

Chapter 829

Confidence Intervals for Intraclass Correlation with Assurance Probability (Two-Sided)

Introduction

This routine calculates the sample size needed to obtain a specified half-width of two-sided confidence interval of the intraclass correlation coefficient (ICC). This procedure allows you to set an *assurance probability* that the requested width is achieved. Note that another **PASS** procedure computes the sample size for a one-sided confidence interval for the ICC.

The ICC is the product-moment correlation calculated among observations on the same subject. For example, if you have three raters rating each subject, it is the average correlation among the ratings of the three raters. This procedure is often used in reliability studies.

The ICC analyzed in this procedure comes from a one-way random effects ANOVA model.

Technical Details

Zou (2012) presents formulas used for constructing a two-sided, $100(1 - \alpha)\%$ confidence interval for the ICC. We adopt his notation as we present these formulas.

Suppose that each of N subjects (or clusters) is rated by K raters. The observations obtained may be from different raters, instruments, or other measurement mechanisms. Such data may be analyzed using a one-way, random-effects, model. The ANOVA model is

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \text{ where } \alpha_i \sim N(0, \sigma_\alpha^2) \text{ and } \varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2).$$

Estimation of ρ

The ICC is defined as

$$\rho = \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma_\varepsilon^2}$$

The ICC is estimated from the mean squares of the ANOVA table as follows

$$r = \frac{MS_B - MS_E}{MS_B + (K - 1)MS_E}$$

where MS_B is the between-subject mean square and MS_E is the within-subject mean square.

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The large sample variance of r is given by

$$\text{Var}(r) = \frac{2(1 - \rho)^2[1 + (K - 1)\rho]^2}{K(K - 1)(N - 1)}$$

This can be estimated by replacing ρ with r as follows

$$\widehat{\text{Var}}(r) = \frac{2(1 - r)^2[1 + (K - 1)r]^2}{K(K - 1)(N - 1)}$$

A large sample, two-sided confidence interval using a Wald statistic is obtained as follows

$$r \mp z_{1-\alpha/2} \sqrt{\widehat{\text{Var}}(r)}$$

This procedure will only provide results for two-sided confidence intervals.

Sample Size Calculation

The sample size is determined so that the probability that the half-width of the requested interval is achieved is above a specified value. This probability is called the *assurance probability* and is referred to as $1 - \gamma$.

The assurance probability requirement is written as

$$1 - \gamma = \Pr\left(z_{1-\alpha/2} \sqrt{\widehat{\text{Var}}(r)} \leq \omega\right)$$

This can be solved for sample size to obtain the following formula

$$N = 1 + \left[\frac{Az_{1-\frac{\alpha}{2}} + \sqrt{\left(Az_{1-\frac{\alpha}{2}}\right)^2 + 4\omega ABz_{1-\gamma}z_{1-\frac{\alpha}{2}}}}{\omega\sqrt{2K(K-1)}} \right]^2$$

where

$$A = (1 - \rho)[1 + (K - 1)\rho]$$

$$B = K - 2 + 2\rho(1 - K)$$

This can be rearranged to solve for the assurance probability or ω as well.

Confidence Level

The confidence level, $1 - \alpha$, has the following interpretation. If thousands of samples of n items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population correlation is $1 - \alpha$.

Example 1 – Calculating Sample Size

Suppose a reliability study is planned to find an estimate of the two-sided 95% confidence interval for the ICC. The researcher would like to examine values of K from 2 to 10. The goal is to determine the necessary sample size, N , when the assurance probability is 0.9, the half width is 0.1, and the planning value of the ICC is 0.6.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **Number of Subjects (N)**
 Confidence Level ($1 - \alpha$) **0.95**
 $1 - \gamma$ (Assurance Probability) **0.9**
 K (Observations per Subject) **2 4 6 8 10**
 ω (Half-Width of Confidence Interval) **0.1**
 ρ_1 (True Value of ρ) **0.6**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: **Number of Subjects (N)**
 Confidence Interval: **Two-Sided**

Confidence Level $1 - \alpha$	Number of Subjects N	Observations per Subject K	Intraclass Correlation Coefficient		Assurance Probability	
			C.I. Half-Width ω	True Value ρ_1	Target $1 - \gamma_T$	Actual $1 - \gamma_A$
0.95	196	2	0.1	0.6	0.9	0.90621
0.95	96	4	0.1	0.6	0.9	0.90443
0.95	77	6	0.1	0.6	0.9	0.90224
0.95	70	8	0.1	0.6	0.9	0.92391
0.95	65	10	0.1	0.6	0.9	0.90879

Confidence Level The proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the true ICC.
 N The number of subjects in the random sample drawn from the population.
 K The number of observations obtained for each subject.
 ω The half-width of the symmetric, two-sided confidence interval of the intraclass correlation coefficient (ρ).
 ρ_1 The true (planning) value of the intraclass correlation coefficient (ρ).
 $1 - \gamma_T$ The target assurance probability that the half-width of the confidence interval will be less than ω .
 $1 - \gamma_A$ The actual assurance probability that is achieved for this sample size.

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Summary Statements

A single-group design, with each subject measured 2 times, will be used to obtain a two-sided 95% confidence interval for a single intraclass correlation coefficient. It is assumed that the data will be analyzed using a one-way ANOVA model. The true intraclass correlation coefficient is assumed to be 0.6. To produce a confidence interval with 0.9 probability that the confidence interval half-width will be no more than 0.1, 196 subjects will be needed.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout- Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	196	245	49
20%	96	120	24
20%	77	97	20
20%	70	88	18
20%	65	82	17

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N	The evaluable sample size at which the confidence interval is computed. If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated confidence interval.
N'	The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. After solving for N, N' is calculated by inflating N using the formula $N' = N / (1 - DR)$, with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D	The expected number of dropouts. $D = N' - N$.

Dropout Summary Statements

Anticipating a 20% dropout rate, 245 subjects should be enrolled to obtain a final sample size of 196 subjects.

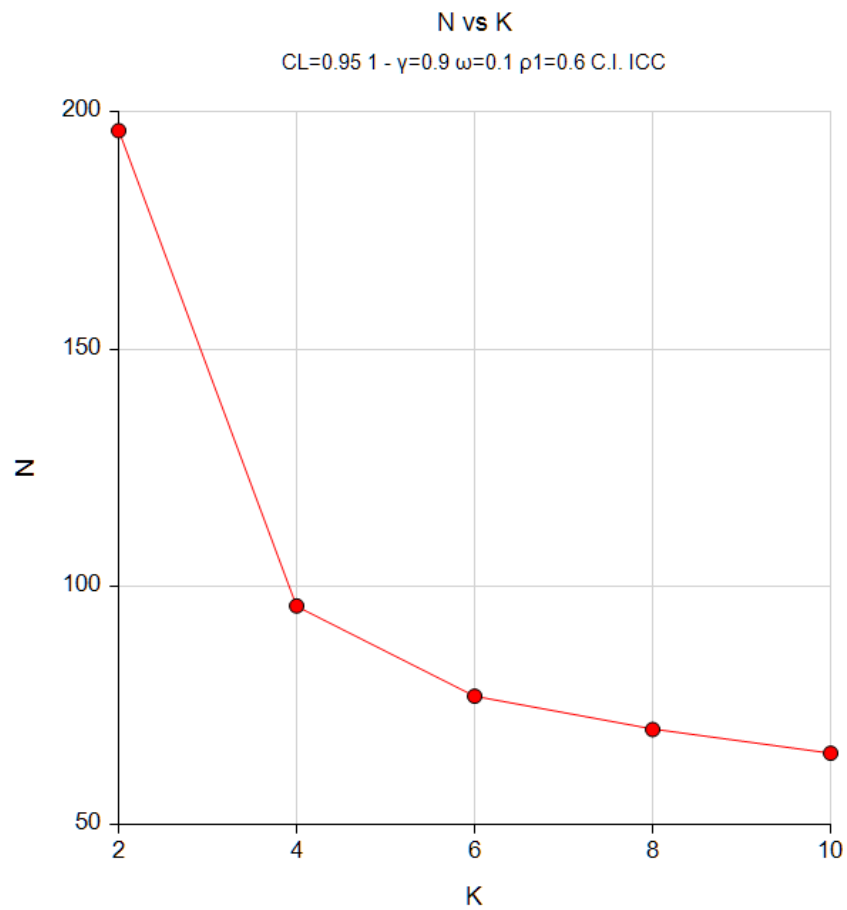
References

- Zou, G.Y. 2012. 'Sample size formulas for estimating intraclass correlation coefficients with precision and assurance.' *Statistics in Medicine*, Vol 31, 3972-3981.
- Bartko, John J. 1966. 'The intraclass correlation coefficient as a measure of reliability.' *Psychological Reports*, Vol 19, 3-11.

This report shows the calculated sample size for each of the scenarios.

Plots Section

Plots



This plot shows the sample size versus the value of K.

Example 2 – Validation using Zou (2012)

Zou (2012), page 3976, in Table 1 gives example calculations of the number of subjects needed of a two-sided confidence interval of ICC when the confidence level is 95%, ρ_1 is 0.60, ω is 0.10, $1 - \gamma$ is 0.80, and K is 2. The result for N is 183.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **Number of Subjects (N)**
 Confidence Level ($1 - \alpha$) **0.95**
 $1 - \gamma$ (Assurance Probability) **0.8**
 K (Observations per Subject) **2**
 ω (Half-Width of Confidence Interval) **0.1**
 ρ_1 (True Value of ρ) **0.6**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: **Number of Subjects (N)**
 Confidence Interval: Two-Sided

Confidence Level $1 - \alpha$	Number of Subjects N	Observations per Subject K	Intraclass Correlation Coefficient		Assurance Probability	
			C.I. Half-Width ω	True Value ρ_1	Target $1 - \gamma_T$	Actual $1 - \gamma_A$
0.95	183	2	0.1	0.6	0.8	0.80198

PASS matches Zou's results exactly.