

## Chapter 287

# Acceptance Sampling for Attributes with Fixed Nonconformities

## Introduction

The *Acceptance Sampling for Attributes with Fixed Nonconformities* procedure is used to determine the number of items to be sampled from a lot to determine whether to accept or reject the lot when the number of nonconformities (defects) is specified.

The number of items in the sample depend upon a number of parameters, including the lot size, the acceptable quality level (AQL), the desired producer's risk, the limiting quality level (LQL, sometimes called the rejectable quality level or lot tolerance percent/proportion defective), the desired consumer's risk, and the number of defective items found in the sample. This procedure permits the user to enter multiple values of any of these parameters to determine the sensitivity of the sample size to that parameter. The cutoff value of acceptance, or acceptance number, is also given as part of the output.

In this procedure, the lot size can be assumed to be infinite (or continuous) and use the binomial distribution for calculations, or the lot can have a fixed size, whereupon the calculations are based on the hypergeometric distribution.

## Technical details

Define  $N$  to be the lot size (possibly infinite),  $n$  as the (unknown) size of the sample to be drawn, and  $c$  to be the acceptance number (the highest number of nonconforming units for which the lot will still be accepted). Let  $X$  denote the number of nonconforming units in the sample. Let  $p_0$  be the AQL, the highest proportion of nonconforming (defective) units for which the lot should still be accepted. Let  $\alpha$  be the producer's risk, the probability of rejecting a lot with a proportion of nonconforming (defective) units that is below the AQL. Let  $p_1$  be the LQL, the proportion of nonconforming (defective) units above which the lot should be routinely rejected. Let  $\beta$  be the probability of accepting a lot with a proportion of nonconforming (defective) units that is above the LQL.

For a given  $N$ ,  $p_0$ ,  $\alpha$ ,  $p_1$ , and  $\beta$ , we desire to obtain an  $n$  and  $c$  such that

$$\Pr\{X \leq c | p_0\} \geq 1 - \alpha$$

and

$$\Pr\{X \leq c | p_1\} \leq \beta$$

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If the lot size is finite,  $n$  and  $c$  should satisfy the hypergeometric distribution inequalities

$$H(c; N, M_0, n) \geq 1 - \alpha$$

and

$$H(c; N, M_1, n) \leq \beta$$

where  $M_0 = [Np_0]$  and  $M_1 = [Np_1]$ . The hypergeometric probability of obtaining exactly  $x$  of  $n$  items with the characteristic of interest is calculated using

$$h(x; N, M, n) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}$$

The cumulative hypergeometric distribution function is

$$H(x; N, M, n) = \sum_{j=0}^x h(j; N, M, n)$$

See Kenett and Zacks (2014) for more details.

If the lot size is infinite (or continuous),  $n$  and  $c$  should satisfy the binomial distribution inequalities

$$\sum_{j=0}^c \frac{n!}{j!(n-j)!} p_0^j (1-p_0)^{n-j} \geq 1 - \alpha$$

and

$$\sum_{j=0}^c \frac{n!}{j!(n-j)!} p_1^j (1-p_1)^{n-j} \leq \beta$$

**PASS** performs a search to determine an  $n$  and  $c$  value that meet the specified inequality requirements.

## Example 1 – Comparing Sample Size and Number Nonconforming

Suppose a quality engineer is to receive lots of 500 units. Because it is not feasible to inspect all units in the lot, the engineer would like to examine an acceptance sample. It is determined that the acceptable quality level is 0.5%, or 0.005, and the limiting quality level is 0.02.

The engineer would like to understand the relationships between producer's risk, consumer's risk, sample size, and number of defective items allowed. To accomplish this, the sample size will range from 20 to 80 and the acceptance number will range from 0 to 2.

Note that even though the *Solve For* parameter will be set to *Consumer's Risk (Beta)*, both consumer's risk and producer's risk will be calculated. However, only consumer's risk will be displayed on the plot.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Consumer's Risk (Beta)</b>
Lot Size (N) .....	<b>500</b>
Sample Size (n) .....	<b>20 40 60 80</b>
Acceptance Number (c) .....	<b>0 1 2</b>
Acceptable Quality Level (AQL, P0) .....	<b>0.005</b>
Limiting Quality Level (LQL, P1) .....	<b>0.02</b>
Approximation Cutoff .....	<b>10000</b>

## Acceptance Sampling for Attributes with Fixed Nonconformities

## Output

Click the Calculate button to perform the calculations and generate the following output.

## Numeric Reports

### Numeric Results

Solve For: **Consumer's Risk (Beta)**

Lot Size N	Sample Size n	# Nonconforming		Acceptable Quality Level P0	Producer's Risk Alpha	Limiting Quality Level P1	Consumer's Risk Beta
		c Accept Lot If # ≤	c + 1 Reject Lot If # ≥				
500	20	0	1	0.005	0.11549	0.02	0.66231
500	20	1	2	0.005	0.00446	0.02	0.94355
500	20	2	3	0.005	0.00006	0.02	0.99449
500	40	0	1	0.005	0.22172	0.02	0.43096
500	40	1	2	0.005	0.01780	0.02	0.81318
500	40	2	3	0.005	0.00048	0.02	0.96159
500	60	0	1	0.005	0.31909	0.02	0.27506
500	60	1	2	0.005	0.03926	0.02	0.65797
500	60	2	3	0.005	0.00165	0.02	0.89330
500	80	0	1	0.005	0.40798	0.02	0.17189
500	80	1	2	0.005	0.06806	0.02	0.50646
500	80	2	3	0.005	0.00397	0.02	0.79515

N Lot Size. The total size of the lot from which the sample drawn.

n Sample Size. The size of the sample drawn from the lot.

c Acceptance Number. The largest number of nonconforming items from the sample of size n for which the lot should still be accepted.

c + 1 Rejection Number. The smallest number of nonconforming items from the sample of size n for which the lot should be rejected.

P0 Acceptable Quality Level (AQL). The highest proportion of nonconforming (defective) units for which the lot is still considered acceptable.

Alpha Producer's Risk. The probability of rejecting a lot with a proportion of nonconforming (defective) units that is below the acceptable quality level. In short, it is the risk of rejecting a good lot.

P1 Limiting Quality Level (LQL) (Also called Rejectable Quality Level or LTPD (for Lot Tolerance Percent/Proportion Defective)). This is the proportion of nonconforming (defective) units above which the lot should be routinely rejected.

Beta Consumer's Risk. The probability of accepting a lot with a proportion of nonconforming (defective) units that is above the limiting quality level. In short, it is the risk of accepting a bad lot.

### Summary Statements

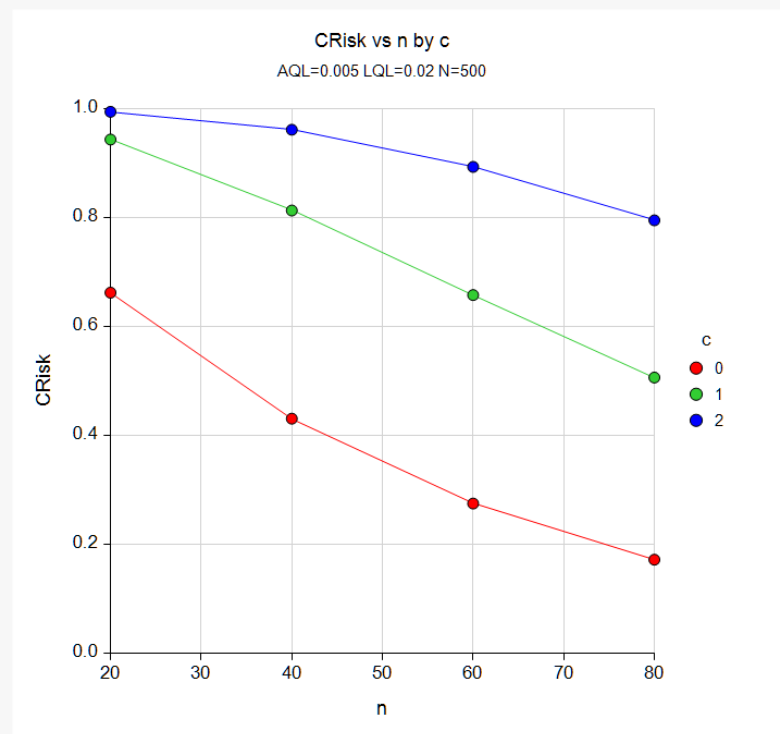
An acceptance sampling design will be used to determine whether to accept or reject a lot of 500 items. The acceptable quality level (P0) is 0.005. The limiting quality level (P1) is assumed to be 0.02. The largest number of non-conforming items (defectives) of the sample for which the lot should still be accepted is fixed at 0. The determination to accept or reject the lot will be based on a one-sided exact test of a single proportion. With a sample size of 20, the producer's risk ( $\alpha$ ) is 0.11549 and the consumer's risk ( $\beta$ ) is 0.66231. From the sample of size 20, if the number of non-conforming items is 0 or fewer, the lot should be accepted. If the number of non-conforming items is 1 or greater, the lot should be rejected.

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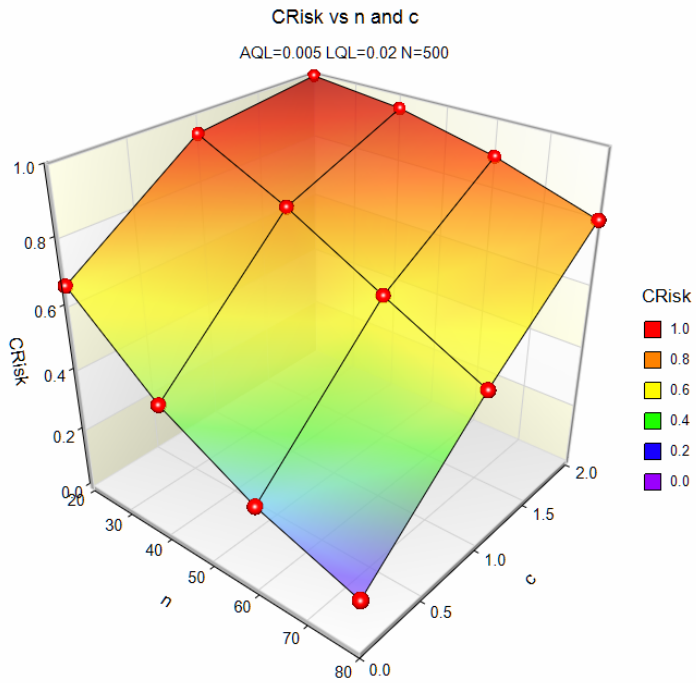
**References**

- Guenther, William C. 1977. Sampling inspection in statistical quality control. Griffin's statistical monographs and courses, no. 37.
- Kenett, R. S. and Zacks, S. 2014. Modern Industrial Statistics, 2nd Edition. John Wiley & Sons. West Sussex, United Kingdom.
- Schilling, E. G. and Neubauer, D. V. 2017. Acceptance Sampling in Quality Control, Third Edition. CRC Press. New York.
- Squeglia, Nicholas L. 2017. Zero Acceptance Number Sampling Plans, Fifth Edition. Quality Press. Milwaukee, WI.

This report shows the values of alpha and beta for various combinations of  $n$  and  $c$ .

**Plots Section****Plots**

## Acceptance Sampling for Attributes with Fixed Nonconformities



This report allows several parameters to be compared. In this example, only the design with  $c = 0$  and  $n = 80$  achieves a consumer's risk that is below 0.2. However, looking at the numeric report, the value of the producer's risk for this design is 0.408 which is way too high.

## Example 2 – Validation of Sample Size Calculation using Kenett and Zacks (2013)

Kenett and Zacks (2013), pp. 264, give an example in which the lot size is 500,  $c$  is 4, alpha and beta (producer's risk and consumer's risk) are both 0.05, the acceptable quality level is 0.01, and the limiting quality level is 0.03. The required sample size is estimated to be 254.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For ..... **Sample Size**  
 Lot Size (N) ..... **500**  
 Acceptance Number (c) ..... **4**  
 Acceptable Quality Level (AQL, P0) ..... **0.01**  
 Producer's Risk (Alpha) ..... **0.05**  
 Limiting Quality Level (LQL, P1) ..... **0.03**  
 Consumer's Risk (Beta) ..... **0.05**  
 Approximation Cutoff ..... **10000**

### Output

Click the Calculate button to perform the calculations and generate the following output.

#### Numeric Results

Solve For: [Sample Size](#)

Lot Size N	Sample Size n	# Nonconforming		Acceptable Quality Level P0	Producer's Risk (Alpha)		Limiting Quality Level P1	Consumer's Risk (Beta)	
		c	c + 1						
		Accept Lot If # ≤	Reject Lot If # ≥						
					Target	Actual		Target	Actual
500	254	4	5	0.01	0.05	0.03318	0.03	0.05	0.04966

The sample size of 254 matches the results of Kenett and Zacks (2013).