

## Chapter 754

# Assurance for Logrank Tests in a Cluster-Randomized Design

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## Introduction

This procedure calculates the assurance of logrank tests for comparing survival curves when the data are collected in a cluster-randomized design. The calculation is based on a user-specified prior distribution of the effect size parameters. This procedure may also be used to determine the needed sample size to obtain a specified assurance.

The methods for assurance calculation in this procedure are based on O'Hagan, Stevens, and Campbell (2005).

Cluster-randomized designs are often adopted when there is a high risk of contamination if cluster members were randomized individually. For example, it may be difficult for an instructor to use two methods of teaching individuals in the same class. The price of randomizing by clusters is a loss of efficiency--the number of subjects needed to obtain a certain level of precision in a cluster-randomized trial is usually much larger than the number needed when the subjects are randomized individually. Hence, the standard methods of sample size estimation cannot be used.

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## Assurance

The assurance of a design is the expected value of the power with respect to one or more prior distributions of the design parameters. Assurance is also referred to as *Bayesian assurance*, *expected power*, *average power*, *statistical assurance*, *hybrid classical-Bayesian procedure*, or *probability of success*.

The power of a design is the probability of rejecting the null hypothesis, conditional on a given set of design attributes, such as the test statistic, the significance level, the sample size, and the effect size to be detected. As the effect size parameters are typically unknown quantities, the stated power may be very different from the true power if the specified parameter values are inaccurate.

While power is conditional on individual design parameter values, and is highly sensitive to those values, assurance is the average power across a presumed prior distribution of the effect size parameters. Thus, assurance adds a Bayesian element to the frequentist framework, resulting in a hybrid approach to the probability of trial or study success. It should be noted that when it comes time to perform the statistical test on the resulting data, these methods for calculating assurance assume that the traditional (frequentist) tests will be used.

The next section describes some of the ways in which the prior distributions for effect size parameters may be determined.

## Elicitation

In order to calculate assurance, a suitable prior distribution for the effect size parameters must be determined. This process is called the *elicitation* of the prior distribution.

The elicitation may be as simple as choosing a distribution that seems plausible for the parameter(s) of interest, or as complex as combining the informed advice of several experts based on experience in the field, available pilot data, or previous studies. The accuracy of the assurance value depends on the accuracy of the elicited prior distribution. The assumption (or hope) is that an informed prior distribution will produce a more accurate estimate of the probability of trial success than a single value estimate. Because clinical trials and other studies are often costly, many institutions now routinely require an elicitation step.

Two reference texts that focus on elicitation are O'Hagan, Buck, Daneshkhah, Eiser, Garthwaite, Jenkinson, Oakley, and Rakow (2006) and Dias, Morton, and Quigley (2018).

## Logrank Tests

Our formulation comes from Campbell and Walters (2014). Denote an observation by  $Y_{ijk}$  where  $i = 1, 2$  gives the group,  $j = 1, 2, \dots, K_i$  gives the cluster within group  $i$ , and  $k = 1, 2, \dots, m_{ij}$  denotes an individual in cluster  $j$  of group  $i$ . In this chapter, we will assume that group 1 is the control group and group 2 is the treatment group.

Let  $\rho$  denote the intraclass correlation coefficient (ICC) among individuals from the same cluster. This correlation is the correlation of censor indicator variable.

The formula for power is found by inverting the following sequence of formulas for sample size. Freedman (1982) showed that the number of events,  $e$ , needed for a power of  $1 - \beta$  and a two-sided significance level of  $\alpha$  to detect a hazard ratio of  $HR$  ( $h_2 / h_1$ ) is given by

$$e = (z_{1-\alpha/2} + z_{1-\beta})^2 \frac{(1 + r HR)^2}{r(1 - HR)^2}$$

where  $r = N_2 / N_1$  and  $z_x = \Phi(x)$  is the standard normal distribution function. Note that for exponential survival,  $HR$  is related to  $S_1$  and  $S_2$ , the probabilities of survival (non-events) in the two groups, by

$$HR = \frac{\ln(S_2)}{\ln(S_1)}$$

Xie and Waksman (2003) showed that, in cluster trials, the above formula could be generalized to give the number of events needed in a cluster-randomized trial as follows.

$$e_c = e(1 + (\bar{M} - 1)\rho)$$

where  $\bar{M}$  is the average cluster size of all clusters given by

$$\bar{M} = \frac{K_1 M_1 + K_2 M_2}{K_1 + K_2}$$

The total number of subjects required is given by

$$N = N_1 + N_2 = (K_1 + K_2)\bar{M} = \frac{e_c(1 + r)}{1 - S_1 + r(1 - S_2)}$$

## Assurance Calculation

The assurance computation described here is based on O'Hagan, Stevens, and Campbell (2005).

Let  $P_p(H|S_1, S_2, \rho, M_1, M_2)$  be the power function described above where  $H$  is the event that null hypothesis is rejected conditional on the values of the parameters. The specification of the parameters is critical to the power calculation, but the actual values are seldom known. Assurance is defined as the expected power where the expectation is with respect to a joint prior distribution for the parameters. Hence, the definition of assurance is

$$\begin{aligned} \text{Assurance} &= E_{S_1, S_2, \rho, M_1, M_2} \left( P_p(H|S_1, S_2, \rho, M_1, M_2) \right) \\ &= \int \int \int \int \int P_p(H|S_1, S_2, \rho, M_1, M_2) f(S_1, S_2, \rho, M_1, M_2) dS_1 \dots dM_2 \end{aligned}$$

where  $f(S_1, S_2, \rho, M_1, M_2)$  is the joint prior distribution of the parameters.

In **PASS**, the joint prior distribution can be specified as either a discrete approximation to the joint prior distribution, or as individual prior distributions, one for each parameter.

### Specifying a Joint Prior Distribution

If the joint prior distribution is to be specified directly, the distribution is specified in **PASS** using a discrete approximation to the function  $f(S_1, S_2, \rho, M_1, M_2)$ . This provides flexibility in specifying the joint prior distribution. In the five-parameter case, six columns are entered on the spreadsheet: five for the parameters and one more for the probability. Each row gives a value for each parameter and the corresponding parameter-combination probability. The accuracy of the distribution approximation is controlled by the number of points (spreadsheet rows) that are used.

An example of entering a joint prior distribution is included at the end of the chapter.

### Specifying Individual Prior Distributions

Ciarleglio, Arendt, and Peduzzi (2016) suggest that more flexibility is available if the joint prior distribution is separated into two independent distributions as follows:

$$f(S_1, S_2, \rho, M_1, M_2) = f_1(S_1)f_2(S_2)f_3(\rho)f_4(M_1)f_5(M_2)$$

where  $f_1(S_1)$  is the prior distribution of  $S_1$  and so forth. This method is also available in **PASS**. In this case, the definition of assurance becomes

$$\begin{aligned} \text{Assurance} &= E_{S_1, S_2, \rho, M_1, M_2} \left( P_p(H|S_1, S_2, \rho, M_1, M_2) \right) \\ &= \int \int \int \int \int P_p(H|S_1, S_2, \rho, M_1, M_2) f_1(S_1)f_2(S_2)f_3(\rho)f_4(M_1)f_5(M_2) dS_1 \dots dM_2 \end{aligned}$$

Using this definition, the assurance can be calculated using numerical integration. There are a variety of pre-programmed, univariate prior distributions available in **PASS**.

## Fixed Values (No Prior) and Custom Values

For any given parameter, **PASS** also provides the option of entering a single fixed value for the prior distribution, or a series of values and corresponding probabilities (using the spreadsheet), rather than one of the pre-programmed distributions.

## Numerical Integration in PASS (and Notes on Computation Speed)

When the prior distribution is specified as independent univariate distributions, **PASS** uses a numerical integration algorithm to compute the assurance value as follows:

The domain of each prior distribution is divided into  $M$  intervals. Since many of the available prior distributions are unbounded on one (e.g., Gamma) or both (e.g., Normal) ends, an approximation is made to make the domain finite. This is accomplished by truncating the distribution to a domain between the two quantiles:  $q_{0.001}$  and  $q_{0.999}$ .

The value of  $M$  controls the accuracy and speed of the algorithm. If only one parameter is to be given a prior distribution, then a value of  $M$  between 50 and 100 usually gives an accurate result in a timely manner. However, if two parameters are given priors, the number of iterations needed increases from  $M$  to  $M^2$ . For example, if  $M$  is 100, 10000 iterations are needed. Reducing  $M$  from 100 to 50 reduces the number of iterations from 10000 to 2500.

The algorithm runtime increases when searching for sample size rather than solving for assurance, as a search algorithm is employed in this case. When solving for sample size, we recommend reducing  $M$  to 20 or less while exploring various scenarios, and then increasing  $M$  to 50 or more for a final, more accurate, result.

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## List of Available Univariate Prior Distributions

This section lists the univariate prior distributions that may be used for any of the applicable parameters when the Prior Entry Method is set to Individual.

### No Prior

If 'No Prior' is chosen for a parameter, the parameter is assumed to take on a single, fixed value with probability one.

## Beta (Shape 1, Shape 2, a, c)

A random variable  $X$  that follows the beta distribution is defined on a finite interval  $[a, c]$ . Two shape parameters ( $\alpha$  and  $\beta$ ) control the shape of this distribution. Two location parameters  $a$  and  $c$  give the minimum and maximum of  $X$ .

The probability density function of the beta distribution is

$$f(x|\alpha, \beta, a, c) = \frac{\left(\frac{x-a}{c-a}\right)^{\alpha-1} \left(\frac{c-x}{c-a}\right)^{\beta-1}}{(c-a)B(\alpha, \beta)}$$

where  $B(\alpha, \beta) = \Gamma(\alpha) \Gamma(\beta) / \Gamma(\alpha + \beta)$  and  $\Gamma(z)$  is the gamma function.

The mean of  $X$  is

$$\mu_X = \frac{\alpha c + \beta a}{\alpha + \beta}$$

Various distribution shapes are controlled by the values of  $\alpha$  and  $\beta$ . These include

### Symmetric and Unimodal

$$\alpha = \beta > 1$$

### U Shaped

$$\alpha = \beta < 1$$

### Bimodal

$$\alpha, \beta < 1$$

### Uniform

$$\alpha = \beta = 1$$

### Parabolic

$$\alpha = \beta = 2$$

### Bell-Shaped

$$\alpha = \beta > 2$$

## Gamma (Shape, Scale)

A random variable  $X$  that follows the gamma distribution is defined on the interval  $(0, \infty)$ . A shape parameter,  $\kappa$ , and a scale parameter,  $\theta$ , control the distribution.

The probability density function of the gamma distribution is

$$f(x|\kappa, \theta) = \frac{x^{\kappa-1} e^{-\frac{x}{\theta}}}{\theta^{\kappa} \Gamma(\kappa)}$$

where  $\Gamma(z)$  is the gamma function.

The mean of  $X$  is

$$\mu_X = \frac{\kappa}{\theta}$$

A truncated version of the distribution is constructed by dividing the density by  $1 - \text{Prob}(\text{Min} \leq X \leq \text{Max})$  where  $\text{Min}$  and  $\text{Max}$  are two truncation bounds.

## Inverse-Gamma (Shape, Scale)

A random variable  $X$  that follows the inverse-gamma distribution is defined on the interval  $(0, \infty)$ . If  $Y \sim \text{gamma}$ , then  $X = 1 / Y \sim \text{inverse-gamma}$ . A shape parameter,  $\alpha$ , and a scale parameter,  $\beta$ , control the distribution.

The probability density function of the inverse-gamma distribution is

$$f(x|\alpha, \beta) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\frac{\beta}{x}}}{\Gamma(\alpha)}$$

where  $\Gamma(z)$  is the gamma function.

The mean of  $X$  is

$$\mu_X = \frac{\beta}{\alpha - 1} \text{ for } \alpha > 1$$

A truncated version of the distribution is constructed by dividing the density by  $1 - \text{Prob}(\text{Min} \leq X \leq \text{Max})$  where  $\text{Min}$  and  $\text{Max}$  are two truncation bounds.

## Logistic (Location, Scale)

A random variable  $X$  that follows the logistic distribution is defined on the interval  $(-\infty, \infty)$ . A location parameter,  $\mu$ , and a scale parameter,  $s$ , control the distribution.

The probability density function of the logistic distribution is

$$f(x|\mu, s) = \frac{e^{-\frac{x-\mu}{s}}}{s \left(1 + e^{-\frac{x-\mu}{s}}\right)^2}$$

The mean of  $X$  is

$$\mu_X = \mu$$

A truncated version of the distribution is constructed by dividing the density by  $1 - \text{Prob}(Min \leq X \leq Max)$  where  $Min$  and  $Max$  are two truncation bounds.

## Lognormal (Mean, SD)

A random variable  $X$  that follows the lognormal distribution is defined on the interval  $(0, \infty)$ . A location parameter,  $\mu_{\log(X)}$ , and a scale parameter,  $\sigma_{\log(X)}$ , control the distribution. If  $Z \sim$  standard normal, then  $X = e^{\mu + \sigma Z} \sim$  lognormal. Note that  $\mu_{\log(X)} = E(\log(X))$  and  $\sigma_{\log(X)} = \text{Standard Deviation}(\log(X))$ .

The probability density function of the lognormal distribution is

$$f(x|\mu, \sigma) = \frac{e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2}}{x\sigma\sqrt{2\pi}}$$

The mean of  $X$  is

$$\mu_X = e^{\mu + \frac{\sigma^2}{2}}$$

A truncated version of the distribution is constructed by dividing the density by  $1 - \text{Prob}(Min \leq X \leq Max)$  where  $Min$  and  $Max$  are two truncation bounds.

## LogT (Mean, SD)

A random variable  $X$  that follows the logT distribution is defined on the interval  $(0, \infty)$ . A location parameter,  $\mu_{\log(X)}$ , a scale parameter,  $\sigma_{\log(X)}$ , and a shape parameter,  $\nu$ , control the distribution. Note that  $\nu$  is referred to as the *degrees of freedom*.

If  $t \sim \text{Student's } t$ , then  $X = e^{\mu + \sigma t} \sim \text{logT}$ .

The probability density function of the logT distribution is

$$f(x|\mu, \sigma, \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{x\Gamma\left(\frac{\nu}{2}\right)\sigma\sqrt{\nu\pi}} \left(1 + \frac{1}{\nu} \left(\frac{\log x - \mu}{\sigma}\right)^2\right)^{\left(\frac{-\nu-1}{2}\right)}$$

The mean of  $X$  is not defined.

A truncated version of the distribution is constructed by dividing the density by  $1 - \text{Prob}(\text{Min} \leq X \leq \text{Max})$  where  $\text{Min}$  and  $\text{Max}$  are two truncation bounds.

## Normal (Mean, SD)

A random variable  $X$  that follows the normal distribution is defined on the interval  $(-\infty, \infty)$ . A location parameter,  $\mu$ , and a scale parameter,  $\sigma$ , control the distribution.

The probability density function of the normal distribution is

$$f(x|\mu, \sigma) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

The mean of  $X$  is

$$\mu_X = \mu$$

A truncated version of the distribution is constructed by dividing the density by  $1 - \text{Prob}(\text{Min} \leq X \leq \text{Max})$  where  $\text{Min}$  and  $\text{Max}$  are two truncation bounds.

## T (Mean, SD, DF)

A random variable  $X$  that follows Student's  $t$  distribution is defined on the interval  $(-\infty, \infty)$ . A location parameter,  $\mu$ , a scale parameter,  $\sigma$ , and a shape parameter,  $\nu$ , control the distribution. Note that  $\nu$  is referred to as the *degrees of freedom* or *DF*.

The probability density function of the Student's  $t$  distribution is

$$f(x|\mu, \sigma, \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sigma\sqrt{\nu\pi}} \left(1 + \frac{1}{\nu} \left(\frac{x-\mu}{\sigma}\right)^2\right)^{\left(\frac{-\nu-1}{2}\right)}$$

The mean of  $X$  is  $\mu$  if  $\nu > 1$ .

A truncated version of the distribution is constructed by dividing the density by  $1 - \text{Prob}(\text{Min} \leq X \leq \text{Max})$  where  $\text{Min}$  and  $\text{Max}$  are two truncation bounds.



### Triangle (Mode, Min, Max)

Let  $a$  = minimum,  $b$  = maximum, and  $c$  = mode. A random variable  $X$  that follows a triangle distribution is defined on the interval  $(a, b)$ .

The probability density function of the triangle distribution is

$$f(x|a, b, c) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & \text{for } a \leq x < c \\ \frac{2}{b-a} & \text{for } x = c \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{for } c < x \leq b \end{cases}$$

The mean of  $X$  is

$$\frac{a + b + c}{3}$$

### Uniform (Min, Max)

Let  $a$  = minimum and  $b$  = maximum. A random variable  $X$  that follows a uniform distribution is defined on the interval  $[a, b]$ .

The probability density function of the uniform distribution is

$$f(x|a, b) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \end{cases}$$

The mean of  $X$  is

$$\frac{a + b}{2}$$

### Weibull (Shape, Scale)

A random variable  $X$  that follows the Weibull distribution is defined on the interval  $(0, \infty)$ . A shape parameter,  $\kappa$ , and a scale parameter,  $\lambda$ , control the distribution.

The probability density function of the Weibull distribution is

$$f(x|\kappa, \lambda) = \frac{\kappa}{\lambda} \left(\frac{x}{\lambda}\right)^{\kappa-1} e^{-\left(\frac{x}{\lambda}\right)^{\kappa}}$$

The mean of  $X$  is

$$\mu_X = \kappa \Gamma\left(1 + \frac{1}{\kappa}\right)$$

A truncated version of the distribution is constructed by dividing the density by  $1 - \text{Prob}(\text{Min} \leq X \leq \text{Max})$  where  $\text{Min}$  and  $\text{Max}$  are two truncation bounds.

## Custom (Values and Probabilities in Spreadsheet)

This custom prior distribution is represented by a set of user-specified points and associated probabilities, entered in two columns of the spreadsheet. The points make up the entire set of values that are used for this parameter in the calculation of assurance. The associated probabilities should sum to one. Note that custom values and probabilities can be used to approximate any continuous distribution.

For example, a prior distribution of X might be

$X_i$	$P_i$
10	0.2
20	0.2
30	0.3
40	0.2
50	0.1

In this example, the mean of X is

$$\mu_X = \sum_{i=1}^5 X_i P_i$$

## Example 1 – Assurance Over a Range of Sample Sizes

Suppose that a cluster randomized study is being planned in which  $S1 = 0.5$ ;  $S2 = 0.6$ ;  $\rho = 0.02$ ;  $M1$  and  $M2 = 7$ ;  $\alpha = 0.05$ ; and  $K1$  and  $K2$  is in the range 20 to 100. Power is to be calculated for a two-sided logrank test.

To complete their sample size study, the researchers want to run an assurance analysis for a range of group sample sizes from 20 to 100. An elicitation exercise determined that the prior distribution of the  $S1$  should be normal with mean 0.5 and standard deviation 0.03, the prior distribution of  $S2$  should be a normal with mean 0.6 and standard deviation 0.05, the prior distribution of the ICC should be a normal with mean 0.02 and standard deviation 0.004, the prior distribution of the cluster size of group 1 should be a normal with mean 7 and standard deviation 1.5, and the prior distribution of the cluster size of group 2 should be a normal with mean 7 and standard deviation 1.5.

To reduce the runtime of this example, the number of computation points is set to 10. In practice, we recommend setting this value to 20 or more for the most accurate results.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Assurance</b>
Prior Entry Method .....	<b>Individual (Enter a prior distribution for each applicable parameter)</b>
Alternative Hypothesis .....	<b>Two-Sided (H1: <math>S1 \neq S2</math>)</b>
Alpha.....	<b>0.05</b>
K1 (Number of Clusters) .....	<b>20 40 60 80 100</b>
Prior Distribution of M1 .....	<b>Normal (Mean, SD)</b>
Mean.....	<b>7</b>
SD.....	<b>1.5</b>
Truncation Boundaries.....	<b>None</b>
K2 (Number of Clusters) .....	<b>K1</b>
Prior Distribution of M2 .....	<b>Normal (Mean, SD)</b>
Mean.....	<b>7</b>
SD.....	<b>1.5</b>
Truncation Boundaries.....	<b>None</b>
Prior Distribution of S1 .....	<b>Normal (Mean, SD)</b>
Mean.....	<b>0.5</b>
SD.....	<b>0.03</b>
Truncation Boundaries.....	<b>None</b>
Prior Distribution of S2 .....	<b>Normal (Mean, SD)</b>
Mean.....	<b>0.6</b>
SD.....	<b>0.05</b>
Truncation Boundaries.....	<b>None</b>

## Assurance for Logrank Tests in a Cluster-Randomized Design

Prior Distribution of  $\rho$  ..... **Normal (Mean, SD)**Mean ..... **0.02**SD ..... **0.004**Truncation Boundaries ..... **None**

## Options Tab

Number of Computation Points for each ..... **10**

Prior Distribution

Maximum K1 in Sample Size Search ..... **1000**

## Output

Click the Calculate button to perform the calculations and generate the following output.

## Numeric Reports

## Numeric Results

Solve For: [Assurance](#)

Groups: 1 = Treatment, 2 = Control

Test Statistic: Logrank Test

Hypotheses:  $H_0: S_1 - S_2 = 0$  vs.  $H_1: S_1 - S_2 \neq 0$ 

Prior Type: Individual Univariate Distributions

## Prior Distributions

M1: Normal (Mean = 7, SD = 1.5).

M2: Normal (Mean = 7, SD = 1.5).

S1: Normal (Mean = 0.5, SD = 0.03).

S2: Normal (Mean = 0.6, SD = 0.05).

 $\rho$ : Normal (Mean = 0.02, SD = 0.004).

Assurance*	Power†	Number of Events		Sample Size		Number of Clusters			Cluster Size		Proportion Surviving		Intraclass Correlation E( $\rho$ )	Alpha
		E1   E2	E	N1   N2	N	K1	K2	K	E(M1)	E(M2)	E(S1)	E(S2)		
0.39400	0.36194	63   63	126	140   140	466	20	20	40	7	7	0.5	0.6	0.02	0.05
0.57040	0.62231	126   126	252	280   280	932	40	40	80	7	7	0.5	0.6	0.02	0.05
0.66495	0.79447	189   189	378	420   420	1398	60	60	120	7	7	0.5	0.6	0.02	0.05
0.72252	0.89479	252   252	504	560   560	1862	80	80	160	7	7	0.5	0.6	0.02	0.05
0.76105	0.94862	315   315	630	700   700	2328	100	100	200	7	7	0.5	0.6	0.02	0.05

\* The number of points used for computation of the prior(s) was 10.

† Power was calculated using  $M1 = E(M1) = 7$ ,  $M2 = E(M2) = 7$ ,  $S1 = E(S1) = 0.5$ ,  $S2 = E(S2) = 0.6$ , and  $\rho = E(\rho) = 0.02$ .

Assurance	The expected power where the expectation is with respect to the prior distribution(s).
Power	The power calculated using the means of the prior distributions as the values of the corresponding parameters.
E1 and E2	The required number of events in groups 1 and 2, respectively.
E	The total number of events required to achieve the indicated power level.
N1 and N2	The number of subjects in groups 1 and 2, respectively. $N1 = K1 \times M1$ . $N2 = K2 \times M2$ .
N	The total sample size. $N = N1 + N2$ .
K1	The number of clusters in group 1.
K2	The number of clusters in group 2.
K	The total number of clusters.
E(M1)	The expected average number of items (subjects) per cluster in group 1.
E(M2)	The expected average number of items (subjects) per cluster in group 2.
E(S1)	The expected proportion surviving (no events) in group 1, the control group.
E(S2)	The expected proportion surviving (no events) in group 2, the treatment group.
E( $\rho$ )	The expected intraclass correlation (ICC). It is the expected correlation between two censor-indicator variables in the same cluster.
Alpha	The probability of rejecting a true null hypothesis.

## Assurance for Logrank Tests in a Cluster-Randomized Design

**Summary Statements**

A parallel two-group cluster-randomized design will be used to test whether the Group 1 (treatment) proportion surviving ( $S_1$ ) is different from the Group 2 (control) proportion surviving ( $S_2$ ) ( $H_0: S_1 - S_2 = 0$  versus  $H_1: S_1 - S_2 \neq 0$ ). The comparison will be made using a two-sided logrank test with a Type I error rate ( $\alpha$ ) of 0.05. The prior distribution used for the average cluster size in Group 1 is Normal (Mean = 7, SD = 1.5). The prior distribution used for the average cluster size in Group 2 is Normal (Mean = 7, SD = 1.5). The prior distribution used for the proportion surviving in Group 1 is Normal (Mean = 0.5, SD = 0.03). The prior distribution used for the proportion surviving in Group 2 is Normal (Mean = 0.6, SD = 0.05). The prior distribution used for the intracluster correlation coefficient is Normal (Mean = 0.02, SD = 0.004). With 20 clusters (140 total subjects) in Group 1 (treatment) and 20 clusters (140 total subjects) in Group 2 (control), the assurance (average power) is 0.394.

**References**

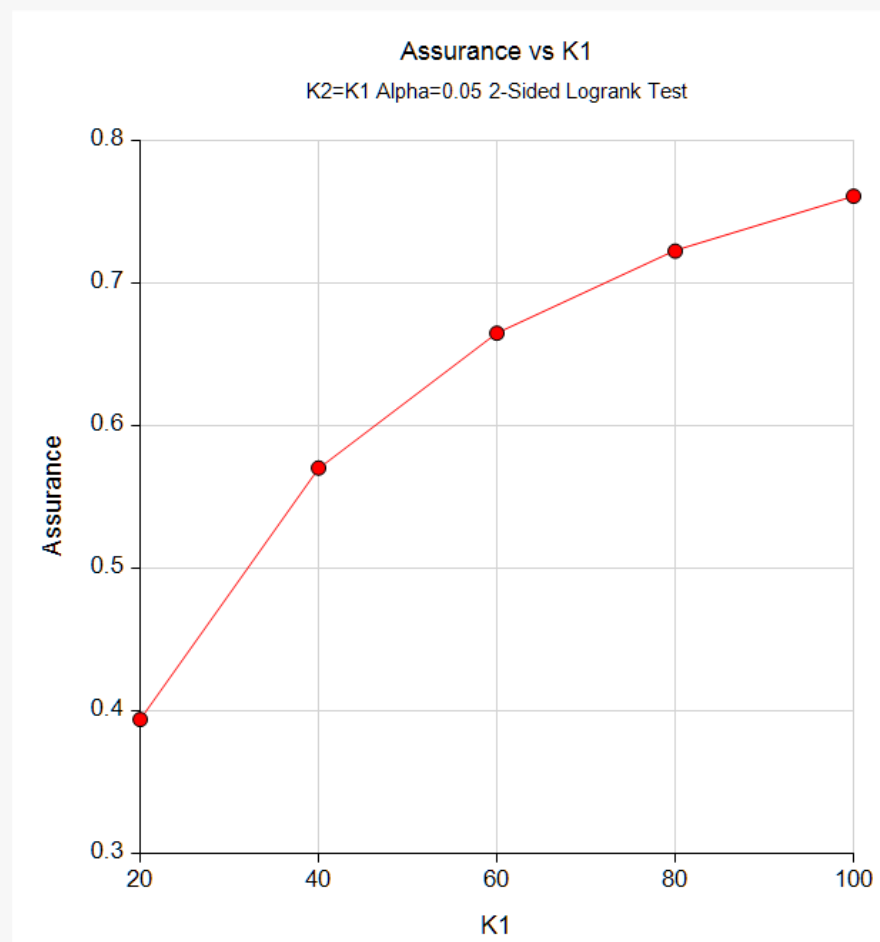
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- Gao, F., Earnest, A., Matchar, D.B., Campbell, M.J., and Machin, D. 2015. 'Sample size calculations for the design of cluster randomized trials: A summary of methodology.' *Contemporary Clinical Trials*, Volume 42, pages 41-50.

This report shows the assurance values obtained by the various sample sizes.

## Assurance for Logrank Tests in a Cluster-Randomized Design

## Plots Section

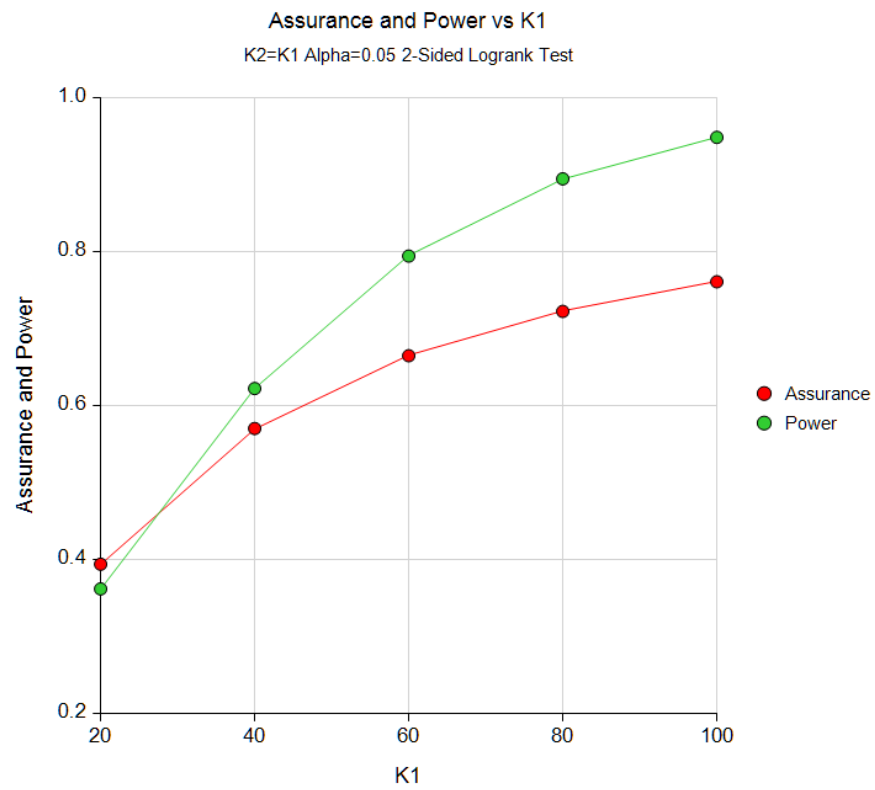
## Plots



This plot shows the relationship between the assurance and sample size.

## Comparison Plots Section

### Comparison Plots



This plot compares the assurance and power across values of sample size.

## Example 2 – Validation using Hand Computation

We could not find a validation example in the literature for procedure, so we have developed a validation example of our own.

Suppose a two-sided logrank test will be used in which  $K1 = K2 = 40$  and the significance level is 0.05.

The prior distribution of S1 is approximated by the following table. These are loaded into C1 and C2.

<b><u>S1</u></b>	<b><u>Prob</u></b>
<b>0.5</b>	<b>0.4</b>
<b>0.55</b>	<b>0.6</b>

The prior distribution of the S2 is approximated by the following table. These are loaded into C3 and C4.

<b><u>S2</u></b>	<b><u>Prob</u></b>
<b>0.6</b>	<b>0.4</b>
<b>0.7</b>	<b>0.6</b>

The prior distribution of the p is approximated by the following table. These are loaded into C5 and C6.

<b><u>p</u></b>	<b><u>Prob</u></b>
<b>0.01</b>	<b>0.5</b>
<b>0.02</b>	<b>0.5</b>

The prior distribution of the M1 is approximated by the following table. These are loaded into C7 and C8.

<b><u>M1</u></b>	<b><u>Prob</u></b>
<b>7</b>	<b>0.5</b>
<b>9</b>	<b>0.5</b>

The prior distribution of the M2 is approximated by the following table. These are loaded into C9 and C10.

<b><u>M2</u></b>	<b><u>Prob</u></b>
<b>7</b>	<b>0.5</b>
<b>9</b>	<b>0.5</b>

To run this example, the spreadsheet will need to be loaded with the following 10 columns corresponding to the values listed above.

<b><u>C1</u></b>	<b><u>C2</u></b>	<b><u>C3</u></b>	<b><u>C4</u></b>	<b><u>C5</u></b>	<b><u>C6</u></b>	<b><u>C7</u></b>	<b><u>C8</u></b>	<b><u>C9</u></b>	<b><u>C10</u></b>
<b>0.5</b>	<b>0.4</b>	<b>0.6</b>	<b>0.4</b>	<b>0.01</b>	<b>0.5</b>	<b>7</b>	<b>0.5</b>	<b>7</b>	<b>0.5</b>
<b>0.55</b>	<b>0.6</b>	<b>0.7</b>	<b>0.6</b>	<b>0.02</b>	<b>0.5</b>	<b>9</b>	<b>0.5</b>	<b>9</b>	<b>0.5</b>



## Assurance for Logrank Tests in a Cluster-Randomized Design

The *Logrank Tests in a Cluster-Randomized Design* procedure is used to compute the power for each of the 32 combinations the parameters. Note that the power calculation used by this procedure ignores the negligible component of the two-sided tests, so the power values are slightly less than those calculated by the more complete formula used in this procedure. The results of these calculations are shown next.

## Numeric Results

Solve For: **Power**  
 Test Type: Logrank Test  
 Groups: 1 = Control, 2 = Treatment  
 Alternative Hypothesis: Two-Sided

Power	Number of Clusters			Cluster Size		Sample Size		Number of Events		Hazard Ratio HR	Proportion Surviving		ICC $\rho$	Alpha
	K1	K2	K	M1	M2	N1	N2	E1	E2		S1	S2		
0.6461	40	40	80	7	7	280	280	126.0	126.0	0.7370	0.50	0.6	0.01	0.05
0.6223	40	40	80	7	7	280	280	126.0	126.0	0.7370	0.50	0.6	0.02	0.05
0.9965	40	40	80	7	7	280	280	112.0	112.0	0.5146	0.50	0.7	0.01	0.05
0.9950	40	40	80	7	7	280	280	112.0	112.0	0.5146	0.50	0.7	0.02	0.05
0.2165	40	40	80	7	7	280	280	119.0	119.0	0.8545	0.55	0.6	0.01	0.05
0.2073	40	40	80	7	7	280	280	119.0	119.0	0.8545	0.55	0.6	0.02	0.05
0.9448	40	40	80	7	7	280	280	105.0	105.0	0.5966	0.55	0.7	0.01	0.05
0.9331	40	40	80	7	7	280	280	105.0	105.0	0.5966	0.55	0.7	0.02	0.05
0.7037	40	40	80	7	9	280	360	124.3	159.8	0.7370	0.50	0.6	0.01	0.05
0.6762	40	40	80	7	9	280	360	124.3	159.8	0.7370	0.50	0.6	0.02	0.05
0.9990	40	40	80	7	9	280	360	108.5	139.5	0.5146	0.50	0.7	0.01	0.05
0.9983	40	40	80	7	9	280	360	108.5	139.5	0.5146	0.50	0.7	0.02	0.05
0.2386	40	40	80	7	9	280	360	118.1	151.9	0.8545	0.55	0.6	0.01	0.05
0.2267	40	40	80	7	9	280	360	118.1	151.9	0.8545	0.55	0.6	0.02	0.05
0.9691	40	40	80	7	9	280	360	102.4	131.6	0.5966	0.55	0.7	0.01	0.05
0.9598	40	40	80	7	9	280	360	102.4	131.6	0.5966	0.55	0.7	0.02	0.05
0.6829	40	40	80	9	7	360	280	164.3	127.8	0.7370	0.50	0.6	0.01	0.05
0.6554	40	40	80	9	7	360	280	164.3	127.8	0.7370	0.50	0.6	0.02	0.05
0.9978	40	40	80	9	7	360	280	148.5	115.5	0.5146	0.50	0.7	0.01	0.05
0.9965	40	40	80	9	7	360	280	148.5	115.5	0.5146	0.50	0.7	0.02	0.05
0.2339	40	40	80	9	7	360	280	154.1	119.9	0.8545	0.55	0.6	0.01	0.05
0.2223	40	40	80	9	7	360	280	154.1	119.9	0.8545	0.55	0.6	0.02	0.05
0.9577	40	40	80	9	7	360	280	138.4	107.6	0.5966	0.55	0.7	0.01	0.05
0.9463	40	40	80	9	7	360	280	138.4	107.6	0.5966	0.55	0.7	0.02	0.05
0.7463	40	40	80	9	9	360	360	162.0	162.0	0.7370	0.50	0.6	0.01	0.05
0.7160	40	40	80	9	9	360	360	162.0	162.0	0.7370	0.50	0.6	0.02	0.05
0.9995	40	40	80	9	9	360	360	144.0	144.0	0.5146	0.50	0.7	0.01	0.05
0.9990	40	40	80	9	9	360	360	144.0	144.0	0.5146	0.50	0.7	0.02	0.05
0.2614	40	40	80	9	9	360	360	153.0	153.0	0.8545	0.55	0.6	0.01	0.05
0.2466	40	40	80	9	9	360	360	153.0	153.0	0.8545	0.55	0.6	0.02	0.05
0.9791	40	40	80	9	9	360	360	135.0	135.0	0.5966	0.55	0.7	0.01	0.05
0.9709	40	40	80	9	9	360	360	135.0	135.0	0.5966	0.55	0.7	0.02	0.05

The assurance calculation is made by summing the quantities

$$[(power_{i,j,k,l,m})p(S1_i)p(S2_j)p(\rho_k)p(M1_l)p(M2_m)]$$

as follows

$$\begin{aligned}
 Assurance &= (0.6461 \times 0.4 \times 0.4 \times 0.5 \times 0.5 \times 0.5) + (0.7037 \times 0.4 \times 0.4 \times 0.5 \times 0.5 \times 0.5) + \dots \\
 &\quad + (0.9709 \times 0.6 \times 0.6 \times 0.5 \times 0.5 \times 0.5) \\
 &= 0.74876.
 \end{aligned}$$

## Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

### Design Tab

Solve For ..... **Assurance**  
 Prior Entry Method ..... **Individual (Enter a prior distribution for each applicable parameter)**  
 Alternative Hypothesis ..... **Two-Sided ( $H_1: S_1 \neq S_2$ )**  
 Alpha ..... **0.05**  
 K1 (Number of Clusters) ..... **40**  
 Prior Distribution of M1 ..... **Custom (Values and Probabilities in Spreadsheet)**  
 Column of Values ..... **C7**  
 Column of Pr(Values) ..... **C8**  
 K2 (Number of Clusters) ..... **K1**  
 Prior Distribution of M2 ..... **Custom (Values and Probabilities in Spreadsheet)**  
 Column of Values ..... **C9**  
 Column of Pr(Values) ..... **C10**  
 Prior Distribution of S1 ..... **Custom (Values and Probabilities in Spreadsheet)**  
 Column of Values ..... **C1**  
 Column of Pr(Values) ..... **C2**  
 Prior Distribution of S2 ..... **Custom (Values and Probabilities in Spreadsheet)**  
 Column of Values ..... **C3**  
 Column of Pr(Values) ..... **C4**  
 Prior Distribution of  $\rho$  ..... **Custom (Values and Probabilities in Spreadsheet)**  
 Column of Values ..... **C5**  
 Column of Pr(Values) ..... **C6**

### Options Tab

Number of Computation Points for each ..... **10**  
 Prior Distribution  
 Maximum K1 in Sample Size Search ..... **1000**

### Input Spreadsheet Data

Row	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	0.50	0.4	0.6	0.4	0.01	0.5	7	0.5	7	0.5
2	0.55	0.6	0.7	0.6	0.02	0.5	9	0.5	9	0.5

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

Solve For: [Assurance](#)  
 Groups: 1 = Treatment, 2 = Control  
 Test Statistic: Logrank Test  
 Hypotheses:  $H_0: S_1 - S_2 = 0$  vs.  $H_1: S_1 - S_2 \neq 0$   
 Prior Type: Individual Univariate Distributions

### Prior Distributions

M1: Point List (Values = C7, Probs = C8).  
 C7: 7 9  
 C8: 0.5 0.5  
 M2: Point List (Values = C9, Probs = C10).  
 C9: 7 9  
 C10: 0.5 0.5  
 S1: Point List (Values = C1, Probs = C2).  
 C1: 0.5 0.55  
 C2: 0.4 0.6  
 S2: Point List (Values = C3, Probs = C4).  
 C3: 0.6 0.7  
 C4: 0.4 0.6  
 p: Point List (Values = C5, Probs = C6).  
 C5: 0.01 0.02  
 C6: 0.5 0.5

Assurance	Power‡	Number of Events		Sample Size		Number of Clusters			Cluster Size		Proportion Surviving		Intraclass Correlation E( $\rho$ )	Alpha
		E1   E2	E	N1   N2	N	K1	K2	K	E(M1)	E(M2)	E(S1)	E(S2)		
0.74894	0.89224	130   130	259	320   320	720	40	40	80	8	8	0.53	0.66	0.015	0.05

‡ Power was calculated using  $M1 = E(M1) = 8$ ,  $M2 = E(M2) = 8$ ,  $S1 = E(S1) = 0.53$ ,  $S2 = E(S2) = 0.66$ , and  $p = E(p) = 0.015$ .

**PASS** has calculated the assurance as 0.74894 which validates the procedure. This value is slightly larger than the 0.74876 which was computed using the slightly less accurate algorithm.

## Example 3 – Finding the Sample Size Needed to Achieve a Specified Assurance

Continuing with Example 1, the researchers want to investigate the sample sizes necessary to achieve assurances of 0.5, 0.6, and 0.7.

In order to reduce the runtime during this exploratory phase of the analysis, the number of points in the prior computation is reduced to 10. This slightly reduces the accuracy, but greatly reduces the runtime.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Sample Size</b>
Prior Entry Method .....	<b>Individual (Enter a prior distribution for each applicable parameter)</b>
Alternative Hypothesis .....	<b>Two-Sided (H1: S1 ≠ S2)</b>
Assurance .....	<b>0.5 0.6 0.7</b>
Alpha .....	<b>0.05</b>
Prior Distribution of M1 .....	<b>Normal (Mean, SD)</b>
Mean .....	<b>7</b>
SD .....	<b>1.5</b>
Truncation Boundaries .....	<b>None</b>
K2 (Number of Clusters) .....	<b>K1</b>
Prior Distribution of M2 .....	<b>Normal (Mean, SD)</b>
Mean .....	<b>7</b>
SD .....	<b>1.5</b>
Truncation Boundaries .....	<b>None</b>
Prior Distribution of S1 .....	<b>Normal (Mean, SD)</b>
Mean .....	<b>0.5</b>
SD .....	<b>0.03</b>
Truncation Boundaries .....	<b>None</b>
Prior Distribution of S2 .....	<b>Normal (Mean, SD)</b>
Mean .....	<b>0.6</b>
SD .....	<b>0.05</b>
Truncation Boundaries .....	<b>None</b>
Prior Distribution of $\rho$ .....	<b>Normal (Mean, SD)</b>
Mean .....	<b>0.02</b>
SD .....	<b>0.004</b>
Truncation Boundaries .....	<b>None</b>

## Assurance for Logrank Tests in a Cluster-Randomized Design

## Options Tab

Number of Computation Points for each ..... **10**

Prior Distribution

Maximum K1 in Sample Size Search..... **1000**

## Output

Click the Calculate button to perform the calculations and generate the following output.

## Numeric Reports

### Numeric Results

Solve For: [Sample Size](#)  
 Groups: 1 = Treatment, 2 = Control  
 Test Statistic: Logrank Test  
 Hypotheses:  $H_0: S_1 - S_2 = 0$  vs.  $H_1: S_1 - S_2 \neq 0$   
 Prior Type: Individual Univariate Distributions

#### Prior Distributions

M1: Normal (Mean = 7, SD = 1.5).  
 M2: Normal (Mean = 7, SD = 1.5).  
 S1: Normal (Mean = 0.5, SD = 0.03).  
 S2: Normal (Mean = 0.6, SD = 0.05).  
 $\rho$ : Normal (Mean = 0.02, SD = 0.004).

Assurance*	Power‡	Number of Events			Sample Size			Number of Clusters			Cluster Size		Proportion Surviving		Intracluster Correlation $E(\rho)$	Alpha
		E1   E2	E		N1   N2	N		K1	K2	K	E(M1)	E(M2)	E(S1)	E(S2)		
0.50579	0.51588	98   98	195		217   217	722		31	31	62	7	7	0.5	0.6	0.02	0.05
0.60446	0.68295	145   145	290		322   322	1072		46	46	92	7	7	0.5	0.6	0.02	0.05
0.70244	0.86161	227   227	454		504   504	1676		72	72	144	7	7	0.5	0.6	0.02	0.05

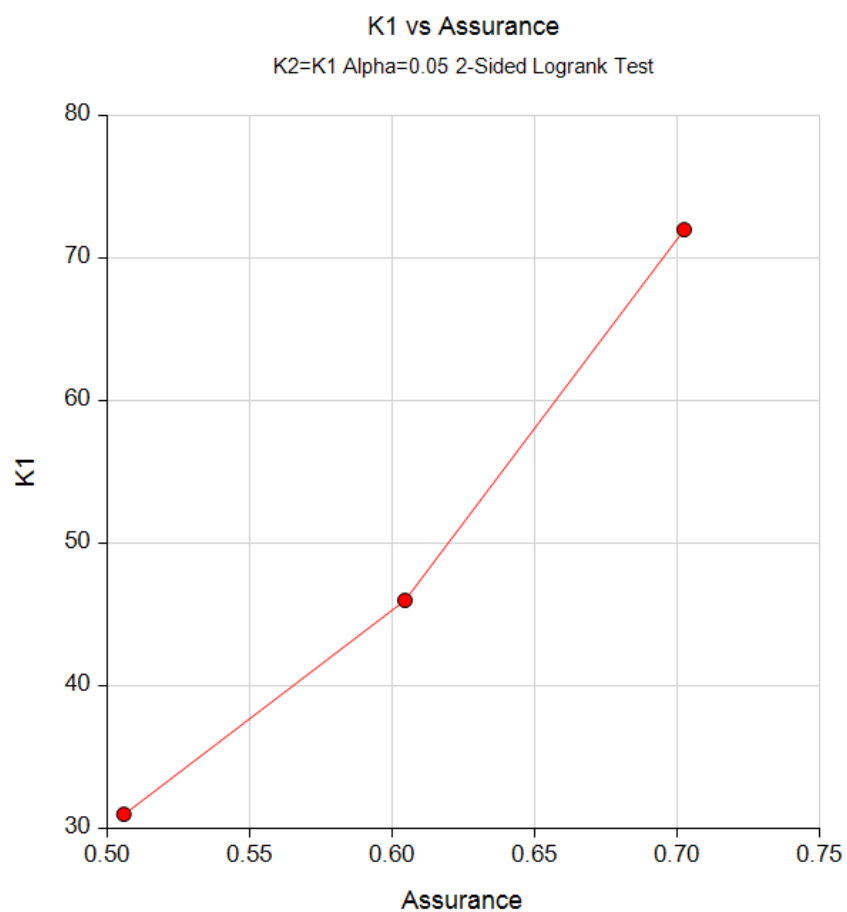
\* The number of points used for computation of the prior(s) was 10.

‡ Power was calculated using  $M1 = E(M1) = 7$ ,  $M2 = E(M2) = 7$ ,  $S1 = E(S1) = 0.5$ ,  $S2 = E(S2) = 0.6$ , and  $\rho = E(\rho) = 0.02$ .

This report shows the required sample size for each assurance target.

## Plots Section

### Plots



This plot shows the relationship between the sample size and assurance.

## Example 4 – Joint Prior Distribution

Suppose a two-sided, pooled z-test will be used in which  $K_1 = K_2 = (20, 40, 60, 80, 100)$  and the significance level is 0.05.

The joint prior distribution of the parameters is approximated by the following table. Note that the labels in parentheses identify the corresponding column of the spreadsheet. Also note that the prior probabilities in C6 will be automatically rescaled so that they sum to one.

<u>S1 (C1)</u>	<u>S2 (C2)</u>	<u>p (C3)</u>	<u>M1 (C4)</u>	<u>M2 (C5)</u>	<u>Prob (C6)</u>
0.5	0.7	0.01	5	5	0.25
0.5	0.7	0.02	10	10	0.2
0.45	0.68	0.01	5	5	0.25
0.45	0.68	0.02	10	10	0.2
0.5	0.65	0.01	5	5	0.65
0.5	0.65	0.02	10	10	0.6
0.45	0.62	0.01	5	5	0.65
0.45	0.62	0.02	10	10	0.6
0.5	0.6	0.01	5	5	0.45
0.5	0.6	0.02	10	10	0.4
0.45	0.58	0.01	5	5	0.45
0.45	0.58	0.02	10	10	0.4
0.5	0.55	0.01	5	5	0.25
0.5	0.55	0.02	10	10	0.2
0.45	0.53	0.01	5	5	0.25
0.45	0.53	0.02	10	10	0.2

To run this example, the spreadsheet will need to be loaded with the above data.

## Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

### Design Tab

Solve For ..... **Assurance**  
 Prior Entry Method ..... **Combined (Enter parameter values and probabilities on spreadsheet)**  
 Alternative Hypothesis ..... **Two-Sided (H1: S1 ≠ S2)**  
 Alpha ..... **0.05**  
 K1 (Number of Clusters) ..... **20 40 60 80 100**  
 Column of M1 Values ..... **C4**  
 K2 (Number of Clusters) ..... **K1**  
 Column of M2 Values ..... **C5**  
 Column of S1 Values ..... **C1**  
 Column of S2 Values ..... **C2**

## Assurance for Logrank Tests in a Cluster-Randomized Design

Column of p Values ..... **C3**Column of Pr(Values)..... **C6**

## Options Tab

Number of Computation Points for each..... **10**

Prior Distribution

Maximum K1 in Sample Size Search..... **1000**

## Input Spreadsheet Data

Row	C1	C2	C3	C4	C5	C6
1	0.50	0.70	0.01	5	5	0.25
2	0.50	0.70	0.02	10	10	0.20
3	0.45	0.68	0.01	5	5	0.25
4	0.45	0.68	0.02	10	10	0.20
5	0.50	0.65	0.01	5	5	0.65
6	0.50	0.65	0.02	10	10	0.60
7	0.45	0.62	0.01	5	5	0.65
8	0.45	0.62	0.02	10	10	0.60
9	0.50	0.60	0.01	5	5	0.45
10	0.50	0.60	0.02	10	10	0.40
11	0.45	0.58	0.01	5	5	0.45
12	0.45	0.58	0.02	10	10	0.40
13	0.50	0.55	0.01	5	5	0.25
14	0.50	0.55	0.02	10	10	0.20
15	0.45	0.53	0.01	5	5	0.25
16	0.45	0.53	0.02	10	10	0.20



## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

Solve For: [Assurance](#)  
 Groups: 1 = Treatment, 2 = Control  
 Test Statistic: Logrank Test  
 Hypotheses:  $H_0: S_1 - S_2 = 0$  vs.  $H_1: S_1 - S_2 \neq 0$   
 Prior Type: Joint Multivariate Distribution

### Prior Distribution

Point Lists

M1: C4: 5 10 5 10 5 10 5 10 5 10 5 10 5 10

M2: C5: 5 10 5 10 5 10 5 10 5 10 5 10 5 10

P1: C1: 0.5 0.5 0.45 0.45 0.5 0.5 0.45 0.45 0.5 0.5 0.45 0.45 0.5 0.5 0.45 0.45

P2: C2: 0.7 0.7 0.68 0.68 0.65 0.65 0.62 0.62 0.6 0.6 0.58 0.58 0.55 0.55 0.53 0.53

$\rho$ : C3: 0.01 0.02 0.01 0.02 0.01 0.02 0.01 0.02 0.01 0.02 0.01 0.02 0.01 0.02 0.01 0.02

Prob: C6: 0.25 0.2 0.25 0.2 0.65 0.6 0.65 0.6 0.45 0.4 0.45 0.4 0.25 0.2 0.25 0.2

Assurance	Power‡	Number of Events			Sample Size			Number of Clusters			Cluster Size		Proportion Surviving		Intracluster Correlation $E(\rho)$	Alpha
		E1   E2	E		N1   N2	N		K1	K2	K	E(M1)	E(M2)	E(S1)	E(S2)		
0.59657	0.64811	67   67	134		147   147	400		20	20	40	7.3	7.3	0.475	0.61625	0.01467	0.05
0.79245	0.91142	134   134	267		294   294	800		40	40	80	7.3	7.3	0.475	0.61625	0.01467	0.05
0.87063	0.98184	200   200	400		440   440	1200		60	60	120	7.3	7.3	0.475	0.61625	0.01467	0.05
0.90952	0.99674	267   267	533		587   587	1600		80	80	160	7.3	7.3	0.475	0.61625	0.01467	0.05
0.93220	0.99947	334   334	667		734   734	2000		100	100	200	7.3	7.3	0.475	0.61625	0.01467	0.05

‡ Power was calculated using  $M1 = E(M1) = 7.3$ ,  $M2 = E(M2) = 7.3$ ,  $S1 = E(S1) = 0.475$ ,  $S2 = E(S2) = 0.61625$ , and  $\rho = E(\rho) = 0.01467$ .

**PASS** has calculated the required number of clusters to achieve each assurance goal.

## Example 5 – Joint Prior Distribution Validation

The problem given in Example 2 will be used to validate the joint prior distribution method. This will be done by running the individual-prior scenario used in that example through the joint-prior method and checking that the assurance values match.

In Example 2, the prior distribution of S1 is given as follows.

<u>S1</u>	<u>Prob</u>
0.5	0.4
0.55	0.6

The prior distribution of the S2 is approximated by the following table.

<u>S2</u>	<u>Prob</u>
0.6	0.4
0.7	0.6

The prior distribution of the  $\rho$  is approximated by the following table.

<u><math>\rho</math></u>	<u>Prob</u>
0.01	0.5
0.02	0.5

The prior distribution of the M1 is approximated by the following table.

<u>M1</u>	<u>Prob</u>
7	0.5
9	0.5

The prior distribution of the M2 is approximated by the following table.

<u>M2</u>	<u>Prob</u>
7	0.5
9	0.5

The joint prior distribution can be found by multiplying the independent probabilities. This results in the following discrete probability distribution.

<u>M1 (C1)</u>	<u>M2(C2)</u>	<u>S1 (C3)</u>	<u>S2 (C4)</u>	<u><math>\rho</math> (C5)</u>	<u>Prob (C6)</u>
7	7	0.5	0.6	0.01	0.02
7	9	0.5	0.6	0.01	0.02
7	7	0.5	0.6	0.02	0.02
7	9	0.5	0.6	0.02	0.02
9	7	0.5	0.6	0.01	0.02
9	9	0.5	0.6	0.01	0.02
9	7	0.5	0.6	0.02	0.02
9	9	0.5	0.6	0.02	0.02
7	7	0.5	0.7	0.01	0.03
7	9	0.5	0.7	0.01	0.03
7	7	0.5	0.7	0.02	0.03
7	9	0.5	0.7	0.02	0.03
9	7	0.5	0.7	0.01	0.03
9	9	0.5	0.7	0.01	0.03
9	7	0.5	0.7	0.02	0.03

## Assurance for Logrank Tests in a Cluster-Randomized Design

9	9	0.5	0.7	0.02	0.03
7	7	0.55	0.6	0.01	0.03
7	9	0.55	0.6	0.01	0.03
7	7	0.55	0.6	0.02	0.03
7	9	0.55	0.6	0.02	0.03
9	7	0.55	0.6	0.01	0.03
9	9	0.55	0.6	0.01	0.03
9	7	0.55	0.6	0.02	0.03
9	9	0.55	0.6	0.02	0.03
7	7	0.55	0.7	0.01	0.045
7	9	0.55	0.7	0.01	0.045
7	7	0.55	0.7	0.02	0.045
7	9	0.55	0.7	0.02	0.045
9	7	0.55	0.7	0.01	0.045
9	9	0.55	0.7	0.01	0.045
9	7	0.55	0.7	0.02	0.045
9	9	0.55	0.7	0.02	0.045

To run this example, the spreadsheet is loaded with the above data.

## Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

### Design Tab

Solve For .....	<b>Assurance</b>
Prior Entry Method .....	<b>Combined (Enter parameter values and probabilities on spreadsheet)</b>
Alternative Hypothesis .....	<b>Two-Sided (<math>H_1: S_1 \neq S_2</math>)</b>
Alpha.....	<b>0.05</b>
K1 (Number of Clusters) .....	<b>40</b>
Column of M1 Values.....	<b>C1</b>
K2 (Number of Clusters) .....	<b>K1</b>
Column of M2 Values.....	<b>C2</b>
Column of S1 Values .....	<b>C3</b>
Column of S2 Values .....	<b>C4</b>
Column of p Values .....	<b>C5</b>
Column of Pr(Values).....	<b>C6</b>

### Options Tab

Number of Computation Points for each.....	<b>10</b>
Prior Distribution	
Maximum K1 in Sample Size Search.....	<b>1000</b>

## Assurance for Logrank Tests in a Cluster-Randomized Design

## Input Spreadsheet Data

Row	C1	C2	C3	C4	C5	C6
1	7	7	0.50	0.6	0.01	0.020
2	7	9	0.50	0.6	0.01	0.020
3	7	7	0.50	0.6	0.02	0.020
4	7	9	0.50	0.6	0.02	0.020
5	9	7	0.50	0.6	0.01	0.020
6	9	9	0.50	0.6	0.01	0.020
7	9	7	0.50	0.6	0.02	0.020
8	9	9	0.50	0.6	0.02	0.020
9	7	7	0.50	0.7	0.01	0.030
10	7	9	0.50	0.7	0.01	0.030
11	7	7	0.50	0.7	0.02	0.030
12	7	9	0.50	0.7	0.02	0.030
13	9	7	0.50	0.7	0.01	0.030
14	9	9	0.50	0.7	0.01	0.030
15	9	7	0.50	0.7	0.02	0.030
16	9	9	0.50	0.7	0.02	0.030
17	7	7	0.55	0.6	0.01	0.030
18	7	9	0.55	0.6	0.01	0.030
19	7	7	0.55	0.6	0.02	0.030
20	7	9	0.55	0.6	0.02	0.030
21	9	7	0.55	0.6	0.01	0.030
22	9	9	0.55	0.6	0.01	0.030
23	9	7	0.55	0.6	0.02	0.030
24	9	9	0.55	0.6	0.02	0.030
25	7	7	0.55	0.7	0.01	0.045
26	7	9	0.55	0.7	0.01	0.045
27	7	7	0.55	0.7	0.02	0.045
28	7	9	0.55	0.7	0.02	0.045
29	9	7	0.55	0.7	0.01	0.045
30	9	9	0.55	0.7	0.01	0.045
31	9	7	0.55	0.7	0.02	0.045
32	9	9	0.55	0.7	0.02	0.045

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

Solve For: [Assurance](#)  
 Groups: 1 = Treatment, 2 = Control  
 Test Statistic: Logrank Test  
 Hypotheses:  $H_0: S_1 - S_2 = 0$  vs.  $H_1: S_1 - S_2 \neq 0$   
 Prior Type: Joint Multivariate Distribution

### Prior Distribution

#### Point Lists

M1: C1: 7 7 7 7 9 9 9 9 7 7 7 7 9 9 9 9 7 7 7 7 9 9 9 9 7 7 7 7 9 9 9 9  
 M2: C2: 7 9 7 9 7 9 7 9 7 9 7 9 7 9 7 9 7 9 7 9 7 9 7 9 7 9 7 9 7 9 7 9  
 P1: C3: 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.55  
 P2: C4: 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.7 0.7 0.7 0.7 0.7 0.7 0.7  
 p: C5: 0.01 0.01 0.02 0.02 0.01 0.01 0.02 0.02 0.01 0.01 0.02 0.02 0.01 0.01 0.02 0.02 0.01 0.01 0.02 0.02 0.01 0.01 0.02 0.02 0.01 0.01 0.02 0.02 0.01 0.01  
 Prob: C6: 0.02 0.02 0.02 0.02 0.02 0.02 0.02 0.02 0.03 0.045 0.045  
 0.045 0.045 0.045 0.045 0.045 0.045 0.045

Assurance	Power‡	Number of Events		Sample Size		Number of Clusters			Cluster Size		Proportion Surviving		Intraclass Correlation	
		E1   E2	E	N1   N2	N	K1	K2	K	E(M1)	E(M2)	E(S1)	E(S2)	E(p)	Alpha
0.74894	0.89224	130   130	259	320   320	720	40	40	80	8	8	0.53	0.66	0.015	0.05

‡ Power was calculated using  $M1 = E(M1) = 8$ ,  $M2 = E(M2) = 8$ ,  $S1 = E(S1) = 0.53$ ,  $S2 = E(S2) = 0.66$ , and  $p = E(p) = 0.015$ .

PASS has calculated the assurance as 0.74894 which matches the result in Example 2. This validates the joint prior method.