Chapter 399

GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Continuous Outcome)

Introduction

This module calculates the power for testing the difference between two slopes from continuous, correlated data that are analyzed using the GEE method. Such data occur in two design types: clustered and longitudinal. Two popular approaches are used to analyze such data, mixed models (MM) and generalized estimation equation (GEE).

GEE is different from MM in that it does not require the full specification of the joint distribution of the repeated measurements, as long as the marginal mean model is correctly specified. Estimation consistency is achieved even if the correlation matrix is incorrect. Also, the correlation matrix of the responses is specified directly, rather than using an intermediate, random effects model as is the case in MM. For clustered designs, GEE often uses a *compound symmetric* (CS) correlation structure. For longitudinal data, an *autoregressive* (AR(1)) correlation structure is often used.

Missing Values

This procedure allows you to specify various patterns of incomplete (or missing) data. Subjects may miss some appointments but attend others. This phenomenon of incomplete data can be accounted for in the sample size calculation which can greatly reduce the overall sample size from that calculated by just omitting subjects with incomplete observations.

Technical Details

Theory and Notation

Technical details are given in Diggle, Heagerty, Liang, and Zeger (2013). The details of the calculation of sample size and power is given in Ahn, Heo, and Zhang (2015), section 4.3.1, pages 90-93, 108. See also Jung and Ahn (2003).

Suppose we have n_1 subjects in group 1 (treatment) and n_2 subjects in group 2 (control) for a total of N subjects, each measured on M occasions at times t_j (j = 1, ..., M). For convenience, we normalize these time points to the proportion of total time so that t_1 = 0 and t_M = 1. The mean of y_{ij} is modeled by

$$\mu_{ij} = \beta_1 + \beta_2 r_i + \beta_3 t_{ij} + \beta_4 r_i t_{ij}$$

where

y_{ij} is the j^{th} response from subject i, with variance given by σ^2.

μ_{ij} is expectation of y_{ij},

r_i is the subject treatment indicator with 0 for control and 1 for treatment,

β_1 is the regression coefficient giving intercept of the control group,

β_2 is the regression coefficient giving slope of the control group,

β_3 is the regression coefficient giving difference in the intercepts of the two groups,

β_4 is the regression coefficient giving difference in the slopes of the two groups.

In this procedure, the primary interest is on β_4, which gives the treatment effect of the rate of change of the two groups.

This mean model is reparametrized as

$$\mu_{ij} = b_1 + b_2(r_i - \bar{r}) + b_3 t_{ij} + \beta_4(r_i - \bar{r})t_{ij}$$

where

$$b_1 = \beta_1 + \bar{r}\beta_2$$

$$b_2 = \beta_2$$

$$b_3 = \beta_3 + \bar{r}\beta_4$$

$$b_4 = \beta_4 = \delta$$

The vector of covariates is given by $x_{ij} = \left(1, r_i - \bar{r}, t_{ij}, (r_i - \bar{r})t_{ij}\right)'$.

GEE is used to estimate and test hypotheses about $\boldsymbol{b}$ with $\hat{\boldsymbol{b}}$.

Interpretation of β4

Note that we could have modeled these data using separate regression equations for each group. The models would be

$$\mu_{kij} = a_k + b_k t_j, k = 1,2$$

It is often realistic to conclude that $a_1 = a_2$ since subjects are randomly assigned to the groups at the beginning of the study and since $t_1 = 0$. Since t_M is one, we have

$$\begin{aligned}\mu_{1iM} - \mu_{2iM} &= (a_1 + b_1 t_M) - (a_2 + b_2 t_M) \\ &= (a_1 + b_1(1)) - (a_1 + b_2(1)) \\ &= b_1 - b_2\end{aligned}$$

This shows that <u>the difference in means at the end of the study is equal to the difference in slopes</u>.

Correlation Patterns

In a longitudinal design with *N* subjects, each measured *M* times, observations from a single subject are correlated, and a pattern of those correlations through time needs to be specified. Several choices are available.

Compound Symmetry

A compound symmetry correlation model assumes that all correlations are equal. That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \rho & \cdots & \rho \\ \rho & \rho & 1 & \rho & \cdots & \rho \\ \rho & \rho & \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \rho & \rho & \cdots & 1 \end{bmatrix}_{M\times M}$$

where ρ is the baseline correlation.

Banded(1)

A Banded(1) (banded order 1) correlation model assumes that correlations for observations one time period apart are equal to ρ, and correlations for measurements greater than one time period apart are equal to zero. That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & 0 & 0 & \cdots & 0 \\ \rho & 1 & \rho & 0 & \cdots & 0 \\ 0 & \rho & 1 & \rho & \cdots & 0 \\ 0 & 0 & \rho & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{M\times M}$$

where ρ is the baseline correlation.

Banded(2)

A Banded(2) (banded order 2) correlation model assumes that correlations for observations one time period or two periods apart are equal to ρ, and correlations for measurements greater than one time period apart are equal to zero. That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho & 0 & \cdots & 0 \\ \rho & 1 & \rho & \rho & \cdots & 0 \\ \rho & \rho & 1 & \rho & \cdots & 0 \\ 0 & \rho & \rho & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{M\times M}$$

where ρ is the baseline correlation.

AR1 (Traditional)

This version of AR1 (autoregressive order 1) correlation model assumes that correlations *t* time periods apart are equal to ρ^t. That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{M-1} \\ \rho & 1 & \rho & \rho^2 & \cdots & \rho^{M-2} \\ \rho^2 & \rho & 1 & \rho & \cdots & \rho^{M-3} \\ \rho^3 & \rho^2 & \rho & 1 & \cdots & \rho^{M-4} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{M-1} & \rho^{M-2} & \rho^{M-3} & \rho^{M-4} & \cdots & 1 \end{bmatrix}_{M\times M}$$

where ρ is the baseline correlation.

AR1 (Proportional)

This version of AR1 (autoregressive order 1) correlation model is described in the book by Ahn et al. (2015). It assumes that correlations $|t_j - t_k|$ time periods apart are equal to $\rho^{|t_j - t_k|}$. That is

$$[\rho_{jk}] = \left[\rho^{|t_j - t_k|}\right]_{M\times M}$$

where ρ is the baseline correlation. Note that in this pattern, the value of ρ is shown in the final column since in this case $t_j = 0$ and $t_k = 1$, so $|t_j - t_k| = 1$.

Damped Exponential

A damped exponential is an extension of the AR(1) correlation model in which the exponents are raised to the power *Dexp* (θ = *Dexp* in the diagram below). This causes the resulting correlations to be reduced (dampened). Here is an example

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho^{2^\theta} & \rho^{3^\theta} & \cdots & \rho^{(M-1)^\theta} \\ \rho & 1 & \rho & \rho^{2^\theta} & \cdots & \rho^{(M-2)^\theta} \\ \rho^{2^\theta} & \rho & 1 & \rho & \cdots & \rho^{(M-3)^\theta} \\ \rho^{3^\theta} & \rho^{2^\theta} & \rho & 1 & \cdots & \rho^{(M-4)^\theta} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{(M-1)^\theta} & \rho^{(M-2)^\theta} & \rho^{(M-3)^\theta} & \rho^{(M-4)^\theta} & \cdots & 1 \end{bmatrix}_{M\times M}$$

where ρ is the baseline correlation.

Damped Exponential (Proportional)

This version of the damped exponential correlation model is described in the book by Ahn et al. (2015). It assumes that correlations $|t_j - t_k|$ time periods apart are equal to $\rho^{|t_j - t_k|^\theta}$. That is

$$[\rho_{jk}] = \left[\rho^{|t_j - t_k|^\theta}\right]_{M\times M}$$

where ρ is the baseline correlation. Note that in this pattern, the value of $\rho^{|t_j - t_k|^\theta}$ turns up in the final column since in this case $t_j = 0$ and $t_k = 1$, so $|t_j - t_k| = 1$.

Linear Exponential Decay

A linear exponential decay correlation model is one in which the exponent of the correlation decays according to a linear equation from 1 at the *Base Time Proportion* to a final value, *Emax*. The resulting pattern looks similar to the damped exponential. Note that the exponents are applied to the absolute difference between the Measurement Time Proportions. This method allows you to easily construct comparable correlation matrices of different dimensions. Otherwise, differences in the resulting power would be more strongly due to differences in the correlation matrices.

Here is an example. Suppose M is 6, ρ = 0.5, *Emax* = 3, the *Base Time Proportion* is 0.20, and the Measurement Time Proportions are (0, 0.2, 0.4, 0.6, 0.8, 1). The following correlation matrix would be obtained

$$[\rho_{jj'}] = \begin{bmatrix} 1 & 0.5 & 0.3536 & 0.25 & 0.1768 & 0.125 \\ 0.5 & 1 & 0.5 & 0.3536 & 0.25 & 0.1768 \\ 0.3536 & 0.5 & 1 & 0.5 & 0.3536 & 0.25 \\ 0.25 & 0.3536 & 0.5 & 1 & 0.5 & 0.3536 \\ 0.1768 & 0.25 & 0.3536 & 0.5 & 0.3536 & 0.5 \\ 0.125 & 0.1768 & 0.25 & 0.3536 & 0.5 & 1 \end{bmatrix}_{M\times M}$$

Note that in the top row, the correlation is 0.5 for the second (0.2 - 0) time point and 0.125 (0.5^3) at the last (1 - 0) time points. The correlations are obtained by raising 0.5 to the appropriate exponent. The linear equation from 1 to 3 results in the exponents 1, 1.5, 2, 2.5, 3 correspondent to the time proportions 0, 0.2, 0.4, 0.6, 0.8, and 1.

As a further example, note that the correlation for the 0.4 time point is, $0.5^{1.5} = 0.35355339 \approx 0.3536$.

This method allows you to compare various values of M while keeping the correlation matrix similar. To see what we mean, consider what the correlation matrix looks like when M is reduced to 4 and the measurement time proportions are set to (0, 0.2, 0.6, 1). It becomes

$$[\rho_{jj'}] = \begin{bmatrix} 1 & 0.5 & 0.25 & 0.125 \\ 0.5 & 1 & 0.5 & 0.25 \\ 0.25 & 0.5 & 1 & 0.5 \\ 0.125 & 0.25 & 0.5 & 1 \end{bmatrix}_{M\times M}$$

Note that the correlation at a measurement time difference of 0.6 is equal to 0.25 in both matrices.

Missing Data Patterns

The problem of missing data occurs for several reasons. In longitudinal studies in which a subject is measured multiple times, missing data becomes more complicated to model because it is possible that a subject is measured only some of the time. In fact, it is probably more common for data to be incomplete than complete. The approach of omitting subjects with incomplete data during the planning phase is very inaccurate. Certainly, subjects with partial measurements are included in the analysis. This procedure provides several missing data patterns to choose from so that your sample size calculations are more realistic.

In the presentation to following, we denote the percent of subjects with a missing response at time point t_j as κ_j. The proportion non-missing at a particular time point is $\phi_j = 1 - \kappa_j$. We will refer to ϕ_j as the *marginal observant probability* at time t_j and $\phi_{jj'}$ as a *joint observant probability* at times t_j and $t_{j'}$.

Pairwise Missing Pattern

The program provides three options for how the pairwise (joint) observant probabilities $\phi_{jj'}$ are calculated. These are

Independent (Ind): $\phi_{jj'} = \phi_j\phi_{j'}, \phi_{jj} = \phi_j$

Monotonic (Mon): $\phi_{jj'} = \phi_k \text{ where } k = \max(j, j')$

Mixture: $\phi_{jj'} = W(\text{Ind}) + (1 - W)(\text{Mon})$ for weighting factor W.

Missing Input Type

The are several ways in which the missing value pattern can be specified. Each missing value pattern is a list of missing proportions at each of the M time points. Each value in the list must be non-negative and less than 1. Possible input choices are

- **Constant = 0**

  All missing proportions are set to 0. That is, there are no missing values.

- **Constant**

  All missing proportions are set to constant value.

- **Piecewise Constant on Spreadsheet**

  A set of missing proportions are defined for several time intervals using the spreadsheet. One column contains the missing proportions for the interval, going down the rows. Another column defines the corresponding upper limit of time proportion of the interval. The lower limit is implied by the limit given immediately above. The program assumes that the first-time interval starts at 0 percent.

- **Linear (Steady Change)**

  The missing proportions fall along a straight-line between 0 and 1 elapsed time. Only the first and last proportions are entered.

- **Piecewise Linear on Spreadsheet**

  The missing proportions fall along a set of connected straight-lines that are defined by two columns on the spreadsheet.

- **List**

  Enter a list of M missing proportions, one for each time point.

- **Multiple Lists on Spreadsheet**

  Select multiple columns containing vertical lists of missing proportions. Each column contains a set of missing proportions in rows, one for each time point.

- **Pairwise Observed Proportions on Spreadsheet**

  Enter an $M \times M$ matrix of observant probabilities by selecting M columns. These observant probabilities are the proportion of the responses for both the row and column time points that are observed.

Sample Size Calculations

The details of the calculation of sample size and power is given in Ahn, Heo, and Zhang (2015), Chapter 4. These are summarized here.

As explained above, GEE is used to estimate the regression coefficients ***b*** with $\hat{\boldsymbol{b}}$. The significance of b_4, the coefficient associated with the difference between the control and treatment slopes, is tested using a Wald statistic from which the following sample size formula is derived

$$n = \frac{\sigma^2 s_t^2 \left(z_{1-\frac{\alpha}{h}} + z_{1-\gamma}\right)^2}{\delta^2 \mu_0^2 \sigma_r^2 \sigma_t^4}$$

where

h = 1 (one-sided test) or 2 (two-sided test)

γ = 1 – power

α = significance level

$s_t^2 = \eta_2 - 2\mu_1\eta_1 + \eta_0\mu_1^2$

$\delta = \beta_4$

$$\mu_0 = \sum_{j=1}^{M} \phi_j$$

$$\mu_1 = \frac{1}{\mu_0}\sum_{j=1}^{M} \phi_j \, t_j$$

$$\mu_2 = \frac{1}{\mu_0}\sum_{j=1}^{M} \phi_j \, t_j^2$$

$$\eta_0 = \sum_{j=1}^{M} \sum_{j'=1}^{M} \phi_{jj'} \rho_{jj'}$$

$$\eta_1 = \sum_{j=1}^{M} \sum_{j'=1}^{M} \phi_{jj'} \rho_{jj'} t_j$$

$$\eta_2 = \sum_{j=1}^{M} \sum_{j'=1}^{M} \phi_{jj'} \rho_{jj'} t_j t_{j'}$$

$$\sigma_t^2 = \mu_2 - \mu_1^2$$

$$\sigma_r^2 = \bar{r}(1 - \bar{r})$$

$$\phi_j = 1 - \kappa_j, \text{where } \kappa_j = \text{proportion missing at the } j^{\text{th}} \text{ time point}$$

$\rho_{jj'}$ is the corresponding element from within-subject correlation matrix

$\phi_{jj'}$ is the joint observant probability of observing both y_{ij} and $y_{ij'}$ for every subject i

Three possible choices are available to calculate $\phi_{jj'}$. These are

Independent: $\phi_{jj'} = \phi_j \phi_{j'}, \phi_{jj} = \phi_j$

Monotonic: $\phi_{jj'} = \phi_k$ where $k = \max(j, j')$

Mixture: $\phi_{jj'} = W(\text{Independent}) + (1 - W)(\text{Monotonic})$ for weighting factor W.

The above formula is easily rearranged to obtain a formula for power.

Example 1 – Determining Sample Size

Researchers are planning a study of the impact of a new drug on heart rate. They want to evaluate the change in heart rate between subjects who take the new drug, and subjects who take a standard drug. Their experimental protocol calls for a baseline heart rate measurement, followed by administration of a certain level of the drug, followed by three additional measurements two days apart. They want to be able to detect a difference in the slopes of the heart rate across time between the two treatments of 5. They want a sensitivity analysis by considering a range of differences from 3 to 8.

Similar studies have found a standard deviation of 9.2 of the residuals within a subject across time. These studies also showed an autocorrelation between adjacent measurements on the same individual of 0.7, so they want to try values of 0.6, 0.7, and 0.8. The researchers assume that first-order autocorrelation adequately represents the autocorrelation pattern. The two-sided test will be conducted at the 0.05 significance level and at 90% power. They are planning on dividing the subjects equally between the treatment and control groups.

The researchers anticipate that the missing pattern across time will begin at 0% missing and increase steadily to 30% at the fourth measurement. They assume that the pairwise missing is *independent*.

What are the sample size requirements for this study?

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Sample Size**
Alternative Hypothesis .... **Two-Sided**
Power... **0.90**
Alpha.. **0.05**
R (Group 1 Allocation %) **50**
Measurement Time Input Type .... **Equally Spaced Measurement Times**
M (Number of Measurement Times) **4**
δ (Difference in Slopes, i.e., Means)............. **3 to 8 by 1**
σ (Standard Deviation).................................. **9.2**
Pattern of ρ's Across Time .... **AR1 (Traditional)**
ρ (Base Correlation)...................................... **0.6, 0.7, 0.8**
Missing Input Type.. **Linear (Steady Change)**
Pairwise Missing Pattern............................... **Independent (Ind)**
First Missing Proportion (Ind)......................... **0**
Last Missing Proportion (Ind)......................... **0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results for a Slope-Difference Test using GEE

Solve For: Sample Size
Measurement Times: Equally spaced
Correlation: AR1: ρ(j,k) = ρ^|j-k|
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Measurement Times M	Difference in Slopes δ	Standard Deviation σ	Base Correlation ρ	First Row of Correlation Matrix	Missing Data Proportions	Measurement Times	Alpha
0.9001	769	50	4	3	9.2	0.6	ρ1(1)	Ms1(1)	T(1)	0.05
0.9002	667	50	4	3	9.2	0.7	ρ2(1)	Ms1(1)	T(1)	0.05
0.9000	529	50	4	3	9.2	0.8	ρ3(1)	Ms1(1)	T(1)	0.05
0.9004	433	50	4	4	9.2	0.6	ρ1(1)	Ms1(1)	T(1)	0.05
0.9001	375	50	4	4	9.2	0.7	ρ2(1)	Ms1(1)	T(1)	0.05
0.9005	298	50	4	4	9.2	0.8	ρ3(1)	Ms1(1)	T(1)	0.05
0.9003	277	50	4	5	9.2	0.6	ρ1(1)	Ms1(1)	T(1)	0.05
0.9001	240	50	4	5	9.2	0.7	ρ2(1)	Ms1(1)	T(1)	0.05
0.9009	191	50	4	5	9.2	0.8	ρ3(1)	Ms1(1)	T(1)	0.05
0.9012	193	50	4	6	9.2	0.6	ρ1(1)	Ms1(1)	T(1)	0.05
0.9006	167	50	4	6	9.2	0.7	ρ2(1)	Ms1(1)	T(1)	0.05
0.9016	133	50	4	6	9.2	0.8	ρ3(1)	Ms1(1)	T(1)	0.05
0.9016	142	50	4	7	9.2	0.6	ρ1(1)	Ms1(1)	T(1)	0.05
0.9013	123	50	4	7	9.2	0.7	ρ2(1)	Ms1(1)	T(1)	0.05
0.9025	98	50	4	7	9.2	0.8	ρ3(1)	Ms1(1)	T(1)	0.05
0.9023	109	50	4	8	9.2	0.6	ρ1(1)	Ms1(1)	T(1)	0.05
0.9008	94	50	4	8	9.2	0.7	ρ2(1)	Ms1(1)	T(1)	0.05
0.9024	75	50	4	8	9.2	0.8	ρ3(1)	Ms1(1)	T(1)	0.05

Item	Values
ρ1(1)	1, 0.6, 0.36, 0.216
ρ2(1)	1, 0.7, 0.49, 0.343
ρ3(1)	1, 0.8, 0.64, 0.512
Ms1(1)	0, 0.1, 0.2, 0.3
T(1)	0, 0.33, 0.67, 1

Power	The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N	The total number of subjects in the study.
R	The treatment group allocation proportion. It is the proportion of subjects that are in the treatment group (group 1).
M	The number of time points at which each subject is measured.
δ	The difference in slopes at which the power is calculated. It is equal to the difference in means at the final measurement.
σ	The standard deviation of a response.
ρ	The base correlation between two responses on the same subject. It may be transformed based on the correlation pattern.
First Row of Correlation Matrix	Presents the top row of the correlation matrix.
Missing Data Proportions	Gives the name of the set containing the missing data proportions across time.
Measurement Times	Gives the name of the set containing the measurement time proportions. These measurement times represent the proportion of the total study time that has elapsed just before the measurement.
Alpha	The probability of rejecting a true null hypothesis.

Summary Statements

A two-group repeated measures design (with a continuous response and with 4 measurements for each subject) will be used to test whether there is a group difference in slopes. The comparison will be made using a two-sided Wald Z-test using GEE methods, with a Type I error rate (α) of 0.05. The (repeated) measurements of each subject will be made at the following 4 times, expressed as proportions of the total study time: 0, 0.33, 0.67, 1. Missing values are assumed to occur completely at random (MCAR). The missing value proportions will be combined to form the pairwise observant probabilities using an independent pairwise missing pattern. The anticipated proportions missing at each measurement time are 0, 0.1, 0.2, 0.3. The first row of the autocorrelation matrix of the responses within a subject is assumed to be 1, 0.6, 0.36, 0.216, with subsequent rows following the same pattern (AR1: ρ(j,k) = ρ^|j-k|). The residual (zero-mean) standard deviation for both groups is assumed to be 9.2. To detect a slope difference (difference in means at the final measurement) of 3 with 90% power, the total number of needed subjects is 769 (with 50% of the subjects in the treatment group (Group 1)).

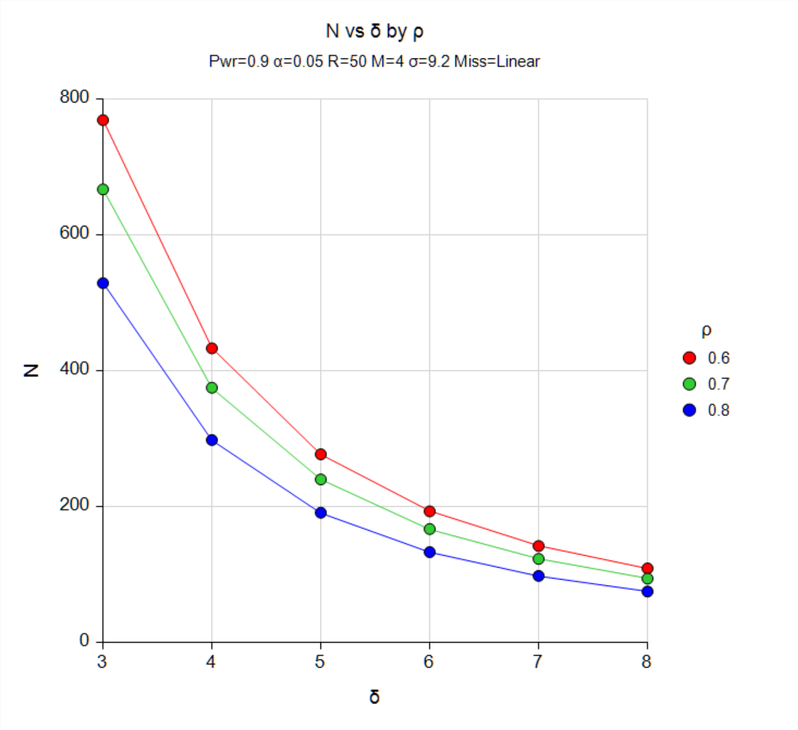
References

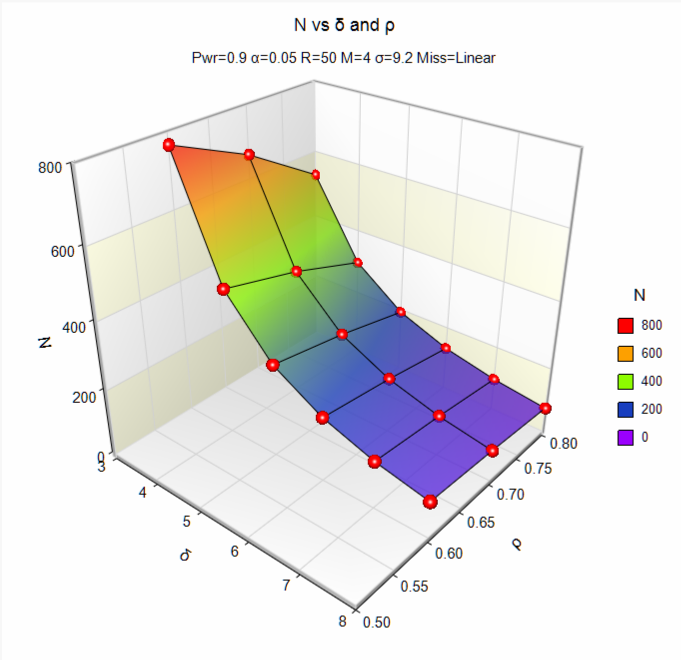
Ahn, C., Heo, M., and Zhang, S. 2015. Sample Size Calculations for Clustered and Longitudinal Outcomes in Clinical Research. CRC Press. New York.
Jung, S.H. and Ahn, C. 2003. Sample size estimation for GEE method for comparing slopes in repeated measures data. Statistics in Medicine, Volume 22, pages 1305-1315.

This report gives the sample size for each value of the other parameters.

Plots Section

Plots





These charts show the relationship between sample size, δ, and ρ when the other parameters in the design are held constant.

Autocorrelation Matrices

Autocorrelation Matrix for Report Row 1

Time	T(0)	T(0.33)	T(0.67)	T(1)
T(0)	1.000	0.60	0.36	0.216
T(0.33)	0.600	1.00	0.60	0.360
T(0.67)	0.360	0.60	1.00	0.600
T(1)	0.216	0.36	0.60	1.000

Autocorrelation Matrix for Report Row 2

Time	T(0)	T(0.33)	T(0.67)	T(1)
T(0)	1.000	0.70	0.49	0.343
T(0.33)	0.700	1.00	0.70	0.490
T(0.67)	0.490	0.70	1.00	0.700
T(1)	0.343	0.49	0.70	1.000

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(More Reports Follow)

These reports show the autocorrelation matrix for the indicated row of the report.

Example 2 – Finding the Power

Continuing with Example 1, the researchers want to determine the power corresponding to sample sizes ranging from 50 to 500 for the main cases of the other parameters.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Two-Sided**
Alpha... **0.05**
N (Subjects) .... **50 to 500 by 50**
R (Group 1 Allocation %) **50**
Measurement Time Input Type .... **Equally Spaced Measurement Times**
M (Number of Measurement Times) **4**
δ (Difference in Slopes, i.e., Means)............. **5**
σ (Standard Deviation).................................. **9.2**
Pattern of ρ's Across Time .... **AR1 (Traditional)**
ρ (Base Correlation)....................................... **0.7**
Missing Input Type.. **Linear (Steady Change)**
Pairwise Missing Pattern............................... **Independent (Ind)**
First Missing Proportion (Ind)........................ **0**
Last Missing Proportion (Ind) **0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

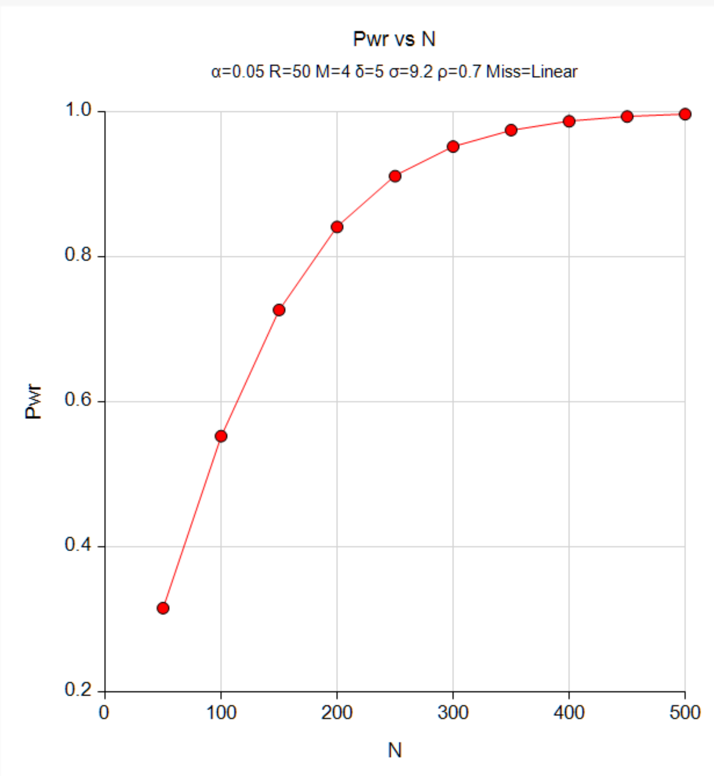
Numeric Results for a Slope-Difference Test using GEE

Solve For: Power
Measurement Times: Equally spaced
Correlation: AR1: ρ(j,k) = ρ^|j-k|
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Measurement Times M	Difference in Slopes δ	Standard Deviation σ	Base Correlation ρ	First Row of Correlation Matrix	Missing Data Proportions	Measurement Times	Alpha
0.3155	50	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.5528	100	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.7267	150	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.8412	200	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9113	250	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9520	300	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9747	350	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9870	400	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9934	450	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9967	500	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05

Item	Values
ρ1(1)	1, 0.7, 0.49, 0.343
Ms1(1)	0, 0.1, 0.2, 0.3
T(1)	0, 0.33, 0.67, 1

Plots



The reports and plot indicate the power for each value of N.

Example 3 – Impact of the Number of Repeated Measurements

Continuing with Examples 1 and 2, the researchers want to study the impact on the sample size of changing the number of measurements made on each individual. Their experimental protocol calls for 4 measurements that are 2 days apart. They want to see the impact of taking measurements daily. To do this, they will need to increase the number of measurements to 7.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Two-Sided**
Alpha... **0.05**
N (Subjects) .... **50 to 500 by 50**
R (Group 1 Allocation %) **50**
Measurement Time Input Type .... **Equally Spaced Measurement Times**
M (Number of Measurement Times) **4 7**
δ (Difference in Slopes, i.e., Means).............. **5**
σ (Standard Deviation).................................. **9.2**
Pattern of ρ's Across Time............................. **AR1 (Traditional)**
ρ (Base Correlation)...................................... **0.7**
Missing Input Type.. **Linear (Steady Change)**
Pairwise Missing Pattern................................ **Independent**
First Missing Proportion (Ind)......................... **0**
Last Missing Proportion (Ind)......................... **0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

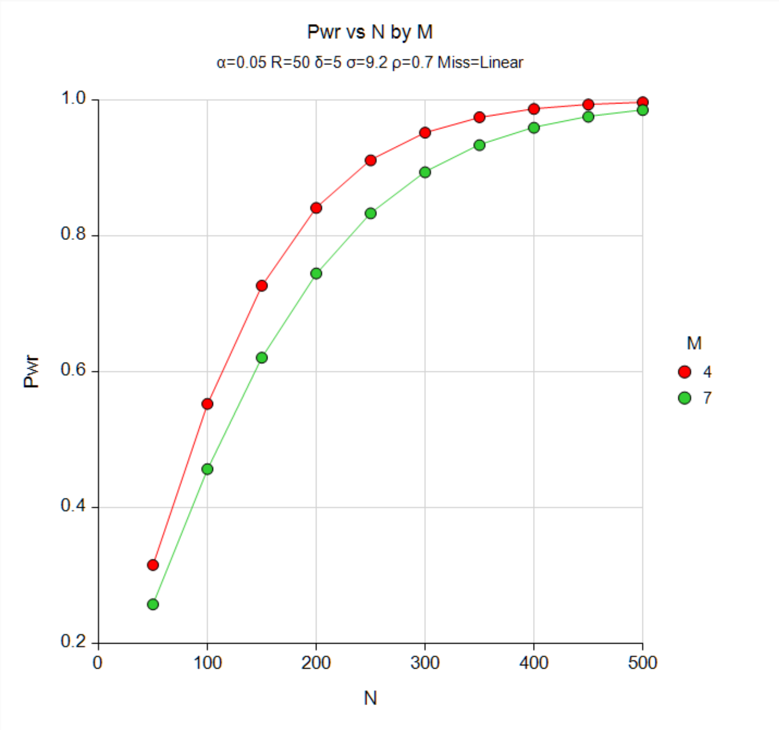
Numeric Results for a Slope-Difference Test using GEE

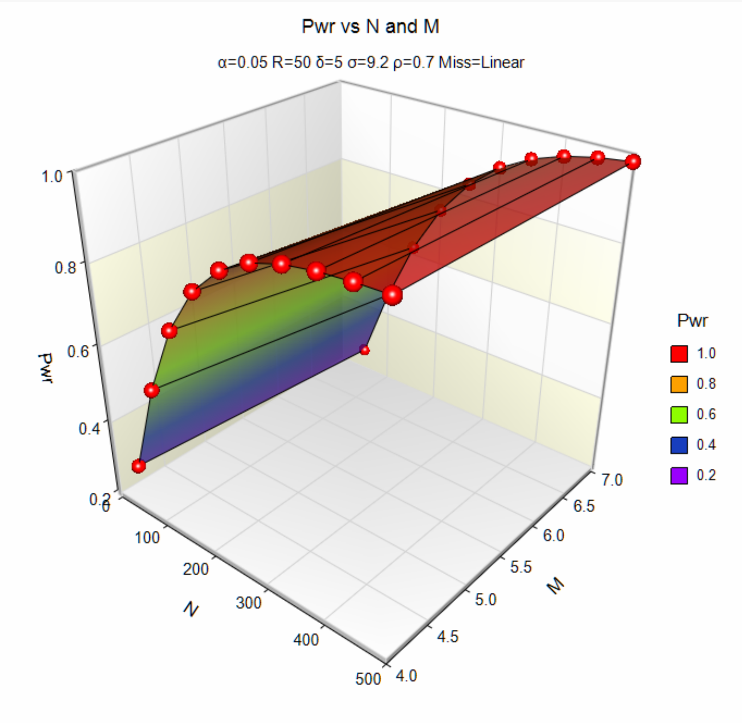
Solve For: Power
Measurement Times: Equally spaced
Correlation: AR1: ρ(j,k) = ρ^|j-k|
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Measurement Times M	Difference in Slopes δ	Standard Deviation σ	Base Correlation ρ	First Row of Correlation Matrix	Missing Data Proportions	Measurement Times	Alpha
0.3155	50	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.2575	50	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.5528	100	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.4567	100	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.7267	150	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.6207	150	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.8412	200	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.7448	200	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9113	250	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.8332	250	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9520	300	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.8937	300	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9747	350	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9336	350	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9870	400	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9593	400	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9934	450	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9754	450	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9967	500	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9854	500	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05

Item	Values
ρ1(1)	1, 0.7, 0.49, 0.343
ρ1(2)	1, 0.7, 0.49, 0.343, 0.2401, 0.1681, 0.1176
Ms1(1)	0, 0.1, 0.2, 0.3
Ms1(2)	0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3
T(1)	0, 0.33, 0.67, 1
T(2)	0, 0.17, 0.33, 0.5, 0.67, 0.83, 1

Plots





Note that increasing the number of measurements has had the surprising result of decreasing the power, probably because the assumption of the AR(1) model for the autocorrelation has changed the way in which the correlations are formed. Note from the footnotes that the final autocorrelation drops from 0.3430 when M = 4 to 0.1176 when M = 7. Look at the next example to see how the autocorrelations can be put on a more equal footing.

Example 4 – Impact of Changing M with Linear Exponential Decay

We saw in Example 3 that the increasing the number of measurements from 4 to 7 had the counter-intuitive result of reducing the power when the sample size was held constant. We surmised that this was partially due to the differing autocorrelation matrices that were used when the AR(1) model as assumed. In this example, we will leave all parameters the same, except that we will use a Linear Exponential Decay model for the autocorrelation. This will keep the autocorrelation matrices more comparable.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Power**
Alternative Hypothesis	**Two-Sided**
Alpha	**0.05**
N (Subjects)	**50 to 500 by 50**
R (Group 1 Allocation %)	**50**
Measurement Time Input Type	**Equally Spaced Measurement Times**
M (Number of Measurement Times)	**4 7**
δ (Difference in Slopes, i.e., Means)	**5**
σ (Standard Deviation)	**9.2**
Pattern of ρ's Across Time	**Linear Exponential Decay**
ρ (Base Correlation)	**0.7**
Base Time Proportion	**0.166666666**
Emax (Max Decay Exponent)	**3**
Missing Input Type	**Linear (Steady Change)**
Pairwise Missing Pattern	**Independent**
First Missing Proportion (Ind)	**0**
Last Missing Proportion (Ind)	**0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

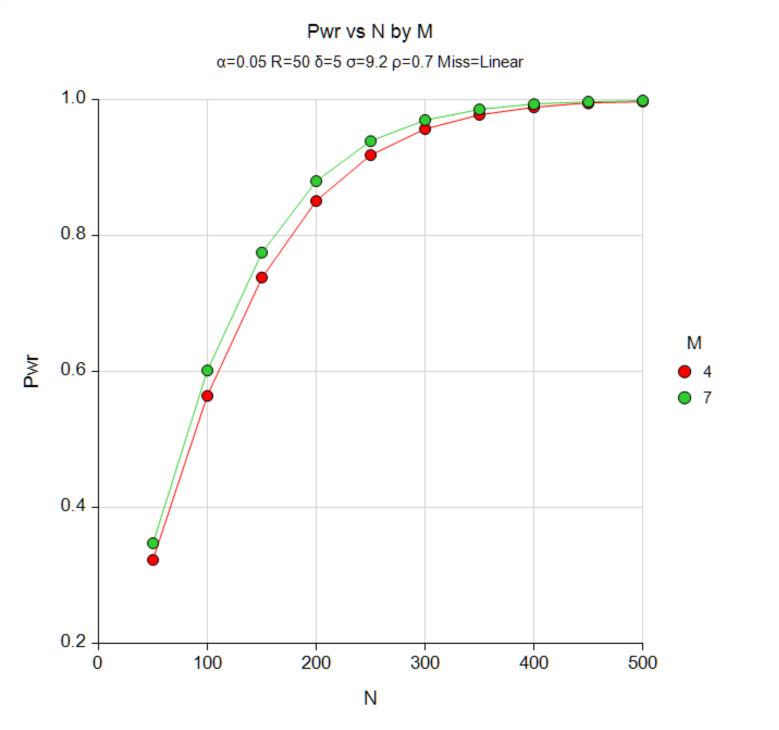
Numeric Results for a Slope-Difference Test using GEE

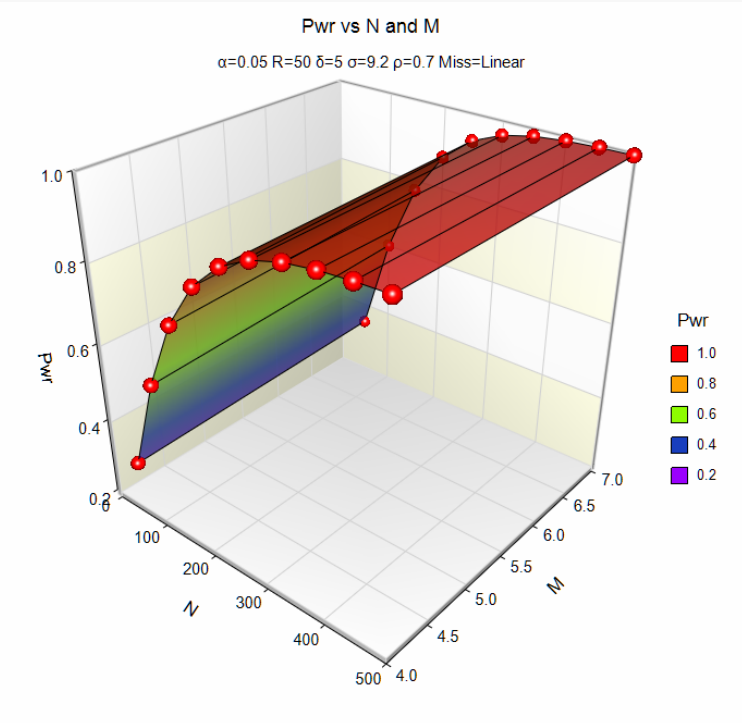
Solve For: Power
Measurement Times: Equally spaced
Correlation: Linear exponential decay, with Emax = 3 and Base Time Prop = 0.16666666
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Measurement Times M	Difference in Slopes δ	Standard Deviation σ	Base Correlation ρ	First Row of Correlation Matrix	Missing Data Proportions	Measurement Times	Alpha
0.3228	50	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.3475	50	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.5642	100	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.6015	100	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.7384	150	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.7750	150	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.8509	200	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.8801	200	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9184	250	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9389	250	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9568	300	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9700	300	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9777	350	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9857	350	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9888	400	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9933	400	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9945	450	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9970	450	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9973	500	50	4	5	9.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9986	500	50	7	5	9.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05

Item	Values
ρ1(1)	1, 0.6069, 0.4563, 0.343
ρ1(2)	1, 0.7, 0.6069, 0.5262, 0.4563, 0.3956, 0.343
Ms1(1)	0, 0.1, 0.2, 0.3
Ms1(2)	0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3
T(1)	0, 0.33, 0.67, 1
T(2)	0, 0.17, 0.33, 0.5, 0.67, 0.83, 1

Plots





Note from the footnotes that the final autocorrelation between the two models is now identical at 0.3430 $(= 0.7^3)$ when M is increased from 4 to 7. Now that the autocorrelation matrices are more comparable, the power values have increased in all cases, although only slightly. We see that increasing M has not had a huge impact on power.

Example 5 – Validation of Sample Size Calculation using Ahn, Heo, and Zhang (2015)

Ahn, Heo, and Zhang (2015) page 110 present sample size results for several scenarios in the following table.

Table of Calculated Sample Sizes

		Monotonic				Independent			
Type	**ρ**	**PM_0**	**PM_1**	**PM_2**	**PM_3**	**PM_0**	**PM_1**	**PM_2**	**PM_3**
CS	0.10	54	88	83	93	54	86	81	90
	0.25	45	82	75	88	45	76	72	80
	0.4	36	77	68	83	36	67	62	71
AR1	0.10	80	127	117	135	80	111	108	114
	0.25	68	117	105	126	68	98	94	101
	0.40	54	105	92	114	54	84	80	87

For this table, the standard deviation is 28.56, the slope difference is 28.6, the significance level is 0.05, the power is 0.90, R is 50%, M is 6, and the two types of correlation matrices are compound symmetry (CS) and AR1. The proportion missing at each time point (PM0 - PM3) are given in the following table.

Table of Proportion Missing Patterns

Time Proportion	PM0	PM1	PM2	PM3
0	0	0	0	0
0.20	0	0.10	0.5	0.20
0.40	0	0.22	0.10	0.40
0.60	0	0.33	0.15	0.46
0.80	0	0.46	0.36	0.52
1.00	0	0.59	0.59	0.59

The sample sizes in the table will be generated in four separate runs.

Setup – Example 5a

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5a** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Alternative Hypothesis ....**Two-Sided**
Power..**0.90**
Alpha..**0.05**
R (Group 1 Allocation %) **50**
Measurement Time Input Type ....**Equally Spaced Measurement Times**
M (Number of Measurement Times) **6**
δ (Difference in Slopes, i.e., Means).............**28.6**
σ (Standard Deviation)..................................**28.56**
Pattern of ρ’s Across Time............................**Compound Symmetry**
ρ (Base Correlation)......................................**0.10, 0.25, 0.40**
Missing Input Type..**Multiple Lists**
Pairwise Missing Pattern...............................**Monotonic (Mon)**
Columns of Missing Proportions (Mon)..........**PM0 – PM3**

Input Spreadsheet Data

Row	PM0	PM1	PM2	PM3
1	0	0.00	0.00	0.00
2	0	0.10	0.05	0.20
3	0	0.22	0.10	0.40
4	0	0.33	0.15	0.46
5	0	0.46	0.37	0.52
6	0	0.59	0.59	0.59

Note that you enter the second table above into the spreadsheet by clicking the spreadsheet icon to the right. You can rename the columns by double-clicking on their original names: C1, C2, C3, C4.

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for a Slope-Difference Test using GEE

Solve For:	Sample Size
Measurement Times:	Equally spaced
Correlation:	Compound symmetry (all ρ's equal)
Missing Pattern:	Lists of missing proportions in columns of the spreadsheet
Observant Proportions:	Assume monotonic decreasing

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Measurement Times M	Difference in Slopes δ	Standard Deviation σ	Base Correlation ρ	First Row of Correlation Matrix	Missing Data Proportions	Measurement Times	Alpha
0.9006	54	50	6	28.6	28.56	0.10	ρ1(1)	PM0	T(1)	0.05
0.9006	88	50	6	28.6	28.56	0.10	ρ1(1)	PM1	T(1)	0.05
0.9020	83	50	6	28.6	28.56	0.10	ρ1(1)	PM2	T(1)	0.05
0.9016	93	50	6	28.6	28.56	0.10	ρ1(1)	PM3	T(1)	0.05
0.9006	45	50	6	28.6	28.56	0.25	ρ2(1)	PM0	T(1)	0.05
0.9003	82	50	6	28.6	28.56	0.25	ρ2(1)	PM1	T(1)	0.05
0.9006	75	50	6	28.6	28.56	0.25	ρ2(1)	PM2	T(1)	0.05
0.9012	88	50	6	28.6	28.56	0.25	ρ2(1)	PM3	T(1)	0.05
0.9006	36	50	6	28.6	28.56	0.40	ρ3(1)	PM0	T(1)	0.05
0.9036	77	50	6	28.6	28.56	0.40	ρ3(1)	PM1	T(1)	0.05
0.9032	68	50	6	28.6	28.56	0.40	ρ3(1)	PM2	T(1)	0.05
0.9008	83	50	6	28.6	28.56	0.40	ρ3(1)	PM3	T(1)	0.05

Item	Values
ρ1(1)	1, 0.1, 0.1, 0.1, 0.1, 0.1
ρ2(1)	1, 0.25, 0.25, 0.25, 0.25, 0.25
ρ3(1)	1, 0.4, 0.4, 0.4, 0.4, 0.4
PM0	0, 0, 0, 0, 0, 0
PM1	0, 0.1, 0.22, 0.33, 0.46, 0.59
PM2	0, 0.05, 0.1, 0.15, 0.37, 0.59
PM3	0, 0.2, 0.4, 0.46, 0.52, 0.59
T(1)	0, 0.2, 0.4, 0.6, 0.8, 1

Note that the sample sizes match those in the table exactly.

Setup – Example 5b

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5b** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Pairwise Missing Pattern................................**Independent (Ind)**
Columns of Missing Proportions (Ind)............**PM0-PM3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for a Slope-Difference Test using GEE

Solve For: Sample Size
Measurement Times: Equally spaced
Correlation: Compound symmetry (all ρ's equal)
Missing Pattern: Lists of missing proportions in columns of the spreadsheet
Observant Proportions: Assume independence

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Measurement Times M	Difference in Slopes δ	Standard Deviation σ	Base Correlation ρ	First Row of Correlation Matrix	Missing Data Proportions	Measurement Times	Alpha
0.9006	54	50	6	28.6	28.56	0.10	ρ1(1)	PM0	T(1)	0.05
0.9022	86	50	6	28.6	28.56	0.10	ρ1(1)	PM1	T(1)	0.05
0.9001	81	50	6	28.6	28.56	0.10	ρ1(1)	PM2	T(1)	0.05
0.9022	90	50	6	28.6	28.56	0.10	ρ1(1)	PM3	T(1)	0.05
0.9006	45	50	6	28.6	28.56	0.25	ρ2(1)	PM0	T(1)	0.05
0.9011	76	50	6	28.6	28.56	0.25	ρ2(1)	PM1	T(1)	0.05
0.9030	72	50	6	28.6	28.56	0.25	ρ2(1)	PM2	T(1)	0.05
0.9010	80	50	6	28.6	28.56	0.25	ρ2(1)	PM3	T(1)	0.05
0.9006	36	50	6	28.6	28.56	0.40	ρ3(1)	PM0	T(1)	0.05
0.9038	67	50	6	28.6	28.56	0.40	ρ3(1)	PM1	T(1)	0.05
0.9024	62	50	6	28.6	28.56	0.40	ρ3(1)	PM2	T(1)	0.05
0.9035	71	50	6	28.6	28.56	0.40	ρ3(1)	PM3	T(1)	0.05

Item	Values
ρ1(1)	1, 0.1, 0.1, 0.1, 0.1, 0.1
ρ2(1)	1, 0.25, 0.25, 0.25, 0.25, 0.25
ρ3(1)	1, 0.4, 0.4, 0.4, 0.4, 0.4
PM0	0, 0, 0, 0, 0, 0
PM1	0, 0.1, 0.22, 0.33, 0.46, 0.59
PM2	0, 0.05, 0.1, 0.15, 0.37, 0.59
PM3	0, 0.2, 0.4, 0.46, 0.52, 0.59
T(1)	0, 0.2, 0.4, 0.6, 0.8, 1

Note that the sample sizes match those in the table exactly.

Setup – Example 5c

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5c** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Pattern of ρ's Across Time............................**AR1 (Proportional)**
Pairwise Missing Pattern...............................**Monotonic (Mon)**
Columns of Missing Proportions (Mon)..........**PM0 – PM3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for a Slope-Difference Test using GEE

Solve For:	Sample Size
Measurement Times:	Equally spaced
Correlation:	AR1 (Proportional): ρ(j,k) = ρ^|t(j)-t(k)|
Missing Pattern:	Lists of missing proportions in columns of the spreadsheet
Observant Proportions:	Assume monotonic decreasing

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Measurement Times M	Difference in Slopes δ	Standard Deviation σ	Base Correlation ρ	First Row of Correlation Matrix	Missing Data Proportions	Measurement Times	Alpha
0.9007	80	50	6	28.6	28.56	0.10	ρ1(1)	PM0	T(1)	0.05
0.9006	127	50	6	28.6	28.56	0.10	ρ1(1)	PM1	T(1)	0.05
0.9002	117	50	6	28.6	28.56	0.10	ρ1(1)	PM2	T(1)	0.05
0.9012	135	50	6	28.6	28.56	0.10	ρ1(1)	PM3	T(1)	0.05
0.9025	68	50	6	28.6	28.56	0.25	ρ2(1)	PM0	T(1)	0.05
0.9010	117	50	6	28.6	28.56	0.25	ρ2(1)	PM1	T(1)	0.05
0.9003	105	50	6	28.6	28.56	0.25	ρ2(1)	PM2	T(1)	0.05
0.9011	126	50	6	28.6	28.56	0.25	ρ2(1)	PM3	T(1)	0.05
0.9003	54	50	6	28.6	28.56	0.40	ρ3(1)	PM0	T(1)	0.05
0.9021	105	50	6	28.6	28.56	0.40	ρ3(1)	PM1	T(1)	0.05
0.9019	92	50	6	28.6	28.56	0.40	ρ3(1)	PM2	T(1)	0.05
0.9003	114	50	6	28.6	28.56	0.40	ρ3(1)	PM3	T(1)	0.05

Item	Values
ρ1(1)	1, 0.631, 0.3981, 0.2512, 0.1585, 0.1
ρ2(1)	1, 0.7579, 0.5743, 0.4353, 0.3299, 0.25
ρ3(1)	1, 0.8326, 0.6931, 0.5771, 0.4804, 0.4
PM0	0, 0, 0, 0, 0, 0
PM1	0, 0.1, 0.22, 0.33, 0.46, 0.59
PM2	0, 0.05, 0.1, 0.15, 0.37, 0.59
PM3	0, 0.2, 0.4, 0.46, 0.52, 0.59
T(1)	0, 0.2, 0.4, 0.6, 0.8, 1

Note that the sample sizes match those in the table exactly.

Setup – Example 5d

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5d** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Pattern of ρ's Across Time............................**AR1 (Proportional)**
Pairwise Missing Pattern...............................**Independent (Ind)**
Columns of Missing Proportions (Ind)...........**PM0-PM3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for a Slope-Difference Test using GEE

Solve For: Sample Size
Measurement Times: Equally spaced
Correlation: AR1 (Proportional): ρ(j,k) = ρ^|t(j)-t(k)|
Missing Pattern: Lists of missing proportions in columns of the spreadsheet
Observant Proportions: Assume independence

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Measurement Times M	Difference in Slopes δ	Standard Deviation σ	Base Correlation ρ	First Row of Correlation Matrix	Missing Data Proportions	Measurement Times	Alpha
0.9007	80	50	6	28.6	28.56	0.10	ρ1(1)	PM0	T(1)	0.05
0.9010	111	50	6	28.6	28.56	0.10	ρ1(1)	PM1	T(1)	0.05
0.9017	108	50	6	28.6	28.56	0.10	ρ1(1)	PM2	T(1)	0.05
0.9019	114	50	6	28.6	28.56	0.10	ρ1(1)	PM3	T(1)	0.05
0.9025	68	50	6	28.6	28.56	0.25	ρ2(1)	PM0	T(1)	0.05
0.9022	98	50	6	28.6	28.56	0.25	ρ2(1)	PM1	T(1)	0.05
0.9014	94	50	6	28.6	28.56	0.25	ρ2(1)	PM2	T(1)	0.05
0.9021	101	50	6	28.6	28.56	0.25	ρ2(1)	PM3	T(1)	0.05
0.9003	54	50	6	28.6	28.56	0.40	ρ3(1)	PM0	T(1)	0.05
0.9030	84	50	6	28.6	28.56	0.40	ρ3(1)	PM1	T(1)	0.05
0.9035	80	50	6	28.6	28.56	0.40	ρ3(1)	PM2	T(1)	0.05
0.9019	87	50	6	28.6	28.56	0.40	ρ3(1)	PM3	T(1)	0.05

Item	Values
ρ1(1)	1, 0.631, 0.3981, 0.2512, 0.1585, 0.1
ρ2(1)	1, 0.7579, 0.5743, 0.4353, 0.3299, 0.25
ρ3(1)	1, 0.8326, 0.6931, 0.5771, 0.4804, 0.4
PM0	0, 0, 0, 0, 0, 0
PM1	0, 0.1, 0.22, 0.33, 0.46, 0.59
PM2	0, 0.05, 0.1, 0.15, 0.37, 0.59
PM3	0, 0.2, 0.4, 0.46, 0.52, 0.59
T(1)	0, 0.2, 0.4, 0.6, 0.8, 1

Note that the sample sizes match those in the table exactly. The program has been validated.

Example 6 – Impact of Measurement Time Distribution

This example will investigate the impact of measurement time on power. It will compare the power of studies that are evenly spaced with those that take more measurements at the beginning of the study, near the middle of the study, and at the end of the study.

In this example the standard deviation is 28.56, the slope difference is 28.6, the significance level is 0.05, the sample size ranges from 40 to 100, and R is 50%. The correlation pattern will be Linear Exponential Decay with a base correlation of 0.4, Base Time Proportion of 10, and Emax set to 3. The missing input type will be set to Linear from 0 to 30% and the pairwise missing assumption will be independent.

The measurement times for five scenarios are given in the following table.

Table of Measurement Times in Proportion of Total Study Time

Tm1	Tm2	Tm3	Tm4	Tm5
0	0	0	0	0
0.20	0.60	0.10	0.10	0.45
0.40	0.70	0.20	0.20	0.50
0.60	0.80	0.30	0.80	0.55
0.80	0.90	0.40	0.90	0.60
1.00	1.00	1.00	1.00	1.00

Note that the measurements in Tm1 are evenly spaced, those in Tm2 are loaded near the end, those of Tm3 occur at the beginning, those of Tm4 occur only at the beginning and the end, and those of Tm5 occur mostly near the middle.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 6** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Two-Sided**
Alpha.. **0.05**
N (Subjects) .... **40 to 100 by 20**
R (Group 1 Allocation %) **50**
Measurement Time Input Type .... **Columns of Measurement Time Proportions**
Column(s) of Time Proportions **Tm1-Tm5**
δ (Difference in Slopes, i.e., Means).............. **28.6**
σ (Standard Deviation).................................. **28.56**
Pattern of ρ's Across Time............................ **Linear Exponential Decay**
ρ (Base Correlation)...................................... **0.4**
Base Time Proportion **0.10**
Emax (Max Decay Exponent) **3**
Missing Input Type.. **Linear (Steady Change)**

Pairwise Missing Pattern................................**Independent (Ind)**
First Missing Proportion (Ind).........................**0**
Last Missing Proportion (Ind).........................**0.3**

Input Spreadsheet Data

Row	PM0	PM1	PM2	PM3	C5	Tm1	Tm2	Tm3	Tm4	Tm5
1	0	0.00	0.00	0.00		0.0	0.0	0.0	0.0	0.00
2	0	0.10	0.05	0.20		0.2	0.6	0.1	0.1	0.45
3	0	0.22	0.10	0.40		0.4	0.7	0.2	0.2	0.50
4	0	0.33	0.15	0.46		0.6	0.8	0.3	0.8	0.55
5	0	0.46	0.37	0.52		0.8	0.9	0.4	0.9	0.60
6	0	0.59	0.59	0.59		1.0	1.0	1.0	1.0	1.00

Output

Click the Calculate button to perform the calculations and generate the following output.

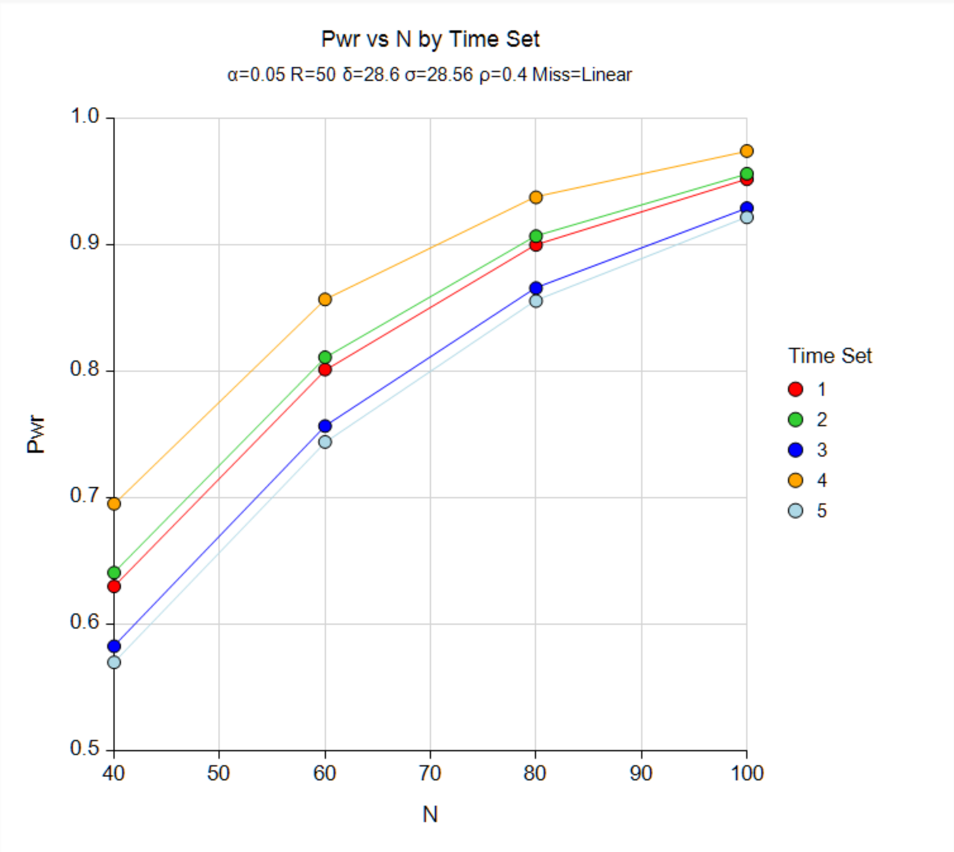
Numeric Results for a Slope-Difference Test using GEE

Solve For: Power
Measurement Times: Lists in spreadsheet columns: {TM1-TM5}
Correlation: Linear exponential decay, with Emax = 3 and Base Time Prop = 0.1
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Measurement Times M	Difference in Slopes δ	Standard Deviation σ	Base Correlation ρ	First Row of Correlation Matrix	Missing Data Proportions	Measurement Times	Alpha
0.6300	40	50	6	28.6	28.56	0.4	ρ1(Tm1)	Ms1(Tm1)	Tm1(1)	0.05
0.6408	40	50	6	28.6	28.56	0.4	ρ1(Tm2)	Ms1(Tm2)	Tm2(2)	0.05
0.5826	40	50	6	28.6	28.56	0.4	ρ1(Tm3)	Ms1(Tm3)	Tm3(3)	0.05
0.6954	40	50	6	28.6	28.56	0.4	ρ1(Tm4)	Ms1(Tm4)	Tm4(4)	0.05
0.5700	40	50	6	28.6	28.56	0.4	ρ1(Tm5)	Ms1(Tm5)	Tm5(5)	0.05
0.8015	60	50	6	28.6	28.56	0.4	ρ1(Tm1)	Ms1(Tm1)	Tm1(1)	0.05
0.8112	60	50	6	28.6	28.56	0.4	ρ1(Tm2)	Ms1(Tm2)	Tm2(2)	0.05
0.7568	60	50	6	28.6	28.56	0.4	ρ1(Tm3)	Ms1(Tm3)	Tm3(3)	0.05
0.8569	60	50	6	28.6	28.56	0.4	ρ1(Tm4)	Ms1(Tm4)	Tm4(4)	0.05
0.7442	60	50	6	28.6	28.56	0.4	ρ1(Tm5)	Ms1(Tm5)	Tm5(5)	0.05
0.8999	80	50	6	28.6	28.56	0.4	ρ1(Tm1)	Ms1(Tm1)	Tm1(1)	0.05
0.9069	80	50	6	28.6	28.56	0.4	ρ1(Tm2)	Ms1(Tm2)	Tm2(2)	0.05
0.8658	80	50	6	28.6	28.56	0.4	ρ1(Tm3)	Ms1(Tm3)	Tm3(3)	0.05
0.9376	80	50	6	28.6	28.56	0.4	ρ1(Tm4)	Ms1(Tm4)	Tm4(4)	0.05
0.8557	80	50	6	28.6	28.56	0.4	ρ1(Tm5)	Ms1(Tm5)	Tm5(5)	0.05
0.9519	100	50	6	28.6	28.56	0.4	ρ1(Tm1)	Ms1(Tm1)	Tm1(1)	0.05
0.9563	100	50	6	28.6	28.56	0.4	ρ1(Tm2)	Ms1(Tm2)	Tm2(2)	0.05
0.9291	100	50	6	28.6	28.56	0.4	ρ1(Tm3)	Ms1(Tm3)	Tm3(3)	0.05
0.9742	100	50	6	28.6	28.56	0.4	ρ1(Tm4)	Ms1(Tm4)	Tm4(4)	0.05
0.9219	100	50	6	28.6	28.56	0.4	ρ1(Tm5)	Ms1(Tm5)	Tm5(5)	0.05

Item	Values
ρ1(Tm1)	1, 0.3263, 0.2172, 0.1445, 0.0962, 0.064
ρ1(Tm2)	1, 0.1445, 0.1179, 0.0962, 0.0785, 0.064
ρ1(Tm3)	1, 0.4, 0.3263, 0.2662, 0.2172, 0.064
ρ1(Tm4)	1, 0.4, 0.3263, 0.0962, 0.0785, 0.064
ρ1(Tm5)	1, 0.1961, 0.1771, 0.16, 0.1445, 0.064
Ms1(Tm1)	0, 0.06, 0.12, 0.18, 0.24, 0.3
Ms1(Tm2)	0, 0.18, 0.21, 0.24, 0.27, 0.3
Ms1(Tm3)	0, 0.03, 0.06, 0.09, 0.12, 0.3
Ms1(Tm4)	0, 0.03, 0.06, 0.24, 0.27, 0.3
Ms1(Tm5)	0, 0.14, 0.15, 0.17, 0.18, 0.3
Tm1(1)	0, 0.2, 0.4, 0.6, 0.8, 1
Tm2(2)	0, 0.6, 0.7, 0.8, 0.9, 1
Tm3(3)	0, 0.1, 0.2, 0.3, 0.4, 1
Tm4(4)	0, 0.1, 0.2, 0.8, 0.9, 1
Tm5(5)	0, 0.45, 0.5, 0.55, 0.6, 1

Plots



The legend, *Time Set*, gives the sequence number of the measurement columns. Thus, 1.0 is Tm1, 2.0 is Tm2, and so on.

The pattern Tm4 consistently produces the highest power across all sample sizes. Remember that Tm4 put the measurements at the beginning and the end, but none in the middle.

Patterns Tm3 and Tm5 are nearly tied for achieving the lowest powers. Tm3 put most of the measurements at the beginning of the study. Tm5 put most of the measurements during the middle of the study.

Note that Tm1, the equally spaced times, is in the middle of the pack.

Example 7 – Entering a Correlation Matrix

This example will show how a correlation matrix can be loaded directly.

In this example, M is 4, the standard deviation is 9.2, the slope difference is 5, the significance level is 0.05, the sample size ranges from 50 to 500, and R is 50%. A correlation matrix (shown below) is available from a previous study. The missing input type will be set to Linear from 0 to 30% and the pairwise missing assumption is independent.

Table of Correlations

C1	C2	C3	C4
1.0000	0.7000	0.4900	0.3430
0.7000	1.0000	0.7000	0.4900
0.4900	0.7000	1.0000	0.7000
0.3430	0.4900	0.7000	1.0000

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 7** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Two-Sided**
Alpha.. **0.05**
N (Subjects) .... **50 to 500 by 50**
R (Group 1 Allocation %) **50**
Measurement Time Input Type .... **Equally Spaced Measurement Times**
M (Number of Measurement Times) **4**
δ (Difference in Slopes, i.e., Means) **5**
σ (Standard Deviation).................................. **9.2**
Pattern of ρ's Across Time............................ **Matrix on Spreadsheet**
Columns Containing the ρjk's **C1-C4**
Missing Input Type....................................... **Linear (Steady Change)**
Pairwise Missing Pattern............................... **Independent (Ind)**
First Missing Proportion (Ind)......................... **0**
Last Missing Proportion (Ind)......................... **0.3**

Input Spreadsheet Data

Row	C1	C2	C3	C4
1	1.000	0.70	0.49	0.343
2	0.700	1.00	0.70	0.490
3	0.490	0.70	1.00	0.700
4	0.343	0.49	0.70	1.000

Output

Click the Calculate button to perform the calculations and generate the following output.

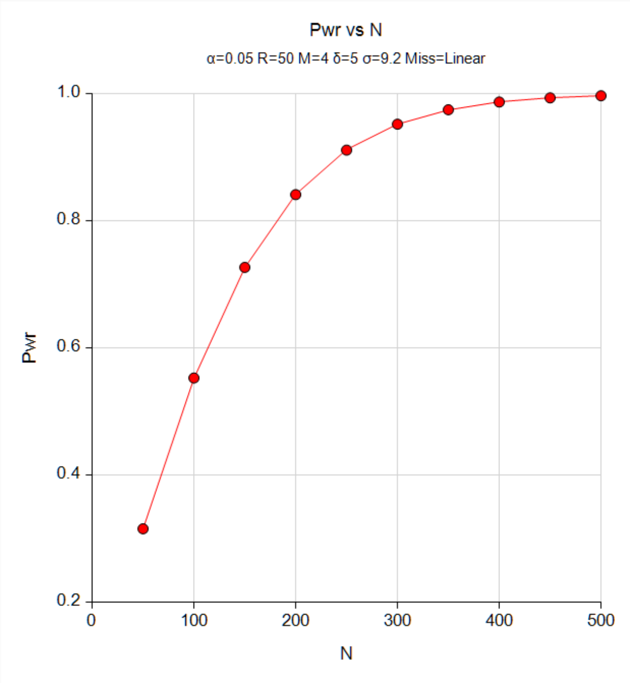
Numeric Results for a Slope-Difference Test using GEE

Solve For: Power
Measurement Times: Equally spaced
Correlation: Matrix stored on spreadsheet in columns C1-C4
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Time Points M	Difference in Slopes δ	Standard Deviation σ	Correlation Matrix Base ρ	Correlation Matrix First Row	Missing Data Proportions	Measurement Times	Alpha
0.3155	50	50	4	5	9.2	n/a	ρ1(1)	Ms1(1)	T(1)	0.05
0.5528	100	50	4	5	9.2	n/a	ρ1(1)	Ms1(1)	T(1)	0.05
0.7267	150	50	4	5	9.2	n/a	ρ1(1)	Ms1(1)	T(1)	0.05
0.8412	200	50	4	5	9.2	n/a	ρ1(1)	Ms1(1)	T(1)	0.05
0.9113	250	50	4	5	9.2	n/a	ρ1(1)	Ms1(1)	T(1)	0.05
0.9520	300	50	4	5	9.2	n/a	ρ1(1)	Ms1(1)	T(1)	0.05
0.9747	350	50	4	5	9.2	n/a	ρ1(1)	Ms1(1)	T(1)	0.05
0.9870	400	50	4	5	9.2	n/a	ρ1(1)	Ms1(1)	T(1)	0.05
0.9934	450	50	4	5	9.2	n/a	ρ1(1)	Ms1(1)	T(1)	0.05
0.9967	500	50	4	5	9.2	n/a	ρ1(1)	Ms1(1)	T(1)	0.05

Item	Values
ρ1(1)	1, 0.7, 0.49, 0.343
Ms1(1)	0, 0.1, 0.2, 0.3
T(1)	0, 0.33, 0.67, 1

Plots



Autocorrelation Matrix for Report Row 1

Time	T(0)	T(0.33)	T(0.67)	T(1)
T(0)	1.000	0.70	0.49	0.343
T(0.33)	0.700	1.00	0.70	0.490
T(0.67)	0.490	0.70	1.00	0.700
T(1)	0.343	0.49	0.70	1.000

Autocorrelation Matrix for Report Row 2

Time	T(0)	T(0.33)	T(0.67)	T(1)
T(0)	1.000	0.70	0.49	0.343
T(0.33)	0.700	1.00	0.70	0.490
T(0.67)	0.490	0.70	1.00	0.700
T(1)	0.343	0.49	0.70	1.000

Autocorrelation Matrix for Report Row 3

Time	T(0)	T(0.33)	T(0.67)	T(1)
T(0)	1.000	0.70	0.49	0.343
T(0.33)	0.700	1.00	0.70	0.490
T(0.67)	0.490	0.70	1.00	0.700
T(1)	0.343	0.49	0.70	1.000

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(More Reports Follow)

The standard reports are displayed.

Example 8 – Entering an Observant Probabilities Matrix

This example will show how an observant probabilities matrix can be loaded directly.

In this example, M is 4, the standard deviation is 9.2, the slope difference is 5, the significance level is 0.05, the sample size ranges from 50 to 500, and R is 50%. The correlation pattern will be Linear Exponential Decay with a base correlation of 0.7, Base Time Proportion of 0.1, and Emax set to 4. The missing input type will be set to Matrix of Pairwise Missing.

Table of Observant Probabilities

Row	C1	C2	C3	C4
1	1.00	0.90	0.80	0.70
2	0.90	0.90	0.72	0.63
3	0.80	0.72	0.80	0.56
4	0.70	0.63	0.56	0.70

This table gives the pairwise observant probabilities. That is, each entry gives the probability of obtaining a response for both the row and column time points. For example, 0.63 is the probability of observing both the second response and the fourth response.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 8** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Two-Sided**
Alpha... **0.05**
N (Subjects) .... **50 to 500 by 50**
R (Group 1 Allocation %) **50**
Measurement Time Input Type .... **Equally Spaced Measurement Times**
M (Number of Measurement Times) **4**
δ (Difference in Slopes, i.e., Means) **5**
σ (Standard Deviation).................................. **9.2**
Pattern of ρ's Across Time............................ **Linear Exponential Decay**
ρ (Base Correlation)....................................... **0.7**
Base Time Proportion **0.1**
Emax (Max Decay Exponent) **4**
Missing Input Type.. **Pairwise Observed Proportions on Spreadsheet**
Columns of Pairwise Observed..................... **C1-C4**

Input Spreadsheet Data

Row	C1	C2	C3	C4
1	1.0	0.90	0.80	0.70
2	0.9	0.90	0.72	0.63
3	0.8	0.72	0.80	0.56
4	0.7	0.63	0.56	0.70

Output

Click the Calculate button to perform the calculations and generate the following output.

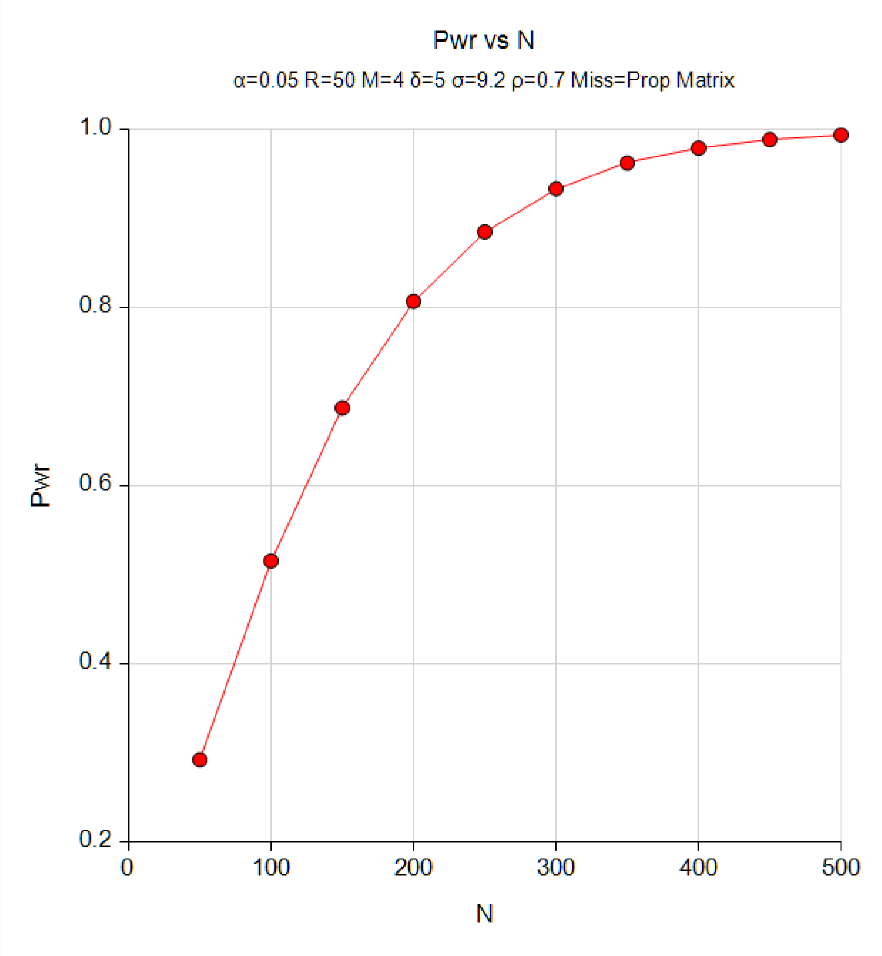
Numeric Results for a Slope-Difference Test using GEE

Solve For: Power
Measurement Times: Equally spaced
Correlation: Linear exponential decay, with Emax = 4 and Base Time Prop = 0.10
Missing Pattern: N/A. Matrix of observant probabilities entered in columns C1-C4.

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Measurement Times M	Difference in Slopes δ	Standard Deviation σ	Base Correlation ρ	First Row of Correlation Matrix	Missing Data Proportions	Measurement Times	Alpha
0.2924	50	50	4	5	9.2	0.7	ρ1(1)	N/A	T(1)	0.05
0.5156	100	50	4	5	9.2	0.7	ρ1(1)	N/A	T(1)	0.05
0.6874	150	50	4	5	9.2	0.7	ρ1(1)	N/A	T(1)	0.05
0.8071	200	50	4	5	9.2	0.7	ρ1(1)	N/A	T(1)	0.05
0.8851	250	50	4	5	9.2	0.7	ρ1(1)	N/A	T(1)	0.05
0.9335	300	50	4	5	9.2	0.7	ρ1(1)	N/A	T(1)	0.05
0.9625	350	50	4	5	9.2	0.7	ρ1(1)	N/A	T(1)	0.05
0.9792	400	50	4	5	9.2	0.7	ρ1(1)	N/A	T(1)	0.05
0.9887	450	50	4	5	9.2	0.7	ρ1(1)	N/A	T(1)	0.05
0.9940	500	50	4	5	9.2	0.7	ρ1(1)	N/A	T(1)	0.05

Item	Values
ρ1(1)	1, 0.5304, 0.3569, 0.2401
T(1)	0, 0.33, 0.67, 1

Plots



The standard reports are displayed.