters can be solved for from any of the other parameters.

Exponential Distribution

The density function of the exponential is defined as

\[ f(t) = he^{-ht} \]

The probability of surviving the first \( t \) years is

\[ S(t) = e^{-ht} \]

The mortality (probability of dying during the first \( t \) years) is

\[ M(t) = 1 - e^{-ht} \]

For an exponential distribution, the mean survival is \( 1/h \) and the median is \( \ln(2)/h \).

Notice that it is easy to translate between the hazard rate, the proportion surviving, the mortality, and the median survival time. The choice of which parameterization is used is arbitrary and is selected according to the convenience of the user.

Hazard Rate from Median Survival Time

Here, the median survival time is specified. These are transformed to hazard rates using the relationship \( h = \ln(2) / MST \).

Hazard Rate from Proportion Surviving

In this case, the proportion surviving until a given time \( T0 \) is specified. These are transformed to hazard rates using the relationship \( h = -\ln(S(T0)) / T0 \). Note that when separate proportions surviving are given for each time period, \( T0 \) is taken to be the time period number.
Hazard Rate from Mortality
Here, the mortality until a given time $T_0$ is specified. These are transformed to hazard rates using the relationship $h = -\ln(1 - M(T_0)) / T_0$. Note that when separate mortalities are given for each time period, $T_0$ is taken to be the time period number.

Hazard Ratio
The hazard ratio is the simple ratio of two hazard rates: $HR = h_1 / h_2$.

Mortality Ratio
The mortality ratio is the simple ratio of two mortalities: $MR = M_1 / M_2$.

Example
Convert a median survival time of 2.3 to the corresponding hazard rate.

1. Load the Survival Parameter Conversion Tool window by clicking on Tools and then clicking on Calculators and then Survival Parameter Conversion Tool. Click on the Survival Parameters tab.
2. Enter 2.3 for Median Survival Time 1 ($T_1$).
3. The value in Hazard Rate 1 ($h_1$) becomes 0.301368339373889. This is the value of the corresponding hazard rate.

Rates and Proportions Tab
Changing the values on this tab allows the user to convert proportions and/or rates from one time unit to another, e.g., annual to monthly.

Main Proportion (e.g., Annual)
This is the proportion corresponding to the main time unit (e.g. annual proportion).

The main proportion and the sub proportion are related by the formula

$$M = 1 - (1 - S) ^ K$$

where $M$ is the main proportion, $S$ is the sub proportion, and $K$ is the number of sub time units in each main time unit.

Range: $0 < $Main Proportion$ < 1$

Sub Proportion (e.g., Monthly)
This is the proportion corresponding to the sub time unit (e.g. monthly proportion).

The sub proportion and the main proportion are related by the formula

$$S = 1 - (1 - M) ^ (1 / K)$$

where $S$ is the sub proportion, $M$ is the main proportion, and $K$ is the number of sub time units in each main time unit.

Range: $0 < $Sub Proportion$ < 1$

Main Rate (e.g., Annual)
This is the rate corresponding to the main time unit (e.g. annual rate).
The main rate and the sub rate are related by the formula
\[ \text{Main Rate} = \text{Sub Rate} \times K \]
where \( K \) is the number of sub time units in each main time unit.
Range: Main Rate > 0

**Sub Rate (e.g., Monthly)**

This is the rate corresponding to the sub time unit (e.g. monthly rate).
The sub rate and the main rate are related by the formula
\[ \text{Sub Rate} = \text{Main Rate} / K \]
where \( K \) is the number of sub time units in each main time unit.
Range: Sub Rate > 0

**Example**

Convert an annual proportion of 0.7 to the corresponding monthly proportion.

1. Load the Survival Parameter Conversion Tool window by clicking on Tools and then Calculators, and the clicking on Survival Parameter Conversion Tool. Click on the Rates and Proportions tab.
2. Enter 12 for Number of Sub Time Units in Each Main Time Unit.
3. Enter 0.7 for Main Proportion.
4. The value in Sub Proportion becomes 0.0954620942160428. If this proportion of the remaining total is allocated each month, the total proportion at the end of 12 months is 0.7.