Chapter 657

Confidence Intervals for the Ratio of Two Variances using Relative Error

Introduction

This routine calculates the necessary sample size such that a variance ratio estimate from two independent samples will achieve a specified relative distance from the true variance ratio at a stated confidence level when the underlying data distributions are normal.

Caution: This procedure controls the relative width of the interval as a proportion of the true variance ratio. For controlling the absolute width of the interval see the procedure Confidence Intervals for the Ratio of Two Variances using Variances.

Technical Details

Following the results of Desu and Raghavarao (1990), let s_1^2 be the variance estimate based on a sample from a normal distribution with unknown μ_1 and unknown σ_1^2. Let s_2^2 be the variance estimate based on a sample from a normal distribution with unknown μ_2 and unknown σ_2^2. Let r be the proportion of σ_1^2/σ_2^2 such that s_1^2/s_2^2 is within $r\sigma_1^2/\sigma_2^2$ of σ_1^2/σ_2^2 with desired confidence (1 - α). That is, the desired condition to be satisfied is

$$\Pr\left(\left|\frac{s_1^2}{s_2^2}-\frac{\sigma_1^2}{\sigma_2^2}\right| \le r\frac{\sigma_1^2}{\sigma_2^2}\right) \ge 1-\alpha$$

which can also be written as

$$\Pr\left(\left|\frac{\frac{s_1^2}{s_2^2}-\frac{\sigma_1^2}{\sigma_2^2}}{\frac{\sigma_1^2}{\sigma_2^2}}\right| \le r\right) \ge 1-\alpha$$

which simplifies to

$$\Pr(1-r \le F(df_1, df_2) \le 1+r) \ge 1-\alpha$$

since

$$\frac{s_1^2}{s_2^2}\bigg/\frac{\sigma_1^2}{\sigma_2^2} \sim F(df_1, df_2)$$

If $G_{df_1,df}(.)$ is the distribution function of $F(df_1, df_2)$, then probability statement can be rewritten as

$$G_{df_1,df_2}(1+r) - G_{df_1,df_2}(1-r) \geq 1-\alpha$$

This equation can be solved for any of the unknown quantities (df_1, df_2, r, α) in terms of the others.

Confidence Level

The confidence level, 1 – α, has the following interpretation. If thousands of random samples of size n_1 and n_2 are drawn from populations 1 and 2, respectively, and a variance ratio is calculated for each pair of samples, the proportion of those estimates that are within $r\sigma_1^2/\sigma_2^2$ of σ_1^2/σ_2^2 is 1 – α.

Example 1 – Calculating Sample Size

Suppose a study is planned in which the researcher wishes to be 95% confident that estimated variance ratio is within 10% of the true variance ratio. In addition to 10% relative error, 5%, 15%, 20% and 25% will also be considered.

The goal is to determine the necessary sample size.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Sample Size**
Interval Type .... **Two-Sided**
Confidence Level (1 - Alpha) **0.95**
Group Allocation .... **Equal (N1 = N2)**
Relative Error.. **0.05 to 0.25 by 0.05**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Sample Size
Interval Type: Two-Sided

Confidence Level		Sample Size			
Target	Actual	N1	N2	N	Relative Error
0.95	0.95	6154	6154	12308	0.05
0.95	0.95	1544	1544	3088	0.10
0.95	0.95	690	690	1380	0.15
0.95	0.95	392	392	784	0.20
0.95	0.95	253	253	506	0.25

Target Confidence Level	The value of the confidence level that is entered into the procedure.
Actual Confidence Level	The value of the confidence level that is obtained from the procedure.
N1 and N2	The number of items sampled from each population.
N	The total sample size. N = N1 + N2.
Relative Error	The distance from the true variance ratio as a proportion of the true variance ratio.

Summary Statements

A parallel two-group design will be used to obtain an estimate of the variance ratio (Variance 1 / Variance 2) with relative precision. The F-distribution-based methods of Desu and Raghavarao (1990) will be used. The underlying data distributions of the two groups are assumed to be Normal. To obtain an estimate of the variance ratio where the probability is 0.95 (95% confidence) that the estimate of the variance ratio will be within 5% of the true population variance ratio (two-sided), the number of subjects needed will be 6154 in Group 1 and 6154 in Group 2.

Dropout-Inflated Sample Size

	Sample Size			Dropout-Inflated Enrollment Sample Size			Expected Number of Dropouts		
Dropout Rate	**N1**	**N2**	**N**	**N1'**	**N2'**	**N'**	**D1**	**D2**	**D**
20%	6154	6154	12308	7693	7693	15386	1539	1539	3078
20%	1544	1544	3088	1930	1930	3860	386	386	772
20%	690	690	1380	863	863	1726	173	173	346
20%	392	392	784	490	490	980	98	98	196
20%	253	253	506	317	317	634	64	64	128

Dropout Rate The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N1, N2, and N The evaluable sample sizes at which the confidence interval is computed. If N1 and N2 subjects are evaluated out of the N1' and N2' subjects that are enrolled in the study, the design will achieve the stated confidence interval.
N1', N2', and N' The number of subjects that should be enrolled in the study in order to obtain N1, N2, and N evaluable subjects, based on the assumed dropout rate. After solving for N1 and N2, N1' and N2' are calculated by inflating N1 and N2 using the formulas N1' = N1 / (1 - DR) and N2' = N2 / (1 - DR), with N1' and N2' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D1, D2, and D The expected number of dropouts. D1 = N1' - N1, D2 = N2' - N2, and D = D1 + D2.

Dropout Summary Statements

Anticipating a 20% dropout rate, 7693 subjects should be enrolled in Group 1, and 7693 in Group 2, to obtain final group sample sizes of 6154 and 6154, respectively.

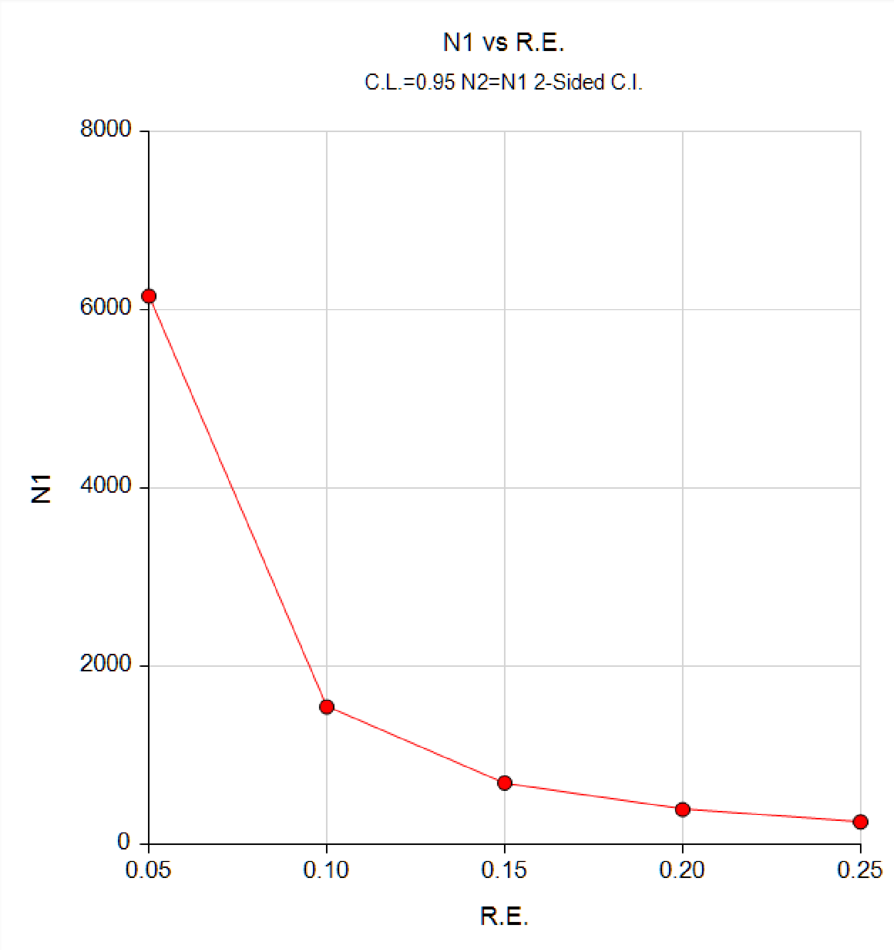
References

Desu, M. M. and Raghavarao, D. 1990. Sample Size Methodology. Academic Press. New York.

This report shows the calculated sample size for each of the scenarios.

Plots Section

Plots



This plot shows the sample size versus the relative error.

Example 2 – Validation using Direct Calculation

Suppose a study is planned in which the researcher wishes to be confident that estimated variance ratio is within 20% of the true variance ratio. For sample sizes of 200 per group, the resulting confidence level is 0.9003379878 - 0.0581710981 = 0.8421668897.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Sample Size**
Interval Type .... **Two-Sided**
Confidence Level (1 - Alpha) **0.8421668897**
Group Allocation .... **Equal (N1 = N2)**
Relative Error.. **0.2**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Interval Type: Two-Sided

Confidence Level		Sample Size			
Target	Actual	N1	N2	N	Relative Error
0.842	0.842	200	200	400	0.2

PASS calculates the necessary sample size to be 200 per group, which is the same as the direct calculation sample size.