

## Chapter 165

# Equivalence Tests for the Difference Between Two Correlated Proportions

## Introduction

The procedure described in this chapter computes power and sample size for testing equivalence using differences in designs in which two dichotomous responses are measured on each subject.

When one is interested in showing that two correlated proportions are different, the data are often analyzed with McNemar's test. However, the procedures discussed here are interested in showing equivalence rather than difference. For example, suppose a diagnostic procedure is accurate, but is expensive to apply or has serious side effects. A replacement procedure may be sought which is equally accurate but is less expensive or has fewer side effects. In this case, we are not interested in showing that the two diagnostic procedures are different, but rather that they are the same. Equivalence tests were designed for this situation.

These tests are often divided into two categories: *equivalence* (two-sided) tests and *non-inferiority* (one-sided) tests. Here, the term *equivalence tests* means that we want to show that two diagnostic procedures are equivalent—that is, their accuracy is about the same. This requires a two-sided hypothesis test. On the other hand, *non-inferiority tests* are used when we want to show that a new (experimental) procedure is no worse than the existing (reference or gold-standard) one. This requires a one-sided hypothesis test.

## Technical Details

The results of a study in which two dichotomous responses are measured on each subject can be displayed in a 2-by-2 table in which one response is shown across the columns and the other is shown down the rows. In the discussion to follow, the columns of the table represent the standard (reference or control) response and the rows represent the treatment (experimental) response. The outcome probabilities can be classified into the following table.

Experimental Diagnosis	Standard Diagnosis		Total
	Yes	No	
Yes	$p_{11}$	$p_{10}$	$P_T$
No	$p_{01}$	$p_{00}$	$1 - P_T$
Total	$P_S$	$1 - P_S$	1

## Equivalence Tests for the Difference Between Two Correlated Proportions

In this table,  $p_{ij} = p_{Treatment, Standard}$ . That is, the first subscript represents the response of the new, experimental procedure while the second subscript represents the response of the standard procedure. Thus,  $p_{01}$  represents the proportion having a negative treatment response and a positive standard response.

## Sensitivity, Specificity, and Prevalence

To aid in interpretation, analysts have developed a few proportions that summarize the table. Three of the most popular ratios are *sensitivity*, *specificity*, and *prevalence*.

### Sensitivity

Sensitivity is the proportion of subjects with a positive standard response who also have a positive experimental response. In terms of proportions from the 2-by-2 table,

$$\text{Sensitivity} = \frac{p_{11}}{(p_{01} + p_{11})} = \frac{p_{11}}{P_S}$$

### Specificity

Specificity is the proportion of subjects with a negative standard response who also have a negative experimental response. In terms of proportions from the 2-by-2 table,

$$\text{Specificity} = \frac{p_{00}}{(p_{10} + p_{00})}$$

### Prevalence

Prevalence is the overall proportion of individuals with the disease (or feature of interest). In terms of proportions from the 2-by-2 table,

$$\text{Prevalence} = P_S$$

## Table Probabilities

The outcome counts from a sample of  $n$  subjects can be classified into the following table.

Experimental Diagnosis	Standard Diagnosis		Total
	Yes	No	
Yes	$n_{11}$	$n_{10}$	$n_T$
No	$n_{01}$	$n_{00}$	$n - n_T$
<b>Total</b>	$n_S$	$n - n_S$	$n$

## Equivalence Tests for the Difference Between Two Correlated Proportions

Note that  $n_{11} + n_{00}$  is the number of matches (*concordant pairs*) and  $n_{01} + n_{10}$  is the number of *discordant pairs*.

The hypothesis of interest concerns the two marginal probabilities  $P_T$  and  $P_S$ .  $P_S$  represents the accuracy or success of the standard test and  $P_T$  represents the accuracy or success of the new, experimental test. Equivalence is defined in terms of either the difference of these two proportions,  $D = P_T - P_S$ , or the relative risk ratio,  $R = P_T/P_S$ . The choice between  $D$  and  $R$  will usually lead to different sample sizes to achieve the same power.

## Equivalence Hypotheses using Differences

This section is based on Liu, Hsueh, Hsieh and Chen (2002). We refer you to that paper for complete details.

If we define  $M_E$  as the positive equivalence margin, then the null and alternative hypotheses of equivalence in terms of the difference are

$$H_0: P_T - P_S \leq -M_E \text{ or } P_T - P_S \geq M_E \quad \text{versus} \quad H_1: -M_E < P_T - P_S < M_E,$$

or equivalently, with  $D_{0,L} = -M_E$  and  $D_{0,U} = |D_{0,L}| = M_E$ ,

$$H_0: P_T - P_S \leq D_{0,L} \text{ or } P_T - P_S \geq D_{0,U} \quad \text{versus} \quad H_1: D_{0,L} < P_T - P_S < D_{0,U}.$$

These hypotheses can be decomposed into two sets of one-sided hypotheses

$$H_{0L}: P_T - P_S \leq D_{0,L} \quad \text{versus} \quad H_{1L}: P_T - P_S > D_{0,L}$$

and

$$H_{0U}: P_T - P_S \geq D_{0,U} \quad \text{versus} \quad H_{1U}: P_T - P_S < D_{0,U}.$$

The hypothesis test of equivalence with type I error rate  $\alpha$  is conducted by computing a  $100(1 - 2\alpha)\%$  confidence interval for  $P_T - P_S$  and determining if this interval is wholly contained between  $D_{0,L}$  and  $D_{0,U}$ . This confidence interval approach is often recommended by regulatory agencies.

Liu et al. (2002) discuss the RMLE-based (score) method for constructing these confidence intervals. This method is based on (developed by, described by) Nam (1997).

## Asymptotic Tests

An asymptotic test for testing  $H_{0L}$  versus  $H_{1L}$  is given by

$$Z_L = \frac{\hat{D} + M_E}{\hat{\sigma}} = \frac{c + nM_E}{\sqrt{d - n\hat{D}^2}} \geq z_{1-\alpha}$$

where

$$\hat{D} = \frac{n_T}{n} - \frac{n_S}{n} = \frac{n_{10}}{n} - \frac{n_{01}}{n}$$

$$d = n_{10} + n_{01}$$

$$c = n_{10} - n_{01}$$

and  $z_{1-\alpha}$  is the standard normal deviate having  $\alpha$  in the right tail.

Similarly, an asymptotic test for testing  $H_{0U}$  versus  $H_{1U}$  is given by

$$Z_U = \frac{\hat{D} - M_E}{\hat{\sigma}} = \frac{c - nM_E}{\sqrt{d - n\hat{D}^2}} \leq -z_{1-\alpha}.$$

Equivalence is concluded if both the tests on  $Z_L$  and  $Z_U$  are rejected.

An estimate of  $\hat{\sigma}$  based on the RMLE-based (score) procedure of Nam (1997) uses the estimates

$$\tilde{\sigma}_L = \sqrt{\frac{\tilde{p}_{L,10} + \tilde{p}_{L,01} - M_E^2}{n}}$$

and

$$\tilde{\sigma}_U = \sqrt{\frac{\tilde{p}_{U,10} + \tilde{p}_{U,01} - M_E^2}{n}}$$

where

$$\tilde{p}_{L,01} = \frac{-\tilde{a}_L + \sqrt{\tilde{a}_L^2 - 8\tilde{b}_L}}{4}$$

$$\tilde{p}_{L,10} = \tilde{p}_{L,01} - M_E$$

$$\tilde{p}_{U,01} = \frac{-\tilde{a}_U + \sqrt{\tilde{a}_U^2 - 8\tilde{b}_U}}{4}$$

$$\tilde{p}_{U,10} = \tilde{p}_{U,01} + M_E$$

## Equivalence Tests for the Difference Between Two Correlated Proportions

$$\tilde{a}_{L,01} = -\hat{D}(1 - M_E) - 2(\hat{p}_{01} + M_E)$$

$$\tilde{b}_{L,01} = M_E(1 + M_E)\hat{p}_{01}$$

$$\tilde{a}_{U,01} = -\hat{D}(1 + M_E) - 2(\hat{p}_{01} - M_E)$$

$$\tilde{b}_{U,01} = -M_E(1 - M_E)\hat{p}_{01}$$

Note that the ICH E9 guideline (see Lewis (1999)) suggests using a significance level of  $\alpha/2$  when testing this hypothesis.

## Power Formula

The power when the actual difference is  $D_1$  can be evaluated exactly using the multinomial distribution. However, when the sample size is above a user-set level, we use a normal approximation to this distribution which leads to

$$\text{Power} = \begin{cases} \Phi(c_U) - \Phi(c_L) & \text{if } c_U - c_L > 0 \\ 0 & \text{otherwise} \end{cases}$$

where

$$c_L = \frac{z_\alpha}{w_L} - \frac{D_1 + M_E}{\sigma} = \frac{z_\alpha}{w_L} - \frac{D_1 - D_{0,L}}{\sigma}$$

$$c_U = -\frac{z_\alpha}{w_U} + \frac{M_E - D_1}{\sigma} = -\frac{z_\alpha}{w_U} + \frac{D_{0,U} - D_1}{\sigma}$$

$$\sigma = \sqrt{\frac{p_{01} + p_{10} - D_1^2}{n}}$$

$$w_L = \sqrt{\frac{2p_{01} + D_1 - D_1^2}{2\bar{p}_{L,01} - M_E - M_E^2}} = \sqrt{\frac{2p_{01} + D_1 - D_1^2}{2\bar{p}_{L,01} + D_{0,L} - D_{0,L}^2}}$$

$$w_U = \sqrt{\frac{2p_{01} + D_1 - D_1^2}{2\bar{p}_{U,01} + M_E - M_E^2}} = \sqrt{\frac{2p_{01} + D_1 - D_1^2}{2\bar{p}_{U,01} + D_{0,U} - D_{0,U}^2}}$$

$$\bar{p}_{L,01} = \frac{-a_L + \sqrt{a_L^2 - 8b_L}}{4}$$

$$\bar{p}_{U,01} = \frac{-a_U + \sqrt{a_U^2 - 8b_U}}{4}$$

## Equivalence Tests for the Difference Between Two Correlated Proportions

$$a_L = -D_1(1 - M_E) - 2(p_{01} + M_E) = -D_1(1 + D_{0.L}) - 2(p_{01} - D_{0.L})$$

$$b_L = M_E(1 + M_E)p_{01} = -D_{0.L}(1 - D_{0.L})p_{01}$$

$$a_U = -D_1(1 + M_E) - 2(p_{01} - M_E) = -D_1(1 + D_{0.U}) - 2(p_{01} - D_{0.U})$$

$$b_U = -M_E(1 - M_E)p_{01} = -D_{0.U}(1 - D_{0.U})p_{01}$$

## Nuisance Parameter

The 2-by-2 table includes four parameters,  $p_{11}$ ,  $p_{10}$ ,  $p_{01}$ , and  $p_{00}$ , but the power calculations only require two parameters:  $P_S$  and  $D_1$ . A third parameter is defined implicitly since the sum of the four parameters is one. Thus, one parameter (known as a nuisance parameter) remains unaccounted for. This parameter must be addressed to fully specify the problem. This fourth parameter can be specified using any one of the following:  $p_{11}$ ,  $p_{10}$ ,  $p_{01}$ ,  $p_{00}$ ,  $p_{10} + p_{01}$ ,  $p_{11} + p_{00}$ , the sensitivity of the experimental response,  $p_{11}/P_S$ , or the within-subject correlation,  $\rho$ .

It may be difficult to specify a reasonable value for the nuisance parameter since its value may not be even approximately known until after the study is conducted. Because of this, we suggest that you calculate power or sample size for a range of values of the nuisance parameter. This will allow you to determine how sensitive the results are to its value.

## Estimating P11, P01, and P10 using Pt, Ps, and $\rho$

Sometimes, obtaining estimates of  $P_{11}$ ,  $P_{01}$ , and/or  $P_{10}$  is problematic. This problem is solved by using the marginal probabilities and the within-subject correlation coefficient, which may be easier to estimate. As outlined in Zhang, Cao, and Ahn (2017), the relationship between  $P_{11}$ ,  $P_t$ ,  $P_s$  and the correlation is

$$\rho = \frac{P_{11} - P_s P_t}{\sqrt{P_s P_t (1 - P_s)(1 - P_t)}}$$

Using this relationship, values of  $\rho$  can be entered and transformed to the corresponding value of  $P_{11}$  using the equation

$$P_{11} = \rho \sqrt{P_s P_t (1 - P_s)(1 - P_t)} + P_s P_t$$

The only concern is that values of  $\rho$  be used that limit  $P_{11}$ ,  $P_{01}$ ,  $P_{10}$ , and  $P_{00}$  to be between 0 and 1. The lower and upper limits of the correlation are

$$\rho_L = \max \left\{ -\sqrt{\frac{P_s P_t}{(1 - P_s)(1 - P_t)}}, -\sqrt{\frac{(1 - P_s)(1 - P_t)}{P_s P_t}} \right\}$$

$$\rho_U = \min \left\{ \sqrt{\frac{P_s(1 - P_t)}{P_t(1 - P_s)}}, \sqrt{\frac{P_t(1 - P_s)}{P_s(1 - P_t)}} \right\}$$

$P_{11}$ , along with  $P_t$  and  $P_s$ , can then be used to calculate  $P_{01}$  and  $P_{10}$ .

## Example 1 – Finding Power

A clinical trial will be conducted to show that a non-invasive MRI test is equivalent to the invasive CTAP reference test. Historical data suggest that the CTAP test is 80% accurate. After careful discussion, the researchers decide that if the MRI test is within five percentage points of the CTAP, it will be considered equivalent. They decide to use a difference test statistic. Thus, the equivalence difference is 0.05. They want to study the power for various sample sizes between 200 and 1000 at the 5% significance level.

They use P01 as the nuisance parameter and look at two values: 0.05 and 0.10.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Power</b>
Power Calculation Method .....	<b>Normal Approximation</b>
Alpha.....	<b>0.05</b>
N (Sample Size).....	<b>200 300 450 600 800 1000</b>
D0.L (Lower Equivalence Difference) .....	<b>-0.05</b>
D1 (Actual Difference).....	<b>0.0</b>
Ps (Standard Proportion) .....	<b>0.80</b>
Nuisance Parameter Type .....	<b>P01 (% -Trt +Std)</b>
Nuisance Parameter Value .....	<b>0.05 0.10</b>

## Equivalence Tests for the Difference Between Two Correlated Proportions

## Output

Click the Calculate button to perform the calculations and generate the following output.

## Numeric Reports

## Numeric Results

Solve For: [Power](#)

Hypotheses:  $H_0: P_t - P_s \leq D_{0.L} \text{ or } P_t - P_s \geq D_{0.U}$  vs.  $H_1: D_{0.L} < P_t - P_s < D_{0.U}$

Power*	Sample Size N	Equivalence Differences		Actual Difference D1	Proportions		Nuisance Parameter P01	Alpha
		Lower D0.L	Upper D0.U		Treatment Pt	Standard Ps		
0.35542	200	-0.05	0.05	0	0.8	0.8	0.05	0.05
0.00000	200	-0.05	0.05	0	0.8	0.8	0.10	0.05
0.66488	300	-0.05	0.05	0	0.8	0.8	0.05	0.05
0.20739	300	-0.05	0.05	0	0.8	0.8	0.10	0.05
0.88574	450	-0.05	0.05	0	0.8	0.8	0.05	0.05
0.51491	450	-0.05	0.05	0	0.8	0.8	0.10	0.05
0.96411	600	-0.05	0.05	0	0.8	0.8	0.05	0.05
0.71314	600	-0.05	0.05	0	0.8	0.8	0.10	0.05
0.99301	800	-0.05	0.05	0	0.8	0.8	0.05	0.05
0.86344	800	-0.05	0.05	0	0.8	0.8	0.10	0.05
0.99874	1000	-0.05	0.05	0	0.8	0.8	0.05	0.05
0.93739	1000	-0.05	0.05	0	0.8	0.8	0.10	0.05

\* Power was computed using the normal approximation method.

Power	The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N	The number of subjects, the sample size.
D0.L and D0.U	The lower and upper difference bounds, respectively, used to construct the equivalence hypotheses.
D1	The actual difference at which the power is calculated. $D1 = P_t - P_s$ .
Pt	The response proportion in the treatment (experimental) group.
Ps	The response proportion in the standard (baseline, reference, or control) group.
Nuisance Parameter	A value that is needed but is not a direct part of the hypotheses.
Alpha	The probability of rejecting a true null hypothesis.

## Summary Statements

A paired design will be used to test whether the treatment proportion ( $P_t$ ) is equivalent to the standard proportion ( $P_s$ ), with equivalence difference bounds of -0.05 and 0.05 ( $H_0: P_t - P_s \leq -0.05 \text{ or } P_t - P_s \geq 0.05$  versus  $H_1: -0.05 < P_t - P_s < 0.05$ ). The comparison will be made using an RMLE-based score test, with a Type I error rate ( $\alpha$ ) of 0.05. The nuisance parameter (P01) is assumed to be 0.05. To detect a difference between the proportions of 0 ( $P_t = 0.8, P_s = 0.8$ ) with a sample size of 200 pairs, the power is 0.35542.



## Equivalence Tests for the Difference Between Two Correlated Proportions

## Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout- Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	200	250	50
20%	300	375	75
20%	450	563	113
20%	600	750	150
20%	800	1000	200
20%	1000	1250	250

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N	The evaluable sample size at which power is computed (as entered by the user). If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated power.
N'	The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. N' is calculated by inflating N using the formula $N' = N / (1 - DR)$ , with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D	The expected number of dropouts. $D = N' - N$ .

## Dropout Summary Statements

Anticipating a 20% dropout rate, 250 subjects should be enrolled to obtain a final sample size of 200 subjects.

## References

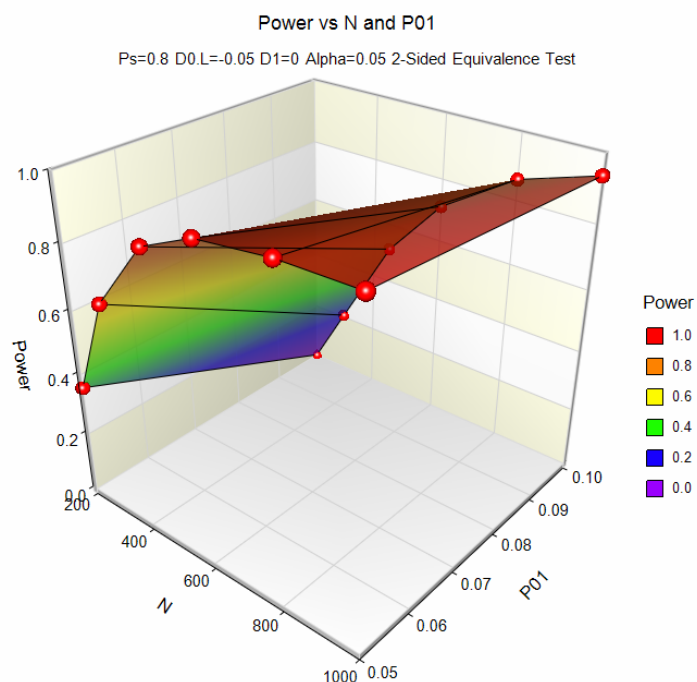
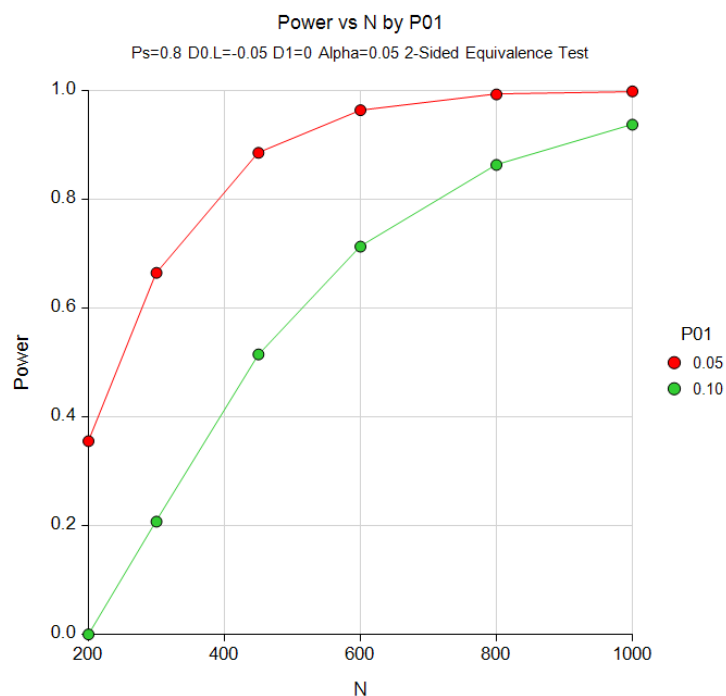
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- Nam, Jun-mo and Blackwelder, W.C. 2002. 'Analysis of the ratio of marginal probabilities in a matched-pair setting', Statistics in Medicine, Volume 21, pages 689-699.
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This report shows the power for the indicated scenarios. All of the columns are defined in the "Report Definitions" section.

## Equivalence Tests for the Difference Between Two Correlated Proportions

## Plots Section

## Plots



These plots show the power versus the sample size for the two values of P01. In this example, we see that the value of the nuisance parameter has a large effect on the calculated sample size.

## Example 2 – Finding Sample Size

Continuing with Example 1, the analysts want to determine the exact sample size necessary to achieve 90% power for both values of the nuisance parameter.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For ..... **Sample Size**  
 Power Calculation Method ..... **Normal Approximation**  
 Power..... **0.90**  
 Alpha..... **0.05**  
 D0.L (Lower Equivalence Difference) ..... **-0.05**  
 D1 (Actual Difference)..... **0.0**  
 Ps (Standard Proportion) ..... **0.80**  
 Nuisance Parameter Type ..... **P01 (% -Trt +Std)**  
 Nuisance Parameter Value ..... **0.05 0.10**

### Output

Click the Calculate button to perform the calculations and generate the following output.

#### Numeric Results

Solve For: [Sample Size](#)

Hypotheses:  $H_0: P_t - P_s \leq D_{0.L} \text{ or } P_t - P_s \geq D_{0.U}$  vs.  $H_1: D_{0.L} < P_t - P_s < D_{0.U}$

Power*	Sample Size N	Equivalence Differences		Actual Difference D1	Proportions		Nuisance Parameter P01	Alpha
		Lower D0.L	Upper D0.U		Treatment Pt	Standard Ps		
0.90019	468	-0.05	0.05	0	0.8	0.8	0.05	0.05
0.90002	881	-0.05	0.05	0	0.8	0.8	0.10	0.05

\* Power was computed using the normal approximation method.

This report shows that the sample size required nearly doubles when P01 is changed from 0.05 to 0.10.

## Example 3 – Finding Power Following an Experiment

An experiment involving a single group of 57 subjects was run to show that a new treatment was equivalent to a previously used standard. Historically, the standard treatment has had a 48% success rate. The new treatment is known to have similar side effects to the standard but is much less expensive. The treatments were to be considered equivalent if the success rate of the new treatment is within 10% of the success rate of the standard.

To compare the new and standard treatments, each of the 57 subjects received both treatments with a washout period between them. Thus, the proportions based on the two treatments are correlated. Of the 57 subjects, 18 responded to both treatments, 20 did not respond to either treatment, 9 responded to the new treatment but not the standard, and 10 responded to the standard but not the new treatment. The proportion responding to the new treatment is  $(18+9)/57 = 0.4737$ . The proportion responding to the standard is  $(18+10)/57 = 0.4912$ . The difference is 0.0175, lower than the threshold for equivalence, but the resulting p-value was 0.3358, indicating the two treatments could not be deemed equivalent at the 0.05 level. Note that McNemar's test only uses the discordant pairs, so the effective size of this study is really only  $9 + 10 = 19$ , although 57 subjects were investigated. The researchers want to know the power of the test they used.

It may be the inclination of the researchers to use the observed difference in proportions for calculating power. The p-value, however, is based on the maximum allowable difference for equivalence, which is 10% of 0.48, or 0.048. This is the number that should be used in the power calculation. The experiment gave a value of P01 of  $10/28 = 0.36$ . The power of the experiment is near zero for all values of P01 less than 0.10. We calculate the power for a variety of nuisance parameter values ( $P01 = 0.01, 0.03, 0.05$ , and  $0.10$ ) to monitor its effect. Because it is in fact believed that the success rates are equivalent for the two treatments, the specified actual difference is set to 0.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Power</b>
Power Calculation Method .....	<b>Multinomial Enumeration</b>
Max N for Multinomial Enumeration .....	<b>1000</b>
Alpha .....	<b>0.05</b>
N (Sample Size) .....	<b>57</b>
D0.L (Lower Equivalence Difference) .....	<b>-0.048</b>
D1 (Actual Difference) .....	<b>0.0</b>
Ps (Standard Proportion) .....	<b>0.48</b>
Nuisance Parameter Type .....	<b>P01 (% -Trt +Std)</b>
Nuisance Parameter Value .....	<b>0.01 0.03 0.05 0.10</b>

## Equivalence Tests for the Difference Between Two Correlated Proportions

## Output

Click the Calculate button to perform the calculations and generate the following output.

## Numeric Results

Solve For: [Power](#)

Hypotheses:  $H_0: P_t - P_s \leq D_{0.L} \text{ or } P_t - P_s \geq D_{0.U}$  vs.  $H_1: D_{0.L} < P_t - P_s < D_{0.U}$

	Sample Size N	Equivalence Differences		Actual Difference D1	Proportions		Nuisance Parameter P01	Target Alpha†
		Lower D0.L	Upper D0.U		Treatment Pt	Standard Ps		
<b>Power*</b>								
0.31614	57	-0.048	0.048	0	0.48	0.48	0.01	0.05
0.02940	57	-0.048	0.048	0	0.48	0.48	0.03	0.05
0.00247	57	-0.048	0.048	0	0.48	0.48	0.05	0.05
0.00000	57	-0.048	0.048	0	0.48	0.48	0.10	0.05

\* Power was computed using multinomial enumeration of all possible outcomes.

† Warning: For small values of N (i.e., less than 100), power computed by Multinomial Enumeration may be overly optimistic because the discrete nature of the multinomial distribution results in the actual alpha value being higher than its target. To be safe, we recommend that you use the power calculation based on the normal approximation.

Note that there is no power for value of P01 greater than 0.05. This is probably due to the low number of discordant pairs.

## Example 4 – Validation using Liu et al. (2002)

Liu *et al.* (2002) page 238 gives a table of power values calculated by multinomial enumeration for sample sizes of 50, 100, and 200 when the significance level is 0.05. From this table, we find that when P01 is 0.10, P10 is 0.10, D1 = P01 - P10 = 0.00, and the equivalence difference is 0.10, the three power values are 0.026, 0.417, and 0.861 for the column head “RMLE-based Without CC” (this is the case we use).

In their calculations, they round the z value to 1.64. This corresponds to an alpha value of 0.0505025835. So that our results match, we will use this value for alpha rather than 0.05. In this example, the value of Ps is not used.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For ..... **Power**  
 Power Calculation Method ..... **Multinomial Enumeration**  
 Max N for Multinomial Enumeration ..... **1000**  
 Alpha ..... **0.0505025835**  
 N (Sample Size) ..... **50 100 200**  
 D0.L (Lower Equivalence Difference) ..... **-0.1**  
 D1 (Actual Difference) ..... **0.0**  
 Ps (Standard Proportion) ..... **0.5**  
 Nuisance Parameter Type ..... **P01 (% -Trt +Std)**  
 Nuisance Parameter Value ..... **0.1**

### Output

Click the Calculate button to perform the calculations and generate the following output.

#### Numeric Results

Solve For: **Power**

Hypotheses: H0:  $P_t - P_s \leq D0.L$  or  $P_t - P_s \geq D0.U$  vs. H1:  $D0.L < P_t - P_s < D0.U$

Power*	Sample Size N	Equivalence Differences		Actual Difference D1	Proportions		Nuisance Parameter P01	Target Alpha†
		Lower D0.L	Upper D0.U		Treatment Pt	Standard Ps		
0.02614	50	-0.1	0.1	0	0.5	0.5	0.1	0.051
0.41741	100	-0.1	0.1	0	0.5	0.5	0.1	0.051
0.86080	200	-0.1	0.1	0	0.5	0.5	0.1	0.051

\* Power was computed using multinomial enumeration of all possible outcomes.

As you can see, the values computed by **PASS** match the results of Liu et al. (2002).