Chapter 285

Acceptance Sampling for Attributes with Optimum Number of Nonconformities

Introduction

The *Acceptance Sampling for Attributes with Optimum Number of Nonconformities* procedure is used to determine the number of items to be sampled from a lot to determine whether to accept or reject the lot. The number of items in the sample depend upon a number of parameters, including the lot size, the acceptable quality level (AQL), the desired producer's risk, the limiting quality level (LQL, sometimes called the rejectable quality level or lot tolerance percent/proportion defective), and the desired consumer's risk. This procedure permits the user to enter multiple values of any of these parameters to determine the sensitivity of the sample size to that parameter. When multiple values are entered for a parameter, a sample size curve is also produced. The cutoff value of acceptance, or acceptance number, is also given as part of the output.

In this procedure, the lot size can be assumed to be infinite (or continuous) and use the binomial distribution for calculations, or the lot can have a fixed size, whereupon the calculations are based on the hypergeometric distribution.

Technical Details

Define *N* to be the lot size (possibly infinite), *n* as the (unknown) size of the sample to be drawn, and *c* to be the acceptance number (the highest number of nonconforming units for which the lot will still be accepted). Let *X* denote the number of nonconforming units in the sample. Let p_0 be the AQL, the highest proportion of nonconforming (defective) units for which the lot should still be accepted. Let α be the producer's risk, the probability of rejecting a lot with a proportion of nonconforming (defective) units that is below the AQL. Let p_1 be the LQL, the proportion of nonconforming (defective) units above which the lot should be routinely rejected. Let β be the probability of accepting a lot with a proportion of nonconforming (defective) units that is above the LQL.

For a given *N*, p_0, α, p_1, and β, we desire to obtain an *n* and *c* such that

$$\Pr\{X \leq c|p_0\} \geq 1 - \alpha$$

and

$$\Pr\{X \leq c|p_1\} \leq \beta$$

If the lot size is finite, *n* and *c* should satisfy the hypergeometric distribution inequalities

$$H(c; N, M_0, n) \geq 1 - \alpha$$

and

$$H(c; N, M_1, n) \leq \beta$$

where M_0 = [Np_0] and M_1 = [Np_1]. The hypergeometric probability of obtaining exactly *x* of *n* items with the characteristic of interest is calculated using

$$h(x; N, M, n) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

The cumulative hypergeometric distribution function is

$$H(x; N, M, n) = \sum_{j=0}^{x} h(j; N, M, n)$$

See Kenett and Zacks (2014) for more details.

If the lot size is infinite (or continuous), n and c should satisfy the binomial distribution inequalities

$$\sum_{j=0}^{c} \frac{n!}{j!\,(n-j)!} p_0^j\,(1-p_0)^{n-j} \geq 1 - \alpha$$

and

$$\sum_{j=0}^{c} \frac{n!}{j!\,(n-j)!} p_1^j\,(1-p_1)^{n-j} \leq \beta$$

PASS performs a search to determine an *n* and *c* value that meet the specified inequality requirements.

Example 1 – Finding a Sample Size

Suppose a quality engineer is to receive a lot of 5,000 units. Because it is not feasible to inspect all 5,000 units, the engineer would like to examine an acceptance sample. It is determined that the acceptable quality level is 0.5%, or 0.005. The desired producer's risk (probability of rejecting a good lot) is 0.05. The limiting quality level (or lot tolerance percent defective or rejectable quality level) is 0.07. The desired consumer's risk (probability of accepting a bad lot) is 0.05. The engineer would like to know the number of units to randomly sample and inspect, as well as the acceptance number (the highest number of nonconforming units of the sample for which the lot should still be accepted).

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Sample Size**
Lot Size (N)	**5000**
Acceptable Quality Level (AQL, P0)	**0.005**
Producer's Risk (Alpha)	**0.05**
Limiting Quality Level (LQL, P1)	**0.07**
Consumer's Risk (Beta)	**0.05**
Approximation Cutoff	**10000**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size

Lot Size N	Sample Size n	# Nonconforming c Accept Lot If # ≤	# Nonconforming c + 1 Reject Lot If # ≥	Acceptable Quality Level P0	Producer's Risk (Alpha) Target	Producer's Risk (Alpha) Actual	Limiting Quality Level P1	Consumer's Risk (Beta) Target	Consumer's Risk (Beta) Actual
5000	66	1	2	0.005	0.05	0.0424	0.07	0.05	0.0486

N	Lot Size. The total size of the lot from which the sample is drawn.
n	Sample Size. The size of the sample drawn from the lot.
c	Acceptance Number. The largest number of nonconforming items from the sample of size n for which the lot should still be accepted.
c + 1	Rejection Number. The smallest number of nonconforming items from the sample of size n for which the lot should be rejected.
P0	Acceptable Quality Level (AQL). The highest proportion of nonconforming (defective) units for which the lot is still considered acceptable.

Alpha	Producer's Risk. The probability of rejecting a lot with a proportion of nonconforming (defective) units that is below the acceptable quality level. In short, it is the risk of rejecting a good lot.
Target Alpha	The producer's risk value entered by the user.
Actual Alpha	The achieved value of the producer's risk.
P1	Limiting Quality Level (LQL) (Also called Rejectable Quality Level or LTPD (for Lot Tolerance Percent/Proportion Defective)). This is the proportion of nonconforming (defective) units above which the lot should be routinely rejected.
Beta	Consumer's Risk. The probability of accepting a lot with a proportion of nonconforming (defective) units that is above the limiting quality level. In short, it is the risk of accepting a bad lot.
Target Beta	The consumer's risk value entered by the user.
Actual Beta	The achieved value of the consumer's risk.

Summary Statements

An acceptance sampling design will be used to determine whether to accept or reject a lot of 5000 items. The acceptable quality level (P0) is 0.005. The determination to accept or reject the lot will be based on a one-sided exact test of a single proportion, with a producer's risk (α) of no more than 0.05. To achieve a consumer's risk (β) of at most 0.05, with a limiting quality level (P1) of 0.07, the needed number of items to sample from the lot is 66. From the sample of size 66, if the number of non-conforming items (defectives) is 1 or fewer, the lot should be accepted. If the number of non-conforming items is 2 or greater, the lot should be rejected.

References

Guenther, William C. 1977. Sampling inspection in statistical quality control. Griffin's statistical monographs and courses, no. 37.

Kenett, R. S. and Zacks, S. 2014. Modern Industrial Statistics, 2nd Edition. John Wiley & Sons. West Sussex, United Kingdom.

Schilling, E. G. and Neubauer, D. V. 2017. Acceptance Sampling in Quality Control, Third Edition. CRC Press. New York.

This report shows that the needed sample size is 66. The lot should be accepted if the number of nonconforming units out of the sample of size 66 is 0 or 1. The lot should be rejected if the number of nonconforming units is 2 or greater.

Example 2 – Finding Sample Sizes for Multiple Parameter Values

Continuing with Example 1, suppose the quality engineer would like to see how the sample size is affected by varying some of the parameters. The acceptable quality level is to stay at 0.005, but the engineer would like to look at Producer's Risk values between 0.01 and 0.1, Limiting Quality Level values between 0.05 and 0.1, and Consumer's Risk values between 0.05 and 0.2.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Sample Size**
Lot Size (N)	**5000**
Acceptable Quality Level (AQL, P0)	**0.005**
Producer's Risk (Alpha)	**0.01 to 0.1 by 0.01**
Limiting Quality Level (LQL, P1)	**0.05 to 0.1 by 0.01**
Consumer's Risk (Beta)	**0.05 to 0.2 by 0.05**
Approximation Cutoff	**10000**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Sample Size

Lot Size N	Sample Size n	# Nonconforming c Accept Lot If # ≤	# Nonconforming c + 1 Reject Lot If # ≥	Acceptable Quality Level P0	Producer's Risk (Alpha) Target	Producer's Risk (Alpha) Actual	Limiting Quality Level P1	Consumer's Risk (Beta) Target	Consumer's Risk (Beta) Actual
5000	152	3	4	0.005	0.01	0.0063	0.05	0.05	0.04871
5000	152	3	4	0.005	0.02	0.0063	0.05	0.05	0.04871
5000	123	2	3	0.005	0.03	0.0225	0.05	0.05	0.04945
5000	123	2	3	0.005	0.04	0.0225	0.05	0.05	0.04945
5000	123	2	3	0.005	0.05	0.0225	0.05	0.05	0.04945
5000	123	2	3	0.005	0.06	0.0225	0.05	0.05	0.04945
5000	123	2	3	0.005	0.07	0.0225	0.05	0.05	0.04945
5000	93	1	2	0.005	0.08	0.0779	0.05	0.05	0.04853
5000	93	1	2	0.005	0.09	0.0779	0.05	0.05	0.04853
5000	93	1	2	0.005	0.10	0.0779	0.05	0.05	0.04853

5000	126	3	4	0.005	0.01	0.0032	0.06	0.05	0.04968
5000	103	2	3	0.005	0.02	0.0141	0.06	0.05	0.04784
5000	103	2	3	0.005	0.03	0.0141	0.06	0.05	0.04784
5000	103	2	3	0.005	0.04	0.0141	0.06	0.05	0.04784
5000	103	2	3	0.005	0.05	0.0141	0.06	0.05	0.04784
5000	77	1	2	0.005	0.06	0.0559	0.06	0.05	0.04924
5000	77	1	2	0.005	0.07	0.0559	0.06	0.05	0.04924
5000	77	1	2	0.005	0.08	0.0559	0.06	0.05	0.04924
5000	77	1	2	0.005	0.09	0.0559	0.06	0.05	0.04924
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N	Lot Size. The total size of the lot from which the sample is drawn.
n	Sample Size. The size of the sample drawn from the lot.
c	Acceptance Number. The largest number of nonconforming items from the sample of size n for which the lot should still be accepted.
c + 1	Rejection Number. The smallest number of nonconforming items from the sample of size n for which the lot should be rejected.
P0	Acceptable Quality Level (AQL). The highest proportion of nonconforming (defective) units for which the lot is still considered acceptable.
Alpha	Producer's Risk. The probability of rejecting a lot with a proportion of nonconforming (defective) units that is below the acceptable quality level. In short, it is the risk of rejecting a good lot.
Target Alpha	The producer's risk value entered by the user.
Actual Alpha	The achieved value of the producer's risk.
P1	Limiting Quality Level (LQL) (Also called Rejectable Quality Level or LTPD (for Lot Tolerance Percent/Proportion Defective)). This is the proportion of nonconforming (defective) units above which the lot should be routinely rejected.
Beta	Consumer's Risk. The probability of accepting a lot with a proportion of nonconforming (defective) units that is above the limiting quality level. In short, it is the risk of accepting a bad lot.
Target Beta	The consumer's risk value entered by the user.
Actual Beta	The achieved value of the consumer's risk.

Summary Statements

An acceptance sampling design will be used to determine whether to accept or reject a lot of 5000 items. The acceptable quality level (P0) is 0.005. The determination to accept or reject the lot will be based on a one-sided exact test of a single proportion, with a producer's risk (α) of no more than 0.01. To achieve a consumer's risk (β) of at most 0.05, with a limiting quality level (P1) of 0.05, the needed number of items to sample from the lot is 152. From the sample of size 152, if the number of non-conforming items (defectives) is 3 or fewer, the lot should be accepted. If the number of non-conforming items is 4 or greater, the lot should be rejected.

References

Guenther, William C. 1977. Sampling inspection in statistical quality control. Griffin's statistical monographs and courses, no. 37.

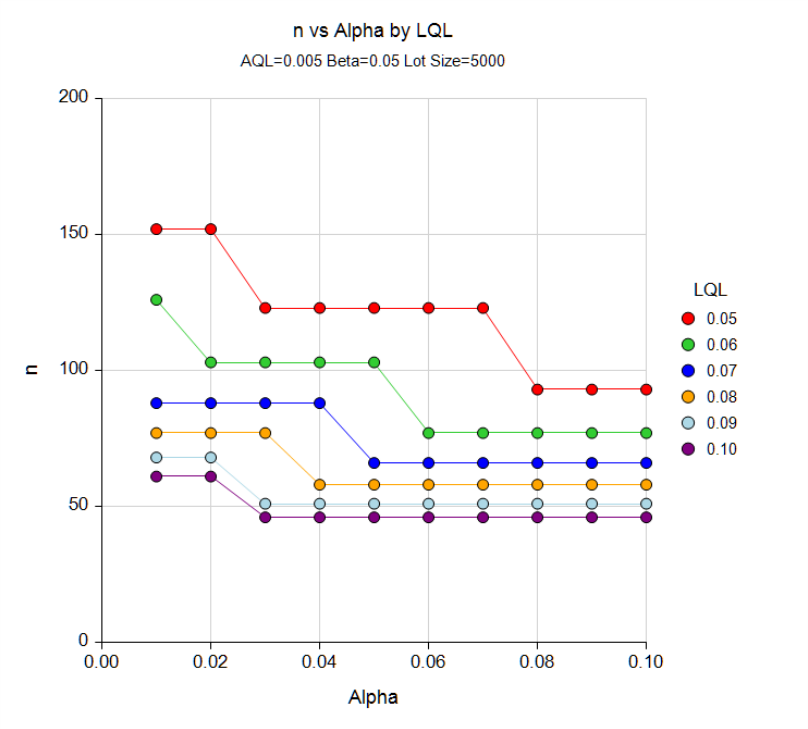
Kenett, R. S. and Zacks, S. 2014. Modern Industrial Statistics, 2nd Edition. John Wiley & Sons. West Sussex, United Kingdom.

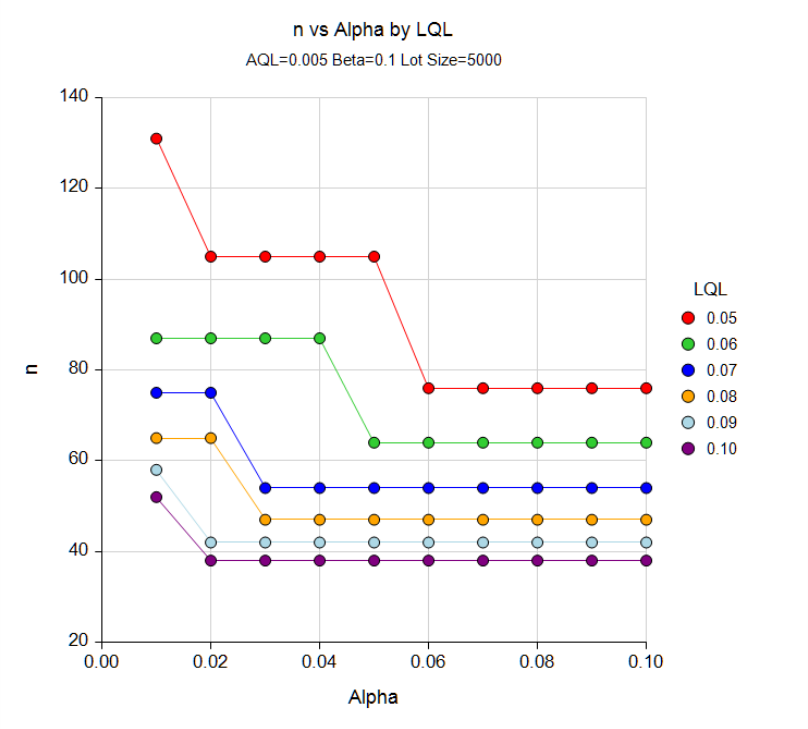
Schilling, E. G. and Neubauer, D. V. 2017. Acceptance Sampling in Quality Control, Third Edition. CRC Press. New York.

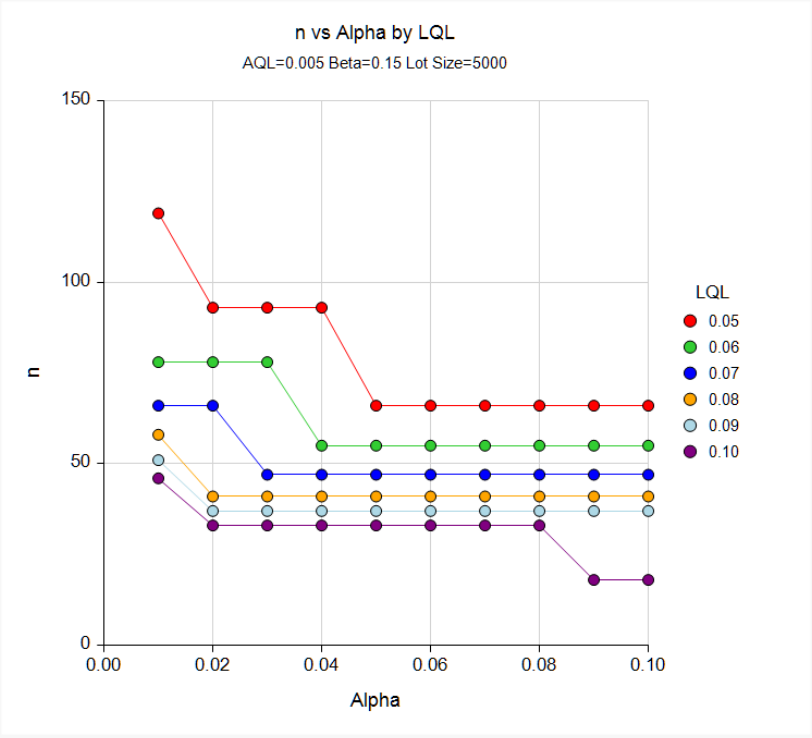
Each line gives a sample size and acceptance number based on the corresponding combination of quality levels and risks.

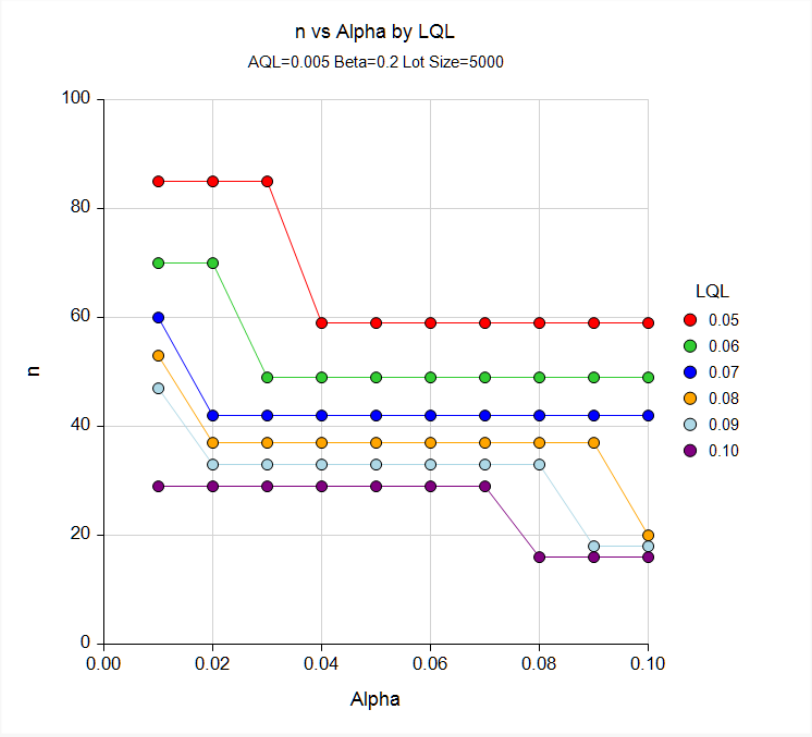
Plots Section

Plots

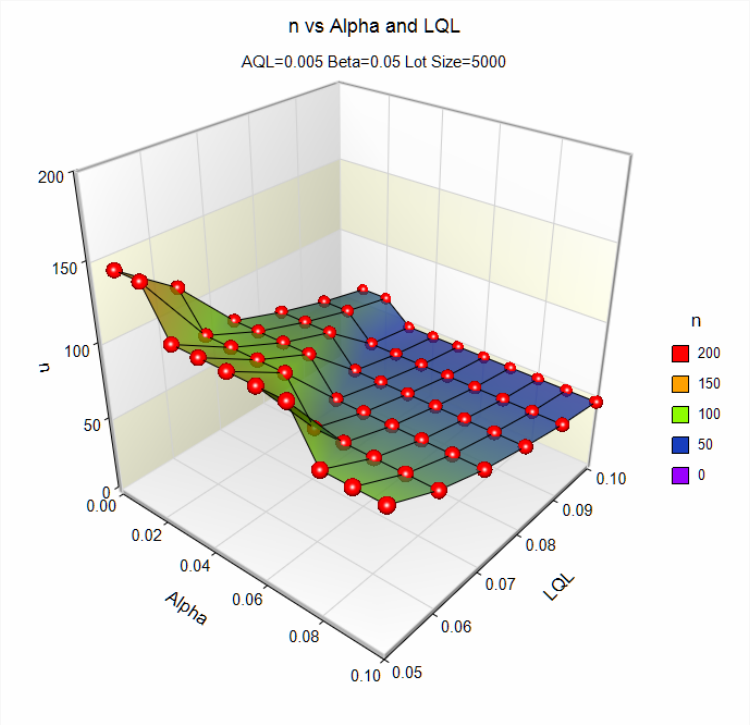


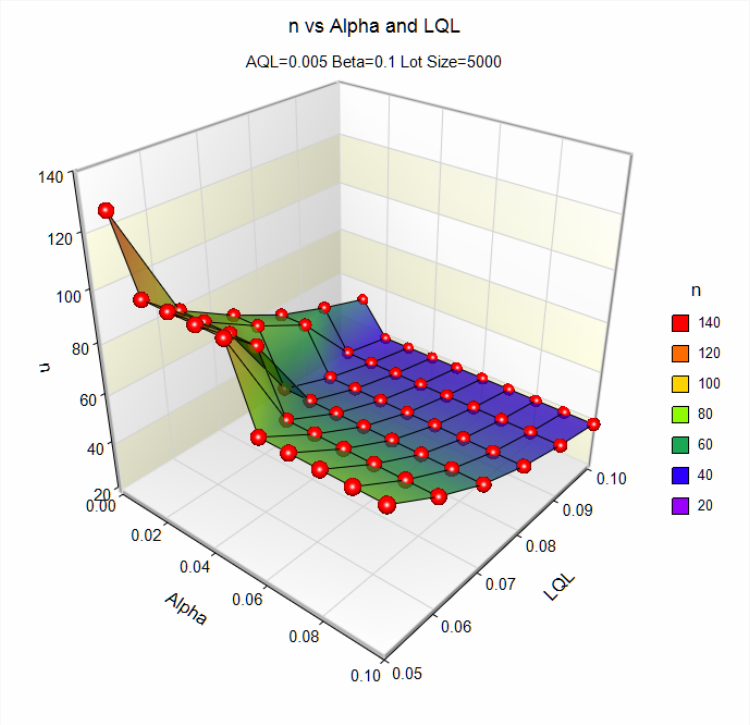




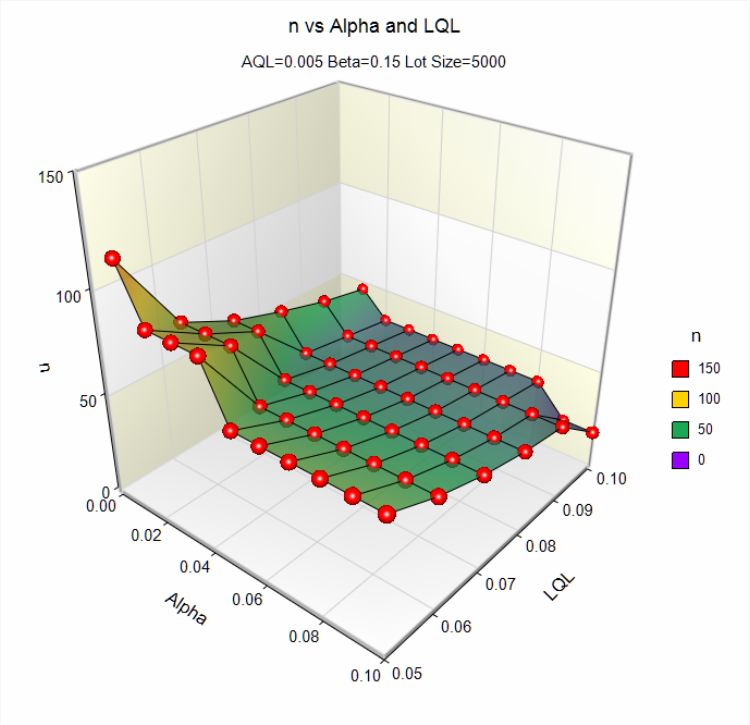


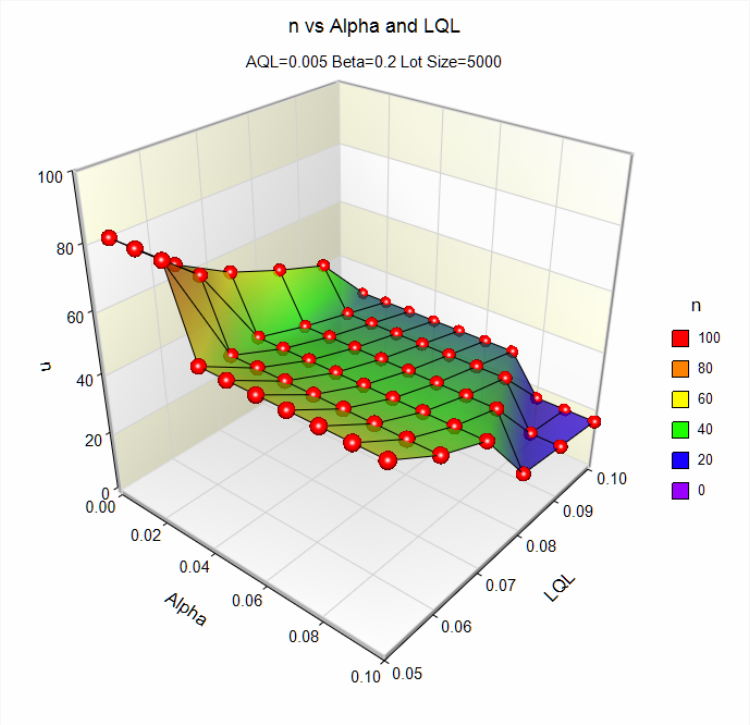
Acceptance Sampling for Attributes with Optimum Number of Nonconformities

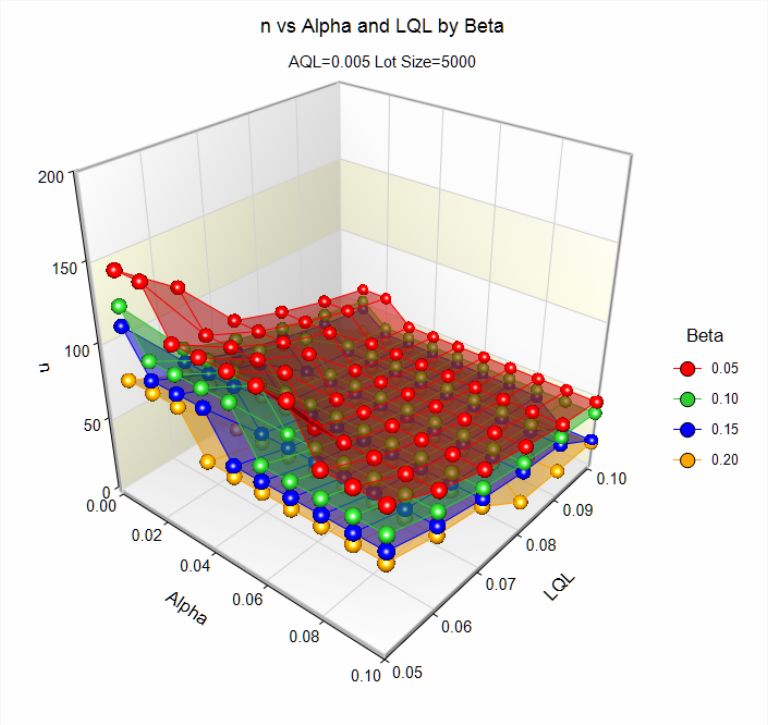




Acceptance Sampling for Attributes with Optimum Number of Nonconformities







These plots show the effect of various combinations of parameters on the sample size needed. The final 3D plot gives a summary of all the combinations in a single plot. This plot can be rotated (if desired) by going to the Plots tab and checking the box 'Live Edit/Rotate' for the X-Y-Z Plots with Groups and then re-running the procedure.

Example 3 – Validation of Sample Size Calculation using Kenett and Zacks (2013)

Kenett and Zacks (2013), pp. 260-261, give an example in which the lot size is 100, alpha and beta (producer's risk and consumer's risk) are both 0.05, the acceptable quality level is 0.01, and the limiting quality level is 0.05. The needed sample size is estimated to be 65, with a *c* of 1.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3(a-c)** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Sample Size**
Lot Size (N)	**100**
Acceptable Quality Level (AQL, P0)	**0.01**
Producer's Risk (Alpha)	**0.05**
Limiting Quality Level (LQL, P1)	**0.05**
Consumer's Risk (Beta)	**0.05**
Approximation Cutoff	**10000**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size

Lot Size N	Sample Size n	# Nonconforming c Accept Lot If # ≤	# Nonconforming c + 1 Reject Lot If # ≥	Acceptable Quality Level P0	Producer's Risk (Alpha) Target	Producer's Risk (Alpha) Actual	Limiting Quality Level P1	Consumer's Risk (Beta) Target	Consumer's Risk (Beta) Actual
100	65	1	2	0.01	0.05	0	0.05	0.05	0.04952

The sample size of 65 and *c* value of 1 matches that of Kenett and Zacks (2013).

In a second example with acceptable quality level of 0.03, and a limiting quality level of 0.05, Kenett and Zacks (2013) estimate a required sample size of 92.

Numeric Results

Solve For: Sample Size

Lot Size N	Sample Size n	# Nonconforming: c Accept Lot If # ≤	# Nonconforming: c + 1 Reject Lot If # ≥	Acceptable Quality Level P0	Producer's Risk (Alpha): Target	Producer's Risk (Alpha): Actual	Limiting Quality Level P1	Consumer's Risk (Beta): Target	Consumer's Risk (Beta): Actual
100	92	3	4	0.03	0.05	0	0.05	0.05	0.0499

PASS also gives a sample size estimate of 92.

In a third example with acceptable quality level of 0.01, and a limiting quality level of 0.2, Kenett and Zacks (2013) estimate a required sample size of 20.

Numeric Results

Solve For: Sample Size

Lot Size N	Sample Size n	# Nonconforming: c Accept Lot If # ≤	# Nonconforming: c + 1 Reject Lot If # ≥	Acceptable Quality Level P0	Producer's Risk Alpha: Target	Producer's Risk Alpha: Actual	Limiting Quality Level P1	Consumer's Risk Beta: Target	Consumer's Risk Beta: Actual
100	20	1	2	0.01	0.05	0	0.2	0.05	0.04985

PASS also estimates a sample size estimate of 20 for this scenario.

Example 4 – Validation of Sample Size Calculation using Guenther (1977)

Guenther (1977), pp. 7-8, gives an example in which the lot size is infinite (binomial calculations), alpha (producer's risk) is 0.05, beta (consumer's risk) is 0.1, the acceptable quality level is 0.05, and the limiting quality level is 0.2. The needed sample size is estimated to be 38, with an acceptance number (*c*) of 4.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Sample Size**
Lot Size (N)	**Infinite**
Acceptable Quality Level (AQL, P0)	**0.05**
Producer's Risk (Alpha)	**0.05**
Limiting Quality Level (LQL, P1)	**0.2**
Consumer's Risk (Beta)	**0.1**
Approximation Cutoff	**10000**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size

Lot Size N	Sample Size n	# Nonconforming c Accept Lot If # ≤	# Nonconforming c + 1 Reject Lot If # ≥	Acceptable Quality Level P0	Producer's Risk (Alpha) Target	Producer's Risk (Alpha) Actual	Limiting Quality Level P1	Consumer's Risk (Beta) Target	Consumer's Risk (Beta) Actual
Infinite	38	4	5	0.05	0.05	0.0397	0.2	0.1	0.09857

The sample size of 38 and *c* value of 4 matches that of Guenther (1977).