Chapter 651

Confidence Intervals for One Variance using Variance

Introduction

This routine calculates the sample size necessary to achieve a specified interval width or distance from the variance to the confidence limit at a stated confidence level for a confidence interval about the variance when the underlying data distribution is normal.

Caution: This procedure assumes that the variance of the future sample will be the same as the variance that is specified. If the variance to be used in the procedure is estimated from a previous sample or represents the population variance, the Confidence Intervals for One Variance with Tolerance Probability procedure should be considered. That procedure controls the probability that the width or the distance from the variance to the confidence limits will be less than or equal to the value specified. The Confidence Intervals for One Variance using Relative Error controls the width or distance from the variance to the limits by controlling the width or distance as a percent of the true variance.

Technical Details

For a single variance from a normal distribution with unknown mean, a two-sided, 100(1 - α)% confidence interval is calculated by

$$\left[\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}\right]$$

A one-sided 100(1 – α)% upper confidence limit is calculated by

$$\frac{(n-1)s^2}{\chi^2_{\alpha,n-1}}$$

Similarly, the one-sided 100(1 – α)% lower confidence limit is

$$\frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}}$$

For two-sided intervals, the distance from the variance to each of the limits is different. Thus, instead of specifying the distance to the limits we specify the width of the interval, *W*.

The basic equation for determining sample size for a two-sided interval when *W* has been specified is

$$W = \frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} - \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

For one-sided intervals, the distance from the variance to limits, *D*, is specified.

The basic equation for determining sample size for a one-sided upper limit when *D* has been specified is

$$D = \frac{(n-1)s^2}{\chi^2_{\alpha,n-1}} - s^2$$

The basic equation for determining sample size for a one-sided lower limit when *D* has been specified is

$$D = s^2 - \frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}}$$

These equations can be solved for any of the unknown quantities in terms of the others.

Confidence Level

The confidence level, 1 – α, has the following interpretation. If thousands of samples of *n* items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population variance is 1 – α.

Example 1 – Calculating Sample Size

Suppose a study is planned in which the researcher wishes to construct a two-sided 95% confidence interval for the variance such that the width of the interval is no wider than 40 units. The confidence level is set at 0.95, but 0.99 is included for comparative purposes. The variance estimate, based on the range of data values, is 24. Instead of examining only the interval width of 40, a series of widths from 30 to 50 will also be considered.

The goal is to determine the necessary sample size.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ..... **Sample Size**
Interval Type ..... **Two-Sided**
Confidence Level (1 - Alpha) **0.95 0.99**
Confidence Interval Width (Two-Sided) ..... **30 to 50 by 5**
Variance..... **24**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Sample Size
Interval Type: Two-Sided

Confidence Level	Sample Size N	Confidence Interval Width Target	Confidence Interval Width Actual	Variance	Confidence Interval Limits Lower	Confidence Interval Limits Upper
0.95	28	30	29.463	24	15.002	44.465
0.99	46	30	29.663	24	14.761	44.424
0.95	22	35	34.808	24	14.206	49.013
0.99	36	35	34.924	24	13.936	48.860
0.95	19	40	38.783	24	13.703	52.486
0.99	30	40	39.745	24	13.299	53.044
0.95	16	45	44.392	24	13.096	57.488
0.99	26	45	44.251	24	12.786	57.036
0.95	14	50	49.678	24	12.613	62.291
0.99	23	50	48.754	24	12.338	61.092

Confidence Level	The proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the population variance.
N	The size of the sample drawn from the population.
Confidence Interval Width	The distance from the lower limit to the upper limit.
Target Width	The value of the width that is entered into the procedure.
Actual Width	The value of the width that is obtained from the procedure.
Variance	The sample variance.
Confidence Interval Limits	The lower and upper limits of the confidence interval.

Summary Statements

A single-group design will be used to obtain a two-sided 95% confidence interval for a single variance. The standard Chi-square-based formula will be used to calculate the confidence interval. The sample variance is assumed to be 24. To produce a confidence interval with a width of no more than 30, 28 subjects will be needed.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	28	35	7
20%	46	58	12
20%	22	28	6
20%	36	45	9
20%	19	24	5
20%	30	38	8
20%	16	20	4
20%	26	33	7
20%	14	18	4
20%	23	29	6

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N	The evaluable sample size at which the confidence interval is computed. If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated confidence interval.
N'	The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. After solving for N, N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D	The expected number of dropouts. D = N' - N.

Dropout Summary Statements

Anticipating a 20% dropout rate, 35 subjects should be enrolled to obtain a final sample size of 28 subjects.

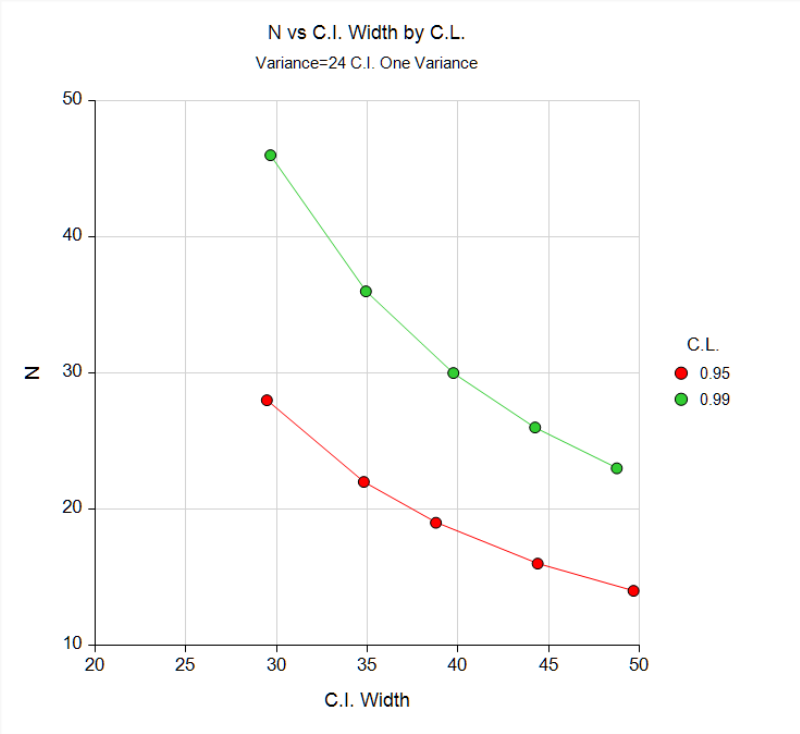
References

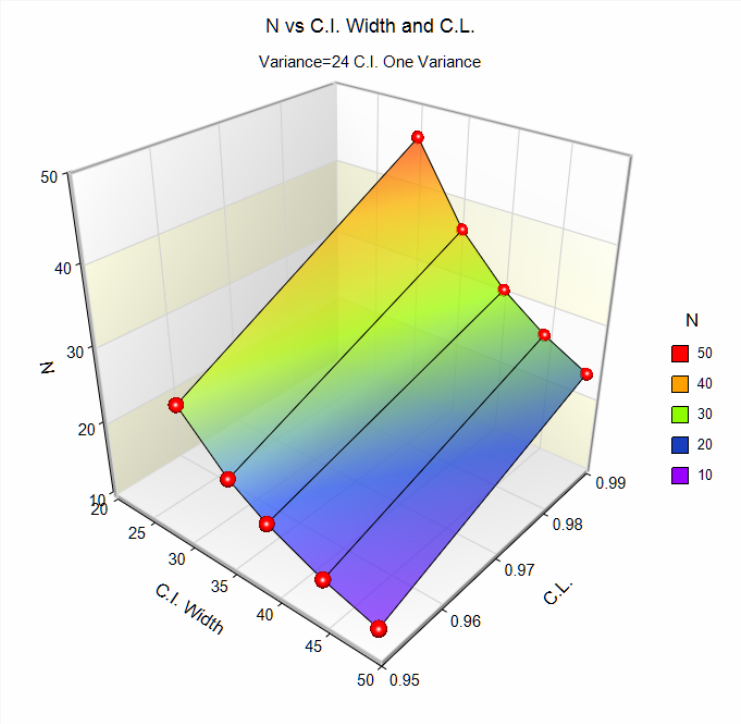
Zar, Jerrold H. 1984. Biostatistical Analysis. Second Edition. Prentice-Hall. Englewood Cliffs, New Jersey.

This report shows the calculated sample size for each of the scenarios.

Plots Section

Plots





These plots show the sample size vs. the confidence interval width for the two confidence levels.

Example 2 – Validation using Zar (1984)

Zar (1984) page 115 give an example of a calculation for a confidence interval on the variance when the confidence level is 95%, the variance is 18.0388, and the interval width is 23.91244. The necessary sample size is 25.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Find (Solve For) ..... **Sample Size**
Interval Type ..... **Two-Sided**
Confidence Level (1 - Alpha) **0.95**
Confidence Interval Width (Two-Sided) **23.91244**
Variance..... **18.0388**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Interval Type: Two-Sided

Confidence Level	Sample Size N	Confidence Interval Width Target	Confidence Interval Width Actual	Variance	Confidence Interval Limits Lower	Confidence Interval Limits Upper
0.95	25	23.912	23.912	18.039	10.998	34.911

PASS also calculated the necessary sample size to be 25.