Chapter 476

Group-Sequential Tests for Two Means (Simulation)

Introduction
This procedure can be used to determine power, sample size and/or boundaries for group sequential tests comparing the means of two groups. The common two-sample T-Test and the Mann-Whitney U test can be simulated in this procedure. For two-sided tests, significance (efficacy) boundaries can be generated. For one-sided tests, significance and futility boundaries can be produced. The spacing of the looks can be equal or custom specified. Boundaries can be computed based on popular alpha- and beta-spending functions (O’Brien-Fleming, Pocock, Hwang-Shih-DeCani Gamma family, linear) or custom spending functions. Boundaries can also be input directly to verify alpha- and/or beta-spending properties. Futility boundaries can be binding or non-binding. Maximum and average (expected) sample sizes are reported as well as the alpha and/or beta spent and incremental power at each look. Corresponding P-Value boundaries are also given for each boundary statistic. Plots of boundaries are also produced.

The distributions of each of the groups under the null and alternative hypotheses can be specified directly using over ten distributions including normal, exponential, Gamma, Uniform, Beta, and Cauchy.

Technical Details
This section outlines many of the technical details of the techniques used in this procedure including the simulation summary, the test statistic details, and the use of spending functions.

An excellent text for the background and details of many group-sequential methods is Jennison and Turnbull (2000).

Simulation Procedure
In this procedure, a large number of simulations are used to calculate boundaries and power using the following steps

1. Based on the specified distributions, random samples of size N1 and N2 are generated under the null distribution and under the alternative distribution. These are simulated samples as though the final look is reached.

2. For each sample, test statistics for each look are produced. For example, if N1 and N2 are 100 and there are 5 equally spaced looks, test statistics are generated from the random samples at N1 = N2 = 20, N1 = N2 = 40, N1 = N2 = 60, N1 = N2 = 80, and N1 = N2 = 100 for both null and alternative samples.
3. To generate the first significance boundary, the null distribution statistics of the first look (e.g., at \( N_1 = N_2 = 20 \)) are ordered and the percent of alpha to be spent at the first look is determined (using either the alpha-spending function or the input value). The statistic for which the percent of statistics above (or below, as the case may be) that value is equal to the percent of alpha to be spent at the first look is the boundary statistic. It is seen here how important a large number of simulations is to the precision of the boundary estimates.

4. All null distribution samples that are outside the first significance boundary at the first look are removed from consideration for the second look. If binding futility boundaries are also being computed, all null distribution samples with statistics that are outside the first futility boundary are also removed from consideration for the second look. If non-binding futility boundaries are being computed, null distribution samples with statistics outside the first futility boundary are not removed.

5. To generate the second significance boundary, the remaining null distribution statistics of the second look (e.g., at \( N_1 = N_2 = 40 \)) are ordered and the percent of alpha to be spent at the second look is determined (again, using either the alpha-spending function or the input value). The percent of alpha to be spent at the second look is multiplied by the total number of simulations to determine the number of the statistic that is to be the second boundary statistic. The statistic for which that number of statistics is above it (or below, as the case may be) is the second boundary statistic. For example, suppose there are initially 1000 simulated samples, with 10 removed at the first look (from, say, alpha spent at Look 1 equal to 0.01), leaving 990 samples considered for the second look. Suppose further that the alpha to be spent at the second look is 0.02. This is multiplied by 1000 to give 20. The 990 still-considered statistics are ordered and the 970th (20 in from 990) statistic is the second boundary.

6. All null distribution samples that are outside the second significance boundary and the second futility boundary, if binding, at the second look are removed from consideration for the third look (e.g., leaving 970 statistics computed at \( N_1 = N_2 = 60 \) to be considered at the third look). Steps 4 and 5 are repeated until the final look is reached.

Futility boundaries are computed in a similar manner using the desired beta-spending function or custom beta-spending values and the alternative hypothesis simulated statistics at each look. For both binding and non-binding futility boundaries, samples for which alternative hypothesis statistics are outside either the significance or futility boundaries of the previous look are excluded from current and future looks.

Because the final futility and significance boundaries are required to be the same, futility boundaries are computed beginning at a small value of beta (e.g., 0.0001) and incrementing beta by that amount until the futility and significance boundaries meet.

When boundaries are entered directly, this procedure uses the null hypothesis and alternative hypothesis simulations to determine the number of test statistics that are outside the boundaries at each look. The cumulative proportion of alternative hypothesis statistics that are outside the significance boundaries is the overall power of the study.

### Generating Random Distributions

Two methods are available in PASS to simulate random samples. The first method generates the random variates directly, one value at a time. The second method generates a large pool (over 10,000) of random values and then draws the random numbers from this pool. This second method can cut the running time of the simulation by 70%.

As mentioned above, the second method begins by generating a large pool of random numbers from the specified distributions. Each of these pools is evaluated to determine if its mean is within a small relative tolerance (0.0001) of the target mean. If the actual mean is not within the tolerance of the target mean, individual members of the population are replaced with new random numbers if the new random number moves the mean towards its target. Only a few hundred such swaps are required to bring the actual mean to within tolerance of the target mean. This population is then sampled with replacement using the uniform distribution. We have found that this method works well as long as the size of the pool is the maximum of twice the number of simulated samples desired and 10,000.
Test Statistics

This section describes the test statistics that are available in this procedure.

**Two-Sample T-Test**

The two-sample t-test assumes that the data are a simple random sample from a population of normally-distributed values that all have the same mean and variance. This assumption implies that the data are continuous and their distribution is symmetric. The calculation of the t statistic is as follows

\[ t_{df} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_{\bar{X}_1-\bar{X}_2}} \]

where

\[ \bar{X}_k = \frac{\sum_{i=1}^{N_k} X_{ki}}{N_k} \]

\[ s_{\bar{X}_1-\bar{X}_2} = \sqrt{\frac{\sum_{i=1}^{N_1} (X_{1i} - \bar{X}_1)^2 + \sum_{i=1}^{N_2} (X_{2i} - \bar{X}_2)^2}{N_1 + N_2 - 2} \left( \frac{1}{N_1} + \frac{1}{N_2} \right)} \]

\[ df = N_1 + N_2 - 2 \]

The significance of the test statistic is determined by computing the p-value based on the t distribution with degrees of freedom \( df \). If this p-value is less than a specified level (often 0.05), the null hypothesis is rejected. Otherwise, no conclusion can be reached.

**Mann-Whitney U Test**

This test is the nonparametric substitute for the equal-variance t-test. Two key assumptions for this test are that the distributions are at least ordinal and that they are identical under H0. This implies that ties (repeated values) are not acceptable. When ties are present, the approximation provided can be used, but know that the theoretic results no longer hold.

The Mann-Whitney test statistic is defined as follows in Gibbons (1985).

\[ z = \frac{W_1 - N_1(N_1 + N_2 + 1)}{2s_w} + C \]

where

\[ W_1 = \sum_{k=1}^{N_1} \text{Rank}(X_{1k}) \]

The ranks are determined after combining the two samples. The standard deviation is calculated as

\[ s_w = \sqrt{\frac{N_1N_2(N_1 + N_2 + 1)}{12} - \frac{N_1N_2\sum_{i=1}^{N_1} (t_i^3 - t_i)}{12(N_1 + N_2)(N_1 + N_2 - 1)}} \]
where \( t_1 \) is the number of observations tied at value one, \( t_2 \) is the number of observations tied at some value two, and so forth.

The correction factor, \( C \), is 0.5 if the rest of the numerator of \( z \) is negative or -0.5 otherwise. The value of \( z \) is then compared to the standard normal distribution.

### Standard Deviations

Care must be used when either the null or alternative distribution is not normal. In these cases, the standard deviation is usually not specified directly. For example, you might use a gamma distribution with a shape parameter of 1.5 and a mean of 4 as the null distribution and a gamma distribution with the same shape parameter and a mean of 5 as the alternative distribution. This allows you to compare the two means. However, although the shape parameters are constant, the standard deviations, which are based on both the shape parameter and the mean, are not. Thus the distributions not only have different means, but also different standard deviations.

### Spending Functions

Spending functions can be used in this procedure to specify the proportion of alpha or beta that is spent at each look without having to specify the proportion directly.

Spending functions have the characteristics that they are increasing and that

\[
\alpha(0) = 0 \\
\alpha(1) = \alpha
\]

The last characteristic guarantees a fixed \( \alpha \) level when the trial is complete. This methodology is very flexible since neither the times nor the number of analyses must be specified in advance. Only the functional form of \( \alpha(\tau) \) must be specified.

\( \text{PASS} \) provides several popular spending functions plus the ability to enter and analyze your own percents of alpha or beta spent. These are calculated as follows (beta may be substituted for alpha for beta-spending functions):

1. Hwang-Shih-DeCani (gamma family) \[ \alpha \left[ \frac{1 - e^{-\gamma \tau}}{1 - e^{-\gamma}} \right], \gamma \neq 0 \; ; \; \alpha, \gamma = 0 \]
2. **O’Brien-Fleming Analog**  
\[
2 - 2\Phi\left(\frac{Z_{\alpha/2}}{\sqrt{t}}\right)
\]

3. **Pocock Analog**  
\[
\alpha \cdot \ln(1 + (e - 1)t)
\]

4. **Alpha * time**  
\[
\alpha \cdot t
\]
5. \textbf{Alpha * time}^{1.5} \quad \alpha \cdot t^{3/2}

6. \textbf{Alpha * time}^{2} \quad \alpha \cdot t^{2}

7. \textbf{Alpha * time}^{C} \quad \alpha \cdot t^{C}

8. \textbf{User Supplied Percents}

A custom set of percents of alpha to be spent at each look may be input directly.

The O'Brien-Fleming Analog spends very little alpha or beta at the beginning and much more at the final looks. The Pocock Analog and \((\text{Alpha or Beta})(\text{Time})\) spending functions spend alpha or beta more evenly across the looks. The Hwang-Shih-DeCani \((C)\) (gamma family) spending functions and \((\text{Alpha or Beta})(\text{Time}^{C})\) spending functions are flexible spending functions that can be used to spend more alpha or beta early or late or evenly, depending on the choice of \(C\).
Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design, Looks & Boundaries, and Options tabs. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options for the general setup of the procedure.

Solve For

Solve For

Solve for either power, sample size, or enter the boundaries directly and solve for power and alpha.

When solving for power or sample size, the early-stopping boundaries are also calculated. High accuracy for early-stopping boundaries requires a very large number of simulations (Recommended 100,000 to 10,000,000).

The parameter selected here is the parameter displayed on the vertical axis of the plot.

Because this is a simulation based procedure, the search for the sample size may take several minutes or hours to complete. You may find it quicker and more informative to solve for Power for a range of sample sizes.

Test and Simulations

Test Type

Specify which test statistic is to be reported on.

The T-Test is the standard T-test based on the pooled variance and SS1 + SS2 - 2 degrees of freedom, where SS1 and SS2 are the sample sizes of groups 1 and 2 at the corresponding look.

The Mann-Whitney Test is the common non-parametric alternative and is based on the ranks of the observations. It is sometimes called the Mann-Whitney U, Mann-Whitney-Wilcoxon, Wilcoxon Rank-Sum, or Wilcoxon-Mann-Whitney test.

Alternative Hypothesis

Specify the alternative hypothesis of the test. Since the null hypothesis is the opposite, specifying the alternative is the only hypothesis needed.

When a two-sided alternative is selected, futility boundaries are not permitted. If you wish to do a two-sided test with futility boundaries you may consider two one-sided tests.

In each case, Mean1 - Mean2 is the true difference between population means. Diff0 is the difference in population means under the null hypothesis, and is usually 0 for a superiority/significance/efficacy test.

Care need be taken in the choice of direction of the test relative to the choice of M0 and M1. For example, for a one-sided test to show that the true mean for group 1 (control) is less than the true mean for group 2 (treatment), M1 should be larger than M0 and the alternative hypothesis chosen should be Mean1 - Mean2 < Diff0.

Simulations

This option specifies the number of iterations, M, used in the simulation. As the number of iterations is increased, the accuracy and running time of the simulation will be increased also.

The precision of the simulated power estimates are calculated from the binomial distribution. Thus, confidence intervals may be constructed for various power values. The following table gives an estimate of the precision that is achieved for various simulation sizes when the power is either 0.50 or 0.95. The table values are interpreted as
follows: a 95% confidence interval of the true power is given by the power reported by the simulation plus and minus the ‘Precision’ amount given in the table.

<table>
<thead>
<tr>
<th>Simulation Size</th>
<th>Precision when Power = 0.50</th>
<th>Precision when Power = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.100</td>
<td>0.044</td>
</tr>
<tr>
<td>500</td>
<td>0.045</td>
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<td>0.006</td>
</tr>
<tr>
<td>10000</td>
<td>0.010</td>
<td>0.004</td>
</tr>
<tr>
<td>50000</td>
<td>0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>100000</td>
<td>0.003</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Notice that a simulation size of 1000 gives a precision of plus or minus 0.01 when the true power is 0.95. Also note that as the simulation size is increased beyond 5000, there is only a small amount of additional accuracy achieved.

However, when solving for Power or N1, the simulations are also used to calculate the look boundaries. To obtain precise boundary estimates, the number of simulations needs to be high. This consideration competes with the length of time to complete the simulation. When solving for power, a large number of simulations (100,000 or 1,000,000) will finish in several minutes. When solving for N1, perhaps 10,000 simulations can be run for each iteration. Then, a final run with the resulting N1 solving for power can be run with more simulations.

### Power and Alpha

#### Power

Power is the probability of rejecting the null hypothesis when it is false. Power is equal to 1-Beta, so specifying power implicitly specifies beta.

Beta is the probability obtaining a false negative on the statistical test. That is, it is the probability of accepting a false null hypothesis.

In the context of simulated group sequential trials, the power is the proportion of the alternative hypothesis simulations that cross any one of the significance (efficacy) boundaries.

The valid range is between 0 and 1.

Different disciplines and protocols have different standards for setting power. A common choice is 0.90, but 0.80 is also popular.

You can enter a range of values such as 0.70 0.80 0.90 or 0.70 to 0.90 by 0.1.

#### Alpha

Alpha is the probability of obtaining a false positive on the statistical test. That is, it is the probability of rejecting a true null hypothesis.

The null hypothesis is usually that the parameters (the means, proportions, etc.) are all equal.

In the context of simulated group sequential trials, alpha is the proportion of the null hypothesis simulations that cross any one of the significance (efficacy) boundaries.

Since Alpha is a probability, it is bounded by 0 and 1. Commonly, it is between 0.001 and 0.250.

Alpha is often set to 0.05 for two-sided tests and to 0.025 for one-sided tests.

You may enter a range of values such as 0.01 0.05 0.10 or 0.01 to 0.10 by 0.01.
Sample Size (When Solving for Sample Size)

Group Allocation
Select the option that describes the constraints on \( N1 \) or \( N2 \) or both.

The options are

- **Equal (N1 = N2)**
  This selection is used when you wish to have equal sample sizes in each group. Since you are solving for both sample sizes at once, no additional sample size parameters need to be entered.

- **Enter N2, solve for N1**
  Select this option when you wish to fix \( N2 \) at some value (or values), and then solve only for \( N1 \). Please note that for some values of \( N2 \), there may not be a value of \( N1 \) that is large enough to obtain the desired power.

- **Enter R = N2/N1, solve for N1 and N2**
  For this choice, you set a value for the ratio of \( N2 \) to \( N1 \), and then PASS determines the needed \( N1 \) and \( N2 \), with this ratio, to obtain the desired power. An equivalent representation of the ratio, \( R \), is
  \[
  N2 = R \times N1. 
  \]

- **Enter percentage in Group 1, solve for N1 and N2**
  For this choice, you set a value for the percentage of the total sample size that is in Group 1, and then PASS determines the needed \( N1 \) and \( N2 \) with this percentage to obtain the desired power.

**N2 (Sample Size, Group 2)**
*This option is displayed if Group Allocation = “Enter N2, solve for N1”*

\( N2 \) is the number of items or individuals sampled from the Group 2 population.

\( N2 \) must be \( \geq 2 \). You can enter a single value or a series of values.

**R (Group Sample Size Ratio)**
*This option is displayed only if Group Allocation = “Enter R = N2/N1, solve for N1 and N2.”*

\( R \) is the ratio of \( N2 \) to \( N1 \). That is,

\[
R = \frac{N2}{N1}. 
\]

Use this value to fix the ratio of \( N2 \) to \( N1 \) while solving for \( N1 \) and \( N2 \). Only sample size combinations with this ratio are considered.

\( N2 \) is related to \( N1 \) by the formula:

\[
N2 = \lceil R \times N1 \rceil, 
\]

where the value \( \lceil Y \rceil \) is the next integer \( \geq Y \).

For example, setting \( R = 2.0 \) results in a Group 2 sample size that is double the sample size in Group 1 (e.g., \( N1 = 10 \) and \( N2 = 20 \), or \( N1 = 50 \) and \( N2 = 100 \)).

\( R \) must be greater than 0. If \( R < 1 \), then \( N2 \) will be less than \( N1 \); if \( R > 1 \), then \( N2 \) will be greater than \( N1 \). You can enter a single or a series of values.
Percent in Group 1

This option is displayed only if Group Allocation = “Enter percentage in Group 1, solve for \(N_1\) and \(N_2\).”

Use this value to fix the percentage of the total sample size allocated to Group 1 while solving for \(N_1\) and \(N_2\). Only sample size combinations with this Group 1 percentage are considered. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single or a series of values.

Sample Size (When Not Solving for Sample Size)

Group Allocation

Select the option that describes how individuals in the study will be allocated to Group 1 and to Group 2.

The options are

- **Equal (\(N_1 = N_2\))**
  This selection is used when you wish to have equal sample sizes in each group. A single per group sample size will be entered.

- **Enter \(N_1\) and \(N_2\) individually**
  This choice permits you to enter different values for \(N_1\) and \(N_2\).

- **Enter \(N_1\) and \(R\), where \(N_2 = R \times N_1\)**
  Choose this option to specify a value (or values) for \(N_1\), and obtain \(N_2\) as a ratio (multiple) of \(N_1\).

- **Enter total sample size and percentage in Group 1**
  Choose this option to specify a value (or values) for the total sample size \((N)\), obtain \(N_1\) as a percentage of \(N\), and then \(N_2\) as \(N - N_1\).

**Sample Size Per Group**

This option is displayed only if Group Allocation = “Equal (\(N_1 = N_2\)).”

The Sample Size Per Group is the number of items or individuals sampled from each of the Group 1 and Group 2 populations. Since the sample sizes are the same in each group, this value is the value for \(N_1\), and also the value for \(N_2\).

The Sample Size Per Group must be \(\geq 2\). You can enter a single value or a series of values.

**\(N_1\) (Sample Size, Group 1)**

This option is displayed if Group Allocation = “Enter \(N_1\) and \(N_2\) individually” or “Enter \(N_1\) and \(R\), where \(N_2 = R \times N_1\).”

\(N_1\) is the number of items or individuals sampled from the Group 1 population.

\(N_1\) must be \(\geq 2\). You can enter a single value or a series of values.

**\(N_2\) (Sample Size, Group 2)**

This option is displayed only if Group Allocation = “Enter \(N_1\) and \(N_2\) individually.”

\(N_2\) is the number of items or individuals sampled from the Group 2 population.

\(N_2\) must be \(\geq 2\). You can enter a single value or a series of values.
R (Group Sample Size Ratio)

This option is displayed only if Group Allocation = “Enter N1 and R, where N2 = R * N1.”

R is the ratio of N2 to N1. That is,

\[ R = \frac{N_2}{N_1} \]

Use this value to obtain N2 as a multiple (or proportion) of N1.

N2 is calculated from N1 using the formula:

\[ N_2 = \lceil R \times N_1 \rceil, \]

where the value \( \lceil Y \rceil \) is the next integer \( \geq Y \).

For example, setting \( R = 2.0 \) results in a Group 2 sample size that is double the sample size in Group 1.

R must be greater than 0. If \( R < 1 \), then \( N_2 \) will be less than \( N_1 \); if \( R > 1 \), then \( N_2 \) will be greater than \( N_1 \). You can enter a single value or a series of values.

Total Sample Size (N)

This option is displayed only if Group Allocation = “Enter total sample size and percentage in Group 1.”

This is the total sample size, or the sum of the two group sample sizes. This value, along with the percentage of the total sample size in Group 1, implicitly defines \( N_1 \) and \( N_2 \).

The total sample size must be greater than one, but practically, must be greater than 3, since each group sample size needs to be at least 2.

You can enter a single value or a series of values.

Percent in Group 1

This option is displayed only if Group Allocation = “Enter total sample size and percentage in Group 1.”

This value fixes the percentage of the total sample size allocated to Group 1. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single value or a series of values.

Effect Size

Group 1 (and 2) Distribution|H0

Specify the distributions of groups 1 and 2 to be used in the simulations for finding the actual significance level. These distributions also specify the value of the mean under the null hypothesis, H0. Usually, these two distributions will be identical. However, if you are planning a non-inferiority test, the means will be different.

The parameters of the distribution can be specified using numbers or letters. If letters are used, their values are specified in the boxes below. The value “M0” is reserved for the value of the mean under the null hypothesis.

Following is a list of the distributions that are available and the syntax used to specify them. Each of the parameters should be replaced with a number or parameter name.
Distributions with Common Parameters
Beta(Shape1, Shape2, Min, Max)
Binomial(P, N)
Cauchy(Mean, Scale)
Constant(Value)
Exponential(Mean)
Gamma(Shape, Scale)
Gumbel(Location, Scale)
Laplace(Location, Scale)
Logistic(Location, Scale)
Lognormal(Mu, Sigma)
Multinomial(P1, P2, P3, ..., Pk)
Normal(Mean, Sigma)
Poisson(Mean)
TukeyGH(Mu, S, G, H)
Uniform(Min, Max)
Weibull(Shape, Scale)

Distributions with Mean and SD Parameters
BetaMS(Mean, SD, Min, Max)
BinomialMS(Mean, N)
GammaMS(Mean, SD)
GumbelMS(Mean, SD)
LaplaceMS(Mean, SD)
LogisticMS(Mean, SD)
LognormalMS(Mean, SD)
UniformMS(Mean, SD)
WeibullMS(Mean, SD)

Details of writing mixture distributions, combined distributions, and compound distributions are found in the chapter on Data Simulation and will not be repeated here.

Finding the Value of the Mean of a Specified Distribution
The mean of a distribution created as a linear combination of other distributions is found by applying the linear combination to the individual means. However, the mean of a distribution created by multiplying or dividing other distributions is not necessarily equal to applying the same function to the individual means. For example, the mean of 4 Normal(4, 5) + 2 Normal(5, 6) is 4*4 + 2*5 = 26, but the mean of 4 Normal(4, 5) * 2 Normal(5, 6) is not exactly 4*4*2*5 = 160 (although it is close).

Group 1 (and 2) Distribution|H1
Specify the distributions of groups 1 and 2 under the alternative hypothesis, H1. The difference between the means of these two distributions will be the actual difference specified under H1.
The parameters of the distribution can be specified using numbers or letters. If letters are used, their values are specified in the boxes below. The value “M1” is reserved for the value of the mean under the alternative hypothesis.

A list of the distributions that are available and the syntax used to specify them is given above.

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**Effect Size – Distribution Parameters**

**M0 (Mean|H0)**

These values are substituted for $M0$ in the distribution specifications given above. $M0$ is intended to be the value of the mean hypothesized by the null hypothesis, H0.

You can enter a list of values using the syntax 0 1 2 3 or 0 to 3 by 1.

**M1 (Mean|H1)**

These values are substituted for $M1$ in the distribution specifications given above. Although it can be used wherever you want, $M1$ is intended to be the value of the mean hypothesized by the alternative hypothesis, H1.

You can enter a list of values using the syntax 0 1 2 3 or 0 to 3 by 1.

**Parameter Values (S, A, B)**

Enter the numeric value(s) of the parameters listed above. These values are substituted for the corresponding letter in all four distribution specifications.

You can enter a list of values for each letter using the syntax 0 1 2 3 or 0 to 3 by 1.

You can also change the letter that is used as the name of this parameter using the pull-down menu to the side.

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**Looks & Boundaries Tab when Solving for Power or Sample Size**

The Looks & Boundaries tab contains settings for the looks and significance boundaries.

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**Looks and Boundaries**

**Specification of Looks and Boundaries**

Choose whether spending functions will be used to divide alpha and beta for each look (Simple Specification), or whether the percents of alpha and beta to be spent at each look will be specified directly (Custom Specification).

Under Simple Specification, the looks are automatically considered to be equally spaced. Under Custom Specification, the looks may be equally spaced or custom defined based on the percent of accumulated information.

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**Looks and Boundaries – Simple Specification**

**Number of Equally Spaced Looks**

Select the total number of looks that will be used if the study is not stopped early for the crossing of a boundary.

**Alpha Spending Function**

Specify the type of alpha spending function to use.

The O'Brien-Fleming Analog spends very little alpha at the beginning and much more at the final looks. The Pocock Analog and (Alpha)(Time) spending functions spend alpha more evenly across the looks. The Hwang-Shih-DeCani (C) (sometimes called the gamma family) spending functions and (Alpha)(Time^C) spending functions are flexible spending functions that can be used to spend more alpha early or late or evenly, depending on the choice of C.
C (Alpha Spending)
C is used to define the Hwang-Shih-DeCani (C) or (Alpha)(Time^C) spending functions.

For the Hwang-Shih-DeCani (C) spending function, negative values of C spend more of alpha at later looks, values near 0 spend alpha evenly, and positive values of C spend more of alpha at earlier looks.

For the (Alpha)(Time^C) spending function, only positive values for C are permitted. Values of C near zero spend more of alpha at earlier looks, values near 1 spend alpha evenly, and larger values of C spend more of alpha at later looks.

Type of Futility Boundary
This option determines whether or not futility boundaries will be created, and if so, whether they are binding or non-binding.

Futility boundaries are boundaries such that, if crossed at a given look, stop the study in favor of H0.

Binding futility boundaries are computed in concert with significance boundaries. They are called binding because they require the stopping of a trial if they are crossed. If the trial is not stopped, the probability of a false positive will exceed alpha.

When Non-binding futility boundaries are computed, the significance boundaries are first computed, ignoring the futility boundaries. The futility boundaries are then computed. These futility boundaries are non-binding because continuing the trial after they are crossed will not affect the overall probability of a false positive declaration.

Number of Skipped Futility Looks
In some trials it may be desirable to wait a number of looks before examining the trial for futility. This option allows the beta to begin being spent after a specified number of looks.

The Number of Skipped Futility Looks should be less than the number of looks.

Beta Spending Function
Specify the type of beta spending function to use.

The O'Brien-Fleming Analog spends very little beta at the beginning and much more at the final looks. The Pocock Analog and (Beta)(Time) spending functions spend beta more evenly across the looks. The Hwang-Shih-DeCani (C) (sometimes called the gamma family) spending functions and (Beta)(Time^C) spending functions are flexible spending functions that can be used to spend more beta early or late or evenly, depending on the choice of C.

C (Beta Spending)
C is used to define the Hwang-Shih-DeCani (C) or (Beta)(Time^C) spending functions.

For the Hwang-Shih-DeCani (C) spending function, negative values of C spend more of beta at later looks, values near 0 spend beta evenly, and positive values of C spend more of beta at earlier looks.

For the (Beta)(Time^C) spending function, only positive values for C are permitted. Values of C near zero spend more of beta at earlier looks, values near 1 spend beta evenly, and larger values of C spend more of beta at later looks.

Looks and Boundaries – Custom Specification

Number of Looks
This is the total number of looks of either type (significance or futility or both).

Equally Spaced
If this box is checked, the Accumulated Information boxes are ignored and the accumulated information is evenly spaced.
Type of Futility Boundary
This option determines whether or not futility boundaries will be created, and if so, whether they are binding or non-binding.

Futility boundaries are boundaries such that, if crossed at a given look, stop the study in favor of H0.

Binding futility boundaries are computed in concert with significance boundaries. They are called binding because they require the stopping of a trial if they are crossed. If the trial is not stopped, the probability of a false positive will exceed alpha.

When Non-binding futility boundaries are computed, the significance boundaries are first computed, ignoring the futility boundaries. The futility boundaries are then computed. These futility boundaries are non-binding because continuing the trial after they are crossed will not affect the overall probability of a false positive declaration.

Accumulated Information
The accumulated information at each look defines the proportion or percent of the sample size that is used at that look.

These values are accumulated information values so they must be increasing.

Proportions, percents, or sample sizes may be entered. All proportions, percents, or sample sizes will be divided by the value at the final look to create an accumulated information proportion for each look.

Percent of Alpha Spent
This is the percent of the total alpha that is spent at the corresponding look. It is not the cumulative value.

Percents, proportions, or alphas may be entered here. Each of the values is divided by the sum of the values to obtain the proportion of alpha that is used at the corresponding look.

Percent of Beta Spent
This is the percent of the total beta (1-power) that is spent at the corresponding look. It is not the cumulative value.

Percents, proportions, or betas may be entered here. Each of the values is divided by the sum of the values to obtain the proportion of beta that is used at the corresponding look.

Looks & Boundaries Tab when Solving for Alpha and Power
The Looks & Boundaries tab contains settings for the looks and significance boundaries.

Looks and Boundaries

Number of Looks
This is the total number of looks of either type (significance or futility or both).

Equally Spaced
If this box is checked, the Accumulated Information boxes are ignored and the accumulated information is evenly spaced.

Types of Boundaries
This option determines whether or not futility boundaries will be entered.

Futility boundaries are boundaries such that, if crossed at a given look, stop the study in favor of H0.

Accumulated Information
The accumulated information at each look defines the proportion or percent of the sample size that is used at that look.
These values are accumulated information values so they must be increasing.

Proportions, percents, or sample sizes may be entered. All proportions, percents, or sample sizes will be divided by the value at the final look to create an accumulated information proportion for each look.

**Significance Boundary**
Enter the value of the significance boundary corresponding to the chosen test statistic. These are sometimes called efficacy boundaries.

**Futility Boundary**
Enter the value of the futility boundary corresponding to the chosen test statistic.

---

### Options Tab
The Options tab contains limits on the number of iterations and various options about individual tests.

---

### Maximum Sample Size Searched

**Maximum N1 Before Search Termination**
Specify the maximum N1 before the search for N1 is aborted.

Since simulations for large sample sizes are very computationally intensive and hence time-consuming, this value can be used to stop searches when N1 is larger than reasonable sample sizes for the study.

This applies only when "Solve For" is set to Sample Size (N1).

The procedure uses a binary search when searching for N1. If a value for N1 is tried that exceeds this value, and the power is not reached, a warning message will be shown on the output indicating the desired power was not reached.

We recommend a value of at least 20000.

---

### Random Numbers

**Random Number Pool Size**
This is the size of the pool of random values from which the random samples will be drawn. Pools should be at least the maximum of 10,000 and twice the number of simulations. You can enter *Automatic* and an appropriate value will be calculated.

If you do not want to draw numbers from a pool, enter 0 here.

---

### Matching Boundaries at Final Look

**Beta Search Increment**
For each simulation, when futility bounds are computed, the appropriate beta is found by searching from 0 to 1 by this increment. Smaller increments are more refined, but the search takes longer.

We recommend 0.001 or 0.0001.
Example 1 – Power and Output

A clinical trial is to be conducted over a two-year period to compare the mean response of a new treatment to that of the current treatment. The current response mean is 108. Although the researchers do not know the true mean of the new treatment, they would like to examine the power that is achieved if the mean of the new treatment is 113. The standard deviation for both groups is assumed to be 25. The sample size at the final look is to be 500 per group. Testing will be done at the 0.05 significance level. A total of five tests are going to be performed on the data as they are obtained. The O’Brien-Fleming (Analog) boundaries will be used.

Find the power and test boundaries assuming equal sample sizes per arm and two-sided hypothesis tests.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the Group-Sequential Tests for Two Means (Simulation) procedure window by expanding Means, then Two Independent Means, then clicking on Group-Sequential, and then clicking on Group-Sequential Tests for Two Means (Simulation). You may then make the appropriate entries as listed below, or open Example 1 by going to the File menu and choosing Open Example Template.

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design Tab</strong></td>
<td></td>
</tr>
<tr>
<td>Solve For</td>
<td>Power</td>
</tr>
<tr>
<td>Test Type</td>
<td>T-Test</td>
</tr>
<tr>
<td>Alternative Hypothesis</td>
<td>Mean1 - Mean2 ≠ Diff0</td>
</tr>
<tr>
<td>Simulations</td>
<td>20000</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.05</td>
</tr>
<tr>
<td>Group Allocation</td>
<td>Equal (N1 = N2)</td>
</tr>
<tr>
<td>Sample Size Per Group</td>
<td>500</td>
</tr>
<tr>
<td>Group 1 Dist’n</td>
<td>H0</td>
</tr>
<tr>
<td>Group 2 Dist’n</td>
<td>H0</td>
</tr>
<tr>
<td>Group 1 Dist’n</td>
<td>H1</td>
</tr>
<tr>
<td>Group 2 Dist’n</td>
<td>H1</td>
</tr>
<tr>
<td>M0 (Mean</td>
<td>H0)</td>
</tr>
<tr>
<td>M1 (Mean</td>
<td>H1)</td>
</tr>
<tr>
<td>S</td>
<td>25</td>
</tr>
</tbody>
</table>

| Looks & Boundaries Tab        |                              |
| Specification of Looks and Boundaries | Simple                      |
| Number of Equally Spaced Looks | 5                           |
| Alpha Spending Function       | O’Brien-Fleming Analog       |
Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results and Plots

Scenario 1 Numeric Results for Group Sequential Testing Mean Difference = Diff0.
Hypotheses: H0: Diff1 = Diff0; H1: Diff1 ≠ Diff0
H0 Dist's: Normal(M0 S) & Normal(M0 S)
H1 Dist's: Normal(M0 S) & Normal(M1 S)
Test Statistic: T-Test
Alpha-Spending Function: O'Brien-Fleming Analog
Beta-Spending Function: None
Futility Boundary Type: None
Number of Looks: 5
Simulations: 20000
Pool Size: 40000

Numeric Summary for Scenario 1

<table>
<thead>
<tr>
<th>Value</th>
<th>95% LCL</th>
<th>95% UCL</th>
<th>Target</th>
<th>Actual</th>
<th>95% LCL</th>
<th>95% UCL</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8770</td>
<td>0.8724</td>
<td>0.8816</td>
<td>0.0500</td>
<td>0.0482</td>
<td>0.0452</td>
<td>0.0512</td>
<td>0.1230</td>
</tr>
</tbody>
</table>

---- Average Sample Size ----

<table>
<thead>
<tr>
<th>N1</th>
<th>N2</th>
<th>Grp1</th>
<th>Grp2</th>
<th>Grp1</th>
<th>Grp2</th>
<th>Diff0</th>
<th>Diff1</th>
<th>M0</th>
<th>M1</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>500</td>
<td>497</td>
<td>497</td>
<td>380</td>
<td>380</td>
<td>0.0</td>
<td>-5.0</td>
<td>108.0</td>
<td>113.0</td>
<td>25.0</td>
</tr>
</tbody>
</table>

Report Definitions

Power is the probability of rejecting a false null hypothesis at one of the looks. It is the total proportion of alternative hypothesis simulations that are outside the significance boundaries.
Power 95% LCL and UCL are the lower and upper confidence limits for the power estimate. The width of the interval is based on the number of simulations.
Target Alpha is the user-specified probability of rejecting a true null hypothesis. It is the total alpha spent.
Alpha or Actual Alpha is the alpha level that was actually achieved by the experiment. It is the total proportion of the null hypothesis simulations that are outside the significance boundaries.
Alpha 95% LCL and UCL are the lower and upper confidence limits for the actual alpha estimate. The width of the interval is based on the number of simulations.
Beta is the probability of accepting a false null hypothesis. It is the total proportion of alternative hypothesis simulations that do not cross the significance boundaries.
N1 and N2 are the sample sizes of each group if the study reaches the final look.
Average Sample Size Given H0 Grp1 and Grp2 are the average or expected sample sizes of each group if H0 is true. These are based on the proportion of null hypothesis simulations that cross the significance or futility boundaries at each look.
Average Sample Size Given H1 Grp1 and Grp2 are the average or expected sample sizes of each group if H1 is true. These are based on the proportion of alternative hypothesis simulations that cross the significance or futility boundaries at each look.
H0 Diff0 is the mean difference between groups (Grp1 - Grp2) assuming the null hypothesis, H0.
H1 Diff1 is the mean difference between groups (Grp1 - Grp2) assuming the alternative hypothesis, H1.
The parameters to the right of H1 Diff1 are the parameters that were set by the user to define the null and alternative simulation distributions.

Summary Statements

Group sequential trials with sample sizes of 500 and 500 at the final look achieve 88% power to detect a difference of 5.0 between the null hypothesis mean difference of 0.0 and the actual mean difference of -5.0 at the 0.0496 significance level (alpha) using a two-sided T-Test.
These results are based on 10000 Monte Carlo samples from the null distributions: Normal(M0 S) and Normal(M0 S), and the alternative distributions: Normal(M0 S) and Normal(M1 S).
Accumulated Information Details for Scenario 1

<table>
<thead>
<tr>
<th>Look</th>
<th>Accumulated Information Percent</th>
<th>Accumulated Sample Size Group 1</th>
<th>Accumulated Sample Size Group 2</th>
<th>Accumulated Sample Size Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.0</td>
<td>100</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>40.0</td>
<td>200</td>
<td>200</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>60.0</td>
<td>300</td>
<td>300</td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>80.0</td>
<td>400</td>
<td>400</td>
<td>800</td>
</tr>
<tr>
<td>5</td>
<td>100.0</td>
<td>500</td>
<td>500</td>
<td>1000</td>
</tr>
</tbody>
</table>

Accumulated Information Details Definitions

Look is the number of the look.
Accumulated Information Percent is the percent of the sample size accumulated up to the corresponding look.
Accumulated Sample Size Group 1 is total number of individuals in group 1 at the corresponding look.
Accumulated Sample Size Group 2 is total number of individuals in group 2 at the corresponding look.
Accumulated Sample Size Total is total number of individuals in the study (group 1 + group 2) at the corresponding look.

Boundaries for Scenario 1

<table>
<thead>
<tr>
<th>Look</th>
<th>T-Value Scale</th>
<th>P-Value Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+/- 4.0302</td>
<td>0.0001</td>
</tr>
<tr>
<td>2</td>
<td>+/- 3.3336</td>
<td>0.0009</td>
</tr>
<tr>
<td>3</td>
<td>+/- 2.7016</td>
<td>0.0071</td>
</tr>
<tr>
<td>4</td>
<td>+/- 2.2941</td>
<td>0.0220</td>
</tr>
<tr>
<td>5</td>
<td>+/- 2.0350</td>
<td>0.0421</td>
</tr>
</tbody>
</table>

Boundaries Definitions

Look is the number of the look.
Significance Boundary T-Value Scale is the value such that statistics outside this boundary at the corresponding look indicate termination of the study and rejection of the null hypothesis. They are sometimes called efficacy boundaries.
Significance Boundary P-Value Scale is the value such that P-Values outside this boundary at the corresponding look indicate termination of the study and rejection of the null hypothesis. This P-Value corresponds to the T-Value Boundary and is sometimes called the nominal alpha.

Boundary Plot
Boundary Plot - P-Value

Significance Boundaries with 95% Simulation Confidence Intervals for Scenario 1

<table>
<thead>
<tr>
<th>Look</th>
<th>T-Value Boundary Value</th>
<th>95% LCL</th>
<th>95% UCL</th>
<th>P-Value Boundary Value</th>
<th>95% LCL</th>
<th>95% UCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+/- 4.0302</td>
<td>-3.5614</td>
<td>-3.1578</td>
<td>0.0001</td>
<td>0.0004</td>
<td>0.0017</td>
</tr>
<tr>
<td>2</td>
<td>+/- 3.3336</td>
<td>-2.8012</td>
<td>-2.6176</td>
<td>0.0071</td>
<td>0.0053</td>
<td>0.0091</td>
</tr>
<tr>
<td>3</td>
<td>+/- 2.7016</td>
<td>-2.3677</td>
<td>-2.2288</td>
<td>0.0220</td>
<td>0.0181</td>
<td>0.0261</td>
</tr>
<tr>
<td>4</td>
<td>+/- 2.2941</td>
<td>-1.9996</td>
<td>-2.0350</td>
<td>0.0421</td>
<td>0.0365</td>
<td>0.0458</td>
</tr>
</tbody>
</table>

Significance Boundary Confidence Limit Definitions

Look is the number of the look.
Look is the number of the look.
T-Value Boundary Value is the value such that statistics outside this boundary at the corresponding look indicate termination of the study and rejection of the null hypothesis. They are sometimes called efficacy boundaries.
P-Value Boundary Value is the value such that P-Values outside this boundary at the corresponding look indicate termination of the study and rejection of the null hypothesis. This P-Value corresponds to the T-Value Boundary and is sometimes called the nominal alpha.
95% LCL and UCL are the lower and upper confidence limits for the boundary at the given look. The width of the interval is based on the number of simulations.
### Alpha-Spending and Null Hypothesis Simulation Details for Scenario 1

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+/- 4.0302</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0043</td>
</tr>
<tr>
<td>2</td>
<td>+/- 3.3336</td>
<td>0.0009</td>
<td>0.0008</td>
<td>0.0008</td>
<td>0.0090</td>
</tr>
<tr>
<td>3</td>
<td>+/- 2.7016</td>
<td>0.0071</td>
<td>0.0068</td>
<td>0.0061</td>
<td>0.0070</td>
</tr>
<tr>
<td>4</td>
<td>+/- 2.2941</td>
<td>0.0220</td>
<td>0.0168</td>
<td>0.0167</td>
<td>0.0237</td>
</tr>
<tr>
<td>5</td>
<td>+/- 2.0350</td>
<td>0.0421</td>
<td>0.0256</td>
<td>0.0246</td>
<td>0.0482</td>
</tr>
</tbody>
</table>

**Alpha-Spending Details Definitions**

- **Look** is the number of the look.
- **Significance Boundary T-Value Scale** is the value such that statistics outside this boundary at the corresponding look indicate termination of the study and rejection of the null hypothesis. They are sometimes called efficacy boundaries.
- **Significance Boundary P-Value Scale** is the value such that P-Values outside this boundary at the corresponding look indicate termination of the study and rejection of the null hypothesis. This P-Value corresponds to the Significance T-Value Boundary and is sometimes called the nominal alpha.
- **Spending Function Alpha** is the intended portion of alpha allocated to the particular look based on the alpha-spending function.
- **Cumulative Spending Function Alpha** is the intended accumulated alpha allocated to the particular look. It is the sum of the Spending Function Alpha up to the corresponding look.
- **Alpha Spent** is the proportion of the null hypothesis simulations resulting in statistics outside the Significance Boundary at this look.
- **Cumulative Alpha Spent** is the proportion of the null hypothesis simulations resulting in Significance Boundary termination up to and including this look. It is the sum of the Alpha Spent up to the corresponding look.
- **Proportion H1 Sims Outside Significance Boundary** is the proportion of the alternative hypothesis simulations resulting in statistics outside the Significance Boundary at this look. It may be thought of as the incremental power.
- **Cumulative H1 Sims Outside Significance Boundary** is the proportion of the alternative hypothesis simulations resulting in Significance Boundary termination up to and including this look. It is the sum of the Proportion H1 Sims Outside Significance Boundary up to the corresponding look.

The values obtained from any given run of this example will vary slightly due to the variation in simulations.
Example 2 – Power for One-Sided Test with Futility Boundaries

Suppose researchers would like to compare two treatments with a one-sided test at each look. Further, suppose they would like to terminate the study early when it can be deemed highly unlikely that the new treatment is better than the standard. Suppose the control group mean is 108. The researchers wish to know the power of the test if the treatment group mean is 113. The sample size at the final look is to be 500 per group. Testing will be done at the 0.05 significance level. A total of five tests are going to be performed on the data as they are obtained. The O’Brien-Fleming (Analog) boundaries will be used for both significance and futility boundaries.

Find the power and test boundaries assuming equal sample sizes per arm and one-sided hypothesis tests.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the Group-Sequential Tests for Two Means (Simulation) procedure window by expanding Means, then Two Independent Means, then clicking on Group-Sequential, and then clicking on Group-Sequential Tests for Two Means (Simulation). You may then make the appropriate entries as listed below, or open Example 2 by going to the File menu and choosing Open Example Template.

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design Tab</strong></td>
<td></td>
</tr>
<tr>
<td>Solve For</td>
<td>Power</td>
</tr>
<tr>
<td>Test Type</td>
<td>T-Test</td>
</tr>
<tr>
<td>Alternative Hypothesis</td>
<td>Mean1 - Mean2 &lt; Diff0</td>
</tr>
<tr>
<td>Simulations</td>
<td>20000</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.05</td>
</tr>
<tr>
<td>Group Allocation</td>
<td>Equal (N1 = N2)</td>
</tr>
<tr>
<td>Sample Size Per Group</td>
<td>500</td>
</tr>
<tr>
<td>Group 1 Dist’n</td>
<td>H0</td>
</tr>
<tr>
<td>Group 2 Dist’n</td>
<td>H0</td>
</tr>
<tr>
<td>Group 1 Dist’n</td>
<td>H1</td>
</tr>
<tr>
<td>Group 2 Dist’n</td>
<td>H1</td>
</tr>
<tr>
<td>M0 (Mean</td>
<td>H0)</td>
</tr>
<tr>
<td>M1 (Mean</td>
<td>H1)</td>
</tr>
<tr>
<td>S</td>
<td>25</td>
</tr>
</tbody>
</table>

| **Looks & Boundaries Tab**    |                                                 |
| Specification of Looks and Boundaries | Simple                                           |
| Number of Equally Spaced Looks | 5                                                |
| Alpha Spending Function       | O’Brien-Fleming Analog                          |
| Type of Futility Boundary     | Non-Binding                                     |
| Number of Skipped Futility Looks | 0                                           |
| Beta Spending Function        | O’Brien-Fleming Analog                          |
Output

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results and Plots**

**Scenario 1 Numeric Results for Group Sequential Testing Mean Difference = Diff0.**

- Hypotheses: H0: Diff1 = Diff0; H1: Diff1 < Diff0
- H0 Dist's: Normal(M0 S) & Normal(M0 S)
- H1 Dist's: Normal(M0 S) & Normal(M1 S)
- Test Statistic: T-Test
- Alpha-Spending Function: O'Brien-Fleming Analog
- Beta-Spending Function: O'Brien-Fleming Analog
- Futility Boundary Type: Non-binding
- Number of Looks: 5
- Simulations: 20000
- Pool Size: 40000

### Numeric Summary for Scenario 1

<table>
<thead>
<tr>
<th>Value</th>
<th>95% LCL</th>
<th>95% UCL</th>
<th>Target</th>
<th>Actual</th>
<th>95% LCL</th>
<th>95% UCL</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9143</td>
<td>0.9104</td>
<td>0.9182</td>
<td>0.0500</td>
<td>0.0455</td>
<td>0.0426</td>
<td>0.0483</td>
<td>0.0857</td>
</tr>
</tbody>
</table>

### Average Sample Size

---

<table>
<thead>
<tr>
<th>N1</th>
<th>N2</th>
<th>Grp1</th>
<th>Grp2</th>
<th>Grp1</th>
<th>Grp2</th>
<th>Diff0</th>
<th>Diff1</th>
<th>M0</th>
<th>M1</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>500</td>
<td>302</td>
<td>302</td>
<td>335</td>
<td>335</td>
<td>0.0</td>
<td>-5.0</td>
<td>108.0</td>
<td>113.0</td>
<td>25.0</td>
</tr>
</tbody>
</table>

### Accumulated Information Details for Scenario 1

<table>
<thead>
<tr>
<th>Look</th>
<th>Percent</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.0</td>
<td>100</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>40.0</td>
<td>200</td>
<td>200</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>60.0</td>
<td>300</td>
<td>300</td>
<td>600</td>
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<tr>
<td>4</td>
<td>80.0</td>
<td>400</td>
<td>400</td>
<td>800</td>
</tr>
<tr>
<td>5</td>
<td>100.0</td>
<td>500</td>
<td>500</td>
<td>1000</td>
</tr>
</tbody>
</table>

### Boundaries for Scenario 1

<table>
<thead>
<tr>
<th>Look</th>
<th>Significance Boundary</th>
<th>Futility Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T-Value</td>
<td>P-Value</td>
</tr>
<tr>
<td>1</td>
<td>-4.5851</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>-2.8811</td>
<td>0.0021</td>
</tr>
<tr>
<td>3</td>
<td>-2.3230</td>
<td>0.0103</td>
</tr>
<tr>
<td>4</td>
<td>-1.9357</td>
<td>0.0266</td>
</tr>
<tr>
<td>5</td>
<td>-1.7361</td>
<td>0.0414</td>
</tr>
</tbody>
</table>
Boundary Plot

Boundary Plot - P-Value

Significance Boundaries with 95% Simulation Confidence Intervals for Scenario 1

<table>
<thead>
<tr>
<th>Look</th>
<th>Value</th>
<th>95% LCL</th>
<th>95% UCL</th>
<th>Value</th>
<th>95% LCL</th>
<th>95% UCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4.5851</td>
<td></td>
<td></td>
<td>0.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-2.8811</td>
<td>-3.0006</td>
<td>-2.8011</td>
<td>0.0021</td>
<td>0.0014</td>
<td>0.0027</td>
</tr>
<tr>
<td>3</td>
<td>-2.3230</td>
<td>-2.3697</td>
<td>-2.2690</td>
<td>0.0103</td>
<td>0.0091</td>
<td>0.0118</td>
</tr>
<tr>
<td>4</td>
<td>-1.9357</td>
<td>-1.9775</td>
<td>-1.9122</td>
<td>0.0266</td>
<td>0.0242</td>
<td>0.0281</td>
</tr>
<tr>
<td>5</td>
<td>-1.7361</td>
<td>-1.7632</td>
<td>-1.7013</td>
<td>0.0414</td>
<td>0.0391</td>
<td>0.0446</td>
</tr>
</tbody>
</table>
### Futility Boundaries with 95% Simulation Confidence Intervals for Scenario 1

<table>
<thead>
<tr>
<th>Look</th>
<th>Value</th>
<th>95% LCL</th>
<th>95% UCL</th>
<th>Value</th>
<th>95% LCL</th>
<th>95% UCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.1482</td>
<td>0.9835</td>
<td></td>
<td>2</td>
<td>0.4558</td>
<td>0.4177</td>
</tr>
<tr>
<td>2</td>
<td>-0.5014</td>
<td>-0.5346</td>
<td>-0.4577</td>
<td>3</td>
<td>-1.1580</td>
<td>-1.1864</td>
</tr>
<tr>
<td>4</td>
<td>1.7361</td>
<td>-1.7688</td>
<td>-1.7055</td>
<td>5</td>
<td>-0.5014</td>
<td>-0.5346</td>
</tr>
</tbody>
</table>

### Alpha-Spending and Null Hypothesis Simulation Details for Scenario 1

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4.5851</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
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<td>0.0021</td>
<td>0.0019</td>
<td>0.0020</td>
<td>0.0020</td>
</tr>
<tr>
<td>3</td>
<td>-2.3230</td>
<td>0.0103</td>
<td>0.0095</td>
<td>0.0114</td>
<td>0.0114</td>
</tr>
<tr>
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<td>0.0264</td>
<td>0.0169</td>
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<td>0.0216</td>
<td>0.0172</td>
<td>0.0455</td>
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</tbody>
</table>

### Beta-Spending and Alternative Hypothesis Simulation Details for Scenario 1

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<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.1482</td>
<td>0.9835</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>2</td>
<td>0.4558</td>
<td>0.6756</td>
<td>0.0065</td>
<td>0.0065</td>
<td>0.0065</td>
</tr>
<tr>
<td>3</td>
<td>-0.5014</td>
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<td>0.0200</td>
<td>0.0267</td>
<td>0.0266</td>
</tr>
<tr>
<td>4</td>
<td>-1.1580</td>
<td>0.1236</td>
<td>0.0283</td>
<td>0.0283</td>
<td>0.0283</td>
</tr>
<tr>
<td>5</td>
<td>-1.7361</td>
<td>0.0414</td>
<td>0.0311</td>
<td>0.0860</td>
<td>0.0309</td>
</tr>
</tbody>
</table>

The values obtained from any given run of this example will vary slightly due to the variation in simulations.
Example 3 – Enter Boundaries

With a set-up similar to Example 2, suppose we wish to investigate the properties of a set of significance (-3, -3, -3, -2, -1) and futility (2, 1, 0, 0, -1) boundaries.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the Group-Sequential Tests for Two Means (Simulation) procedure window by expanding Means, then Two Independent Means, then clicking on Group-Sequential, and then clicking on Group-Sequential Tests for Two Means (Simulation). You may then make the appropriate entries as listed below, or open Example 3 by going to the File menu and choosing Open Example Template.

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design Tab</strong></td>
<td></td>
</tr>
<tr>
<td>Solve For</td>
<td>Alpha and Power (Enter Boundaries)</td>
</tr>
<tr>
<td>Test Type</td>
<td>T-Test</td>
</tr>
<tr>
<td>Alternative Hypothesis</td>
<td>Mean1 - Mean2 &lt; Diff0</td>
</tr>
<tr>
<td>Simulations</td>
<td>20000</td>
</tr>
<tr>
<td>Group Allocation</td>
<td>Equal (N1 = N2)</td>
</tr>
<tr>
<td>Sample Size Per Group</td>
<td>500</td>
</tr>
<tr>
<td>Group 1 Dist’n</td>
<td>H0</td>
</tr>
<tr>
<td>Group 2 Dist’n</td>
<td>H0</td>
</tr>
<tr>
<td>Group 1 Dist’n</td>
<td>H1</td>
</tr>
<tr>
<td>Group 2 Dist’n</td>
<td>H1</td>
</tr>
<tr>
<td>M0 (Mean</td>
<td>H0)</td>
</tr>
<tr>
<td>M1 (Mean</td>
<td>H1)</td>
</tr>
<tr>
<td>S</td>
<td>25</td>
</tr>
</tbody>
</table>

| **Looks & Boundaries Tab**    |                                            |
| Number of Looks              | 5                                          |
| Equally Spaced               | Checked                                    |
| Types of Boundaries          | Significance and Futility Boundaries        |
| Significance Boundary        | -3 -3 -3 -2 -1 (for looks 1 through 5)      |
| Futility Boundary            | 2 1 0 0 -1 (for looks 1 through 5)          |
Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results and Plots

**Scenario 1 Numeric Results for Group Sequential Testing Mean Difference = Diff0.**

Hypotheses: H0: Diff1 = Diff0; H1: Diff1 < Diff0  
H0 Dist's: Normal(M0 S) & Normal(M0 S)  
H1 Dist's: Normal(M0 S) & Normal(M1 S)  
Test Statistic: T-Test  
Type of Boundaries: Significance and Futility Boundaries

Number of Looks: 5  
Simulations: 20000  
Pool Size: 40000

<table>
<thead>
<tr>
<th>Scenario 1 Numeric Results</th>
<th>Power</th>
<th>Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.9801</td>
<td>0.0199</td>
</tr>
<tr>
<td>95% LCL</td>
<td>0.9781</td>
<td>0.1479</td>
</tr>
<tr>
<td>95% UCL</td>
<td>0.9820</td>
<td>0.1578</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Sample Size</th>
<th>N1</th>
<th>N2</th>
<th>Grp1</th>
<th>Grp2</th>
<th>Diff0</th>
<th>Diff1</th>
<th>M0</th>
<th>M1</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given H0</td>
<td>500</td>
<td>500</td>
<td>370</td>
<td>370</td>
<td>0.0</td>
<td>-5.0</td>
<td>108.0</td>
<td>113.0</td>
<td>25.0</td>
</tr>
<tr>
<td>Given H1</td>
<td>-----</td>
<td>Average Sample Size -----</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Accumulated Information Details for Scenario 1</th>
<th>Accumulated Information</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Look</td>
<td>Percent</td>
<td>100</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>1</td>
<td>20.0</td>
<td>200</td>
<td>400</td>
<td>600</td>
</tr>
<tr>
<td>2</td>
<td>40.0</td>
<td>200</td>
<td>400</td>
<td>800</td>
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<tr>
<td>3</td>
<td>60.0</td>
<td>400</td>
<td>400</td>
<td>800</td>
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<td>4</td>
<td>80.0</td>
<td>500</td>
<td>500</td>
<td>1000</td>
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<tr>
<td>5</td>
<td>100.0</td>
<td>500</td>
<td>500</td>
<td>1000</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Boundaries for Scenario 1</th>
<th>Significance Boundary</th>
<th>Futility Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Look</td>
<td>T-Value</td>
<td>P-Value</td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>1</td>
<td>-3.0000</td>
<td>0.0015</td>
</tr>
<tr>
<td>2</td>
<td>-3.0000</td>
<td>0.0014</td>
</tr>
<tr>
<td>3</td>
<td>-3.0000</td>
<td>0.0014</td>
</tr>
<tr>
<td>4</td>
<td>-2.0000</td>
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</tr>
<tr>
<td>5</td>
<td>-1.0000</td>
<td>0.1588</td>
</tr>
</tbody>
</table>
Group-Sequential Tests for Two Means (Simulation)

Boundary Plot

Boundary Plot - P-Value
Example 4 – Validation Using Simulation

With a set-up similar to Example 1, we examine the power and alpha generated by the set of two-sided significance boundaries (+/- 4.0302, +/- 3.3336, +/- 2.7016, +/- 2.2941, +/- 2.0350).

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the Group-Sequential Tests for Two Means (Simulation) procedure window by expanding Means, then Two Independent Means, then clicking on Group-Sequential, and then clicking on Group-Sequential Tests for Two Means (Simulation). You may then make the appropriate entries as listed below, or open Example 4 by going to the File menu and choosing Open Example Template.

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design Tab</strong></td>
<td></td>
</tr>
<tr>
<td>Solve For</td>
<td>Alpha and Power (Enter Boundaries)</td>
</tr>
<tr>
<td>Test Type</td>
<td>T-Test</td>
</tr>
<tr>
<td>Alternative Hypothesis</td>
<td>Mean1 - Mean2 ≠ Diff0</td>
</tr>
<tr>
<td>Simulations</td>
<td>20000</td>
</tr>
<tr>
<td>Group Allocation</td>
<td>Equal (N1 = N2)</td>
</tr>
<tr>
<td>Sample Size Per Group</td>
<td>500</td>
</tr>
<tr>
<td>Group 1 Dist’n</td>
<td>H0</td>
</tr>
<tr>
<td>Group 2 Dist’n</td>
<td>H0</td>
</tr>
<tr>
<td>Group 1 Dist’n</td>
<td>H1</td>
</tr>
<tr>
<td>Group 2 Dist’n</td>
<td>H1</td>
</tr>
<tr>
<td>M0 (Mean</td>
<td>H0)</td>
</tr>
<tr>
<td>M1 (Mean</td>
<td>H1)</td>
</tr>
<tr>
<td>S</td>
<td>25</td>
</tr>
</tbody>
</table>
Looks & Boundaries Tab
Number of Looks ........................................... 5
Equally Spaced ............................................. Checked
Types of Boundaries ........................................ Significance Boundaries
Significance Boundary ................................. 4.0302, 3.3336, 2.7016, 2.2941, 2.0350

Output
Click the Calculate button to perform the calculations and generate the following output.

Numeric Results and Plots

Scenario 1 Numeric Results for Group Sequential Testing Mean Difference = Diff0.
Hypotheses: H0: Diff1 = Diff0; H1: Diff1 ≠ Diff0
H0 Dist's: Normal(M0 S) & Normal(M0 S)
H1 Dist's: Normal(M0 S) & Normal(M1 S)
Test Statistic: T-Test
Type of Boundaries: Significance Boundaries Only
Number of Looks: 5
Simulations: 20000
Pool Size: 40000

Numeric Summary for Scenario 1

<table>
<thead>
<tr>
<th>Value</th>
<th>95% LCL</th>
<th>95% UCL</th>
<th>95% LCL</th>
<th>95% UCL</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8827</td>
<td>0.8792</td>
<td>0.8872</td>
<td>0.0530</td>
<td>0.0498</td>
<td>0.0561</td>
</tr>
</tbody>
</table>

----- Average Sample Size ----

<table>
<thead>
<tr>
<th>N1</th>
<th>N2</th>
<th>Grp1</th>
<th>Grp2</th>
<th>Grp1</th>
<th>Grp2</th>
<th>Diff0</th>
<th>Diff1</th>
<th>M0</th>
<th>M1</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>500</td>
<td>496</td>
<td>496</td>
<td>378</td>
<td>378</td>
<td>0.0</td>
<td>-5.0</td>
<td>108.0</td>
<td>113.0</td>
<td>25.0</td>
</tr>
</tbody>
</table>

The values obtained from any given run of this example will vary slightly due to the variation in simulations. The power and alpha generated with these boundaries are very close to the values of Example 1.