

Chapter 369

Mixed Models Tests for Interaction in a 2×2 Factorial 2-Level Hierarchical Design (Level-1 Randomization)

Introduction

This procedure calculates power and sample size for a two-level hierarchical mixed model in which clusters of subjects are measured one time (cross-sectional) on a continuous variable. The study uses a two-by-two factorial design with two binary factors, each with two possible values (0 and 1). This results in four treatment arms. The goal of the study is to test the significance of the two-way interaction between the two factors.

In this two-level hierarchical design, the subjects are the level-one units, and the clusters are the level-two units. Each subject is individually randomized to receive one of the four possible treatments. Hence, all four treatments occur within each level-2 unit.

Technical Details

Our formulation comes from Ahn, Heo, and Zhang (2015), chapter 5, section 5.5.2, pages 170-172. The hierarchical mixed model that is adopted is

$$Y_{ij} = \beta_0 + \delta_{X(1)}X_{ij} + \delta_{Z(1)}Z_{ij} + \delta_{XZ(1)}X_{ij}Z_{ij} + u_i + e_{ij}$$

where

- Y_{ij} is the continuous response of the j^{th} subject in the i^{th} cluster.
- β_0 is the fixed intercept.
- $\delta_{X(1)}$ is the treatment effect of X.
- X_{ij} is an indicator variable that is = 1 if cluster i is assigned to receive the X intervention and 0 otherwise.
- $\delta_{Z(1)}$ is the treatment effect of Z.
- Z_{ij} is an indicator variable that is = 1 if cluster i is assigned to receive the Z intervention and 0 otherwise.
- $\delta_{XZ(1)}$ is the interaction effect of X and Z. In terms of the four group means, this effect is equal to $(\mu_{1,1} - \mu_{1,0}) - (\mu_{0,1} - \mu_{0,0})$.
- u_i is a random effect term for the i^{th} cluster that is distributed as $N(0, \sigma_u^2)$.
- e_{ij} is a random effect for the j^{th} subject in the i^{th} cluster that is distributed as $N(0, \sigma_e^2)$.

Mixed Models Tests for Interaction in a 2x2 Factorial 2-Level Hierarchical Design (Level-1 Randomization)

σ_u^2 is variance of the level(cluster) random effects.

σ_e^2 is variance of the level one (subject) random effects.

σ^2 is the variance of Y , where $\sigma^2 = \sigma_u^2 + \sigma_e^2$.

ρ is the intraclass correlation (ICC). This is the correlation between any two level-1 units within a specific level-2 unit.

K is the number of level-2 units in the study.

$M_{0,0}$ is the number of level-1 units per level-2 unit for which $X = 0$ and $Z = 0$.

$M_{0,1}$ is the number of level-1 units per level-2 unit for which $X = 0$ and $Z = 1$.

$M_{1,0}$ is the number of level-1 units per level-2 unit for which $X = 1$ and $Z = 0$.

$M_{1,1}$ is the number of level-1 units per level-2 unit for which $X = 1$ and $Z = 1$.

The test of significance of the product $X_{ij}Z_{ij}$ term in the mixed model analysis is the test statistic of interest. It tests whether the difference between the two levels of one factor at the high level of the other factor is equal to the corresponding difference at the low level of the second factor.

Assume that $\delta_{XZ(1)}$ is to be tested using a Wald test. The statistical hypotheses are $H_0: \delta_{XZ(2)} = 0$ vs. $H_a: \delta_{XZ(2)} \neq 0$.

The power is calculated using

$$Power = \Phi \left\{ \frac{\delta_{XZ(2)}}{\sigma} \sqrt{\frac{K}{f \left(\frac{1}{M_{0,0}} + \frac{1}{M_{1,1}} + \frac{1}{M_{1,0}} + \frac{1}{M_{0,1}} \right)}} - \Phi^{-1}(1 - \alpha/2) \right\}$$

where $f = 1 - \rho$.

Example 1 – Calculating Power

Suppose that a two-level hierarchical design is planned in which there will be two interventions. Each intervention will be whether one of two drugs is administered. There will be only one measurement per subject and each subject (level-1 unit) will be randomized to a group individually. The analysis will be a mixed model of continuous data using the model given earlier in this chapter. The following parameter settings are to be used for the power analysis: $\delta = 7$; $\sigma = 9.7$; $\rho = 0.06$; $K = 4$ or 6 ; $\alpha = 0.05$; and $M00 = M01 = M10 = M11 = 5$ to 20 by 5 . Find the power of each combination of parameter settings.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	Power
Alpha.....	0.05
K (Level-2 Unit Count)	4 6
M00 (Group 00 Count (X=0, Z=0))	5 10 15 20
M01 (Group 01 Count (X=0, Z=1))	M00
M10 (Group 10 Count (X=1, Z=0))	M00
M11 (Group 11 Count (X=1, Z=1))	M00
δ (Interaction = $(\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00})$)	7
σ (Standard Deviation).....	9.7
ρ (Intraclass Correlation, ICC)	0.06

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results for an Interaction in a 2x2 Factorial

Solve For: **Power**

Power	Total Sample Size N	Number of Level-2 Units K	Number of Level-1 Units per Level-2 Unit				Interaction Difference δ	Standard Deviation σ	Intraclass Correlation ρ	Alpha
			Group 00 M00	Group 01 M01	Group 10 M10	Group 11 M11				
0.3838	80	4	5	5	5	5	7	9.7	0.06	0.05
0.6531	160	4	10	10	10	10	7	9.7	0.06	0.05
0.8219	240	4	15	15	15	15	7	9.7	0.06	0.05
0.9145	320	4	20	20	20	20	7	9.7	0.06	0.05
0.5313	120	6	5	5	5	5	7	9.7	0.06	0.05
0.8219	240	6	10	10	10	10	7	9.7	0.06	0.05
0.9419	360	6	15	15	15	15	7	9.7	0.06	0.05
0.9829	480	6	20	20	20	20	7	9.7	0.06	0.05

Power The probability of rejecting a false null hypothesis when the alternative hypothesis is true.

N The total number of Level-1 units in the study.

K The number of Level-2 units in the study.

M00 The number of Level-1 units per Level-2 unit in Group (0,0) (the group in which F1 = 0 and F2 = 0).

M01 The number of Level-1 units per Level-2 unit in Group (0,1) (the group in which F1 = 0 and F2 = 1).

M10 The number of Level-1 units per Level-2 unit in Group (1,0) (the group in which F1 = 1 and F2 = 0).

M11 The number of Level-1 units per Level-2 unit in Group (1,1) (the group in which F1 = 1 and F2 = 1).

δ The interaction difference at which the power is calculated. $\delta = (\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00})$.

σ The standard deviation of the subject responses.

ρ The intraclass correlation (ICC) among Level-1 units within a single Level-2 unit.

Alpha The probability of rejecting a true null hypothesis.

Summary Statements

A 2x2 factorial ($X = 0,1$ and $Z = 0,1$) 2-level hierarchical design will have level-1 units (e.g., students, subjects, or patients) in level-2 units (e.g., classes, clinics, or hospitals) with random assignment of level-1 units to each of the 4 treatment arms (Groups 00, 01, 10, and 11) (level-1 randomization). This design will be used to test the two-way interaction term ($X \times Z$) of the linear mixed-effects model, with a Type I error rate (α) of 0.05. This interaction is formed from the four group means using the following formula: $\delta = (\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00})$. The standard deviation of responses (σ) is assumed to be 9.7 (where σ^2 equals the sum of the error term variance and the level-2 random intercept variance). The intraclass correlation coefficient of level-1 units is assumed to be 0.06. To detect a two-way interaction effect (δ) of at least 7, with 4 level-2 units, and within each level-2 unit, 5 level-1 units in Group 00 ($X = 0, Z = 0$), 5 level-1 units in Group 01 ($X = 0, Z = 1$), 5 level-1 units in Group 10 ($X = 1, Z = 0$), and 5 level-1 units in Group 11 ($X = 1, Z = 1$) (for a grand total of 80 level-1 units), the power is 0.3838.

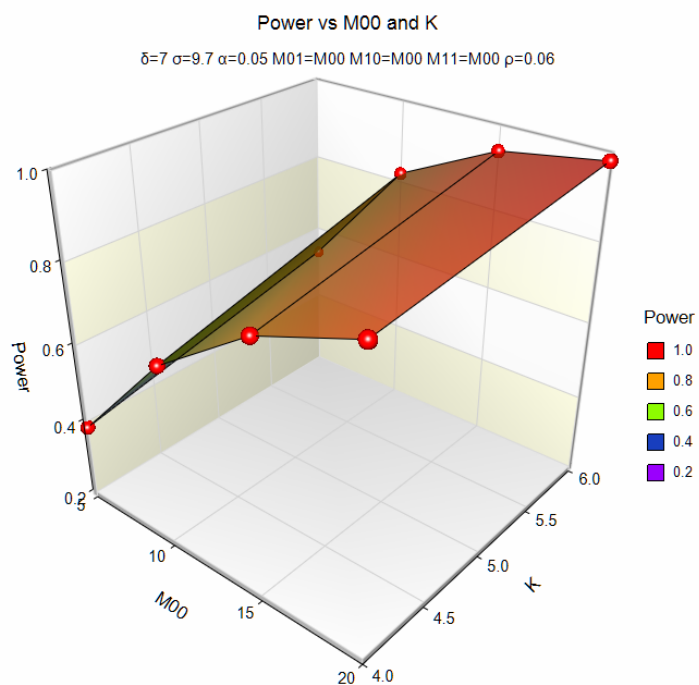
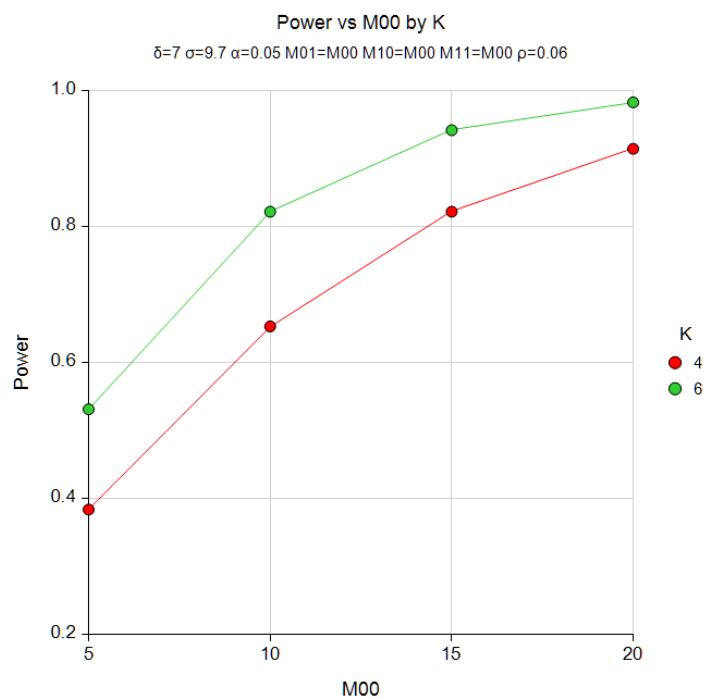
References

Ahn, C., Heo, M., and Zhang, S. 2015. Sample Size Calculations for Clustered and Longitudinal Outcomes in Clinical Research. CRC Press. New York.

This report shows the power for each of the scenarios.

Plots Section

Plots



These plots show the power for the various parameter settings.

Example 2 – Calculating Sample Size (M00)

Continuing with the last example, suppose the researchers want to determine the value of M00 needed to achieve 90% power for both values of K.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **M00 (Group 00 Count of Level-1 Units)**
 Power..... **0.90**
 Alpha..... **0.05**
 K (Level-2 Unit Count) **4 6**
 M01 (Group 01 Count (X=0, Z=1)) **M00**
 M10 (Group 10 Count (X=1, Z=0)) **M00**
 M11 (Group 11 Count (X=1, Z=1)) **M00**
 δ (Interaction = $(\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00})$) **7**
 σ (Standard Deviation)..... **9.7**
 ρ (Intraclass Correlation, ICC) **0.06**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for an Interaction in a 2x2 Factorial

Solve For: **M00 (Group 00 Count of Level-1 Units)**

Power	Total Sample Size N	Number of Level-2 Units K	Number of Level-1 Units per Level-2 Unit				Interaction Difference δ	Standard Deviation σ	Intraclass Correlation ρ	Alpha
			Group 00 M00	Group 01 M01	Group 10 M10	Group 11 M11				
0.9005	304	4	19	19	19	19	7	9.7	0.06	0.05
0.9077	312	6	13	13	13	13	7	9.7	0.06	0.05

This report shows the power for each of the scenarios.

Example 3 – Validation using Ahn, Heo, and Zhang (2015)

Ahn, Heo, and Zhang (2015) page 173 provide a table in which several scenarios are reported. We will validate this procedure by duplicating the top entry. The following parameter settings are used for the power analysis: Power = 0.80; $\delta = 0.4$; $\sigma = 1$; $\rho = 0.1$; M00, M01, M10, M11 are 10; and $\alpha = 0.05$. The value of K is found to be 18 and the attained power at 0.807.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **K (Level-2 Unit Count)**
 Power..... **0.80**
 Alpha..... **0.05**
 M00 (Group 00 Count (X=0, Z=0))..... **10**
 M01 (Group 01 Count (X=0, Z=1))..... **M00**
 M10 (Group 10 Count (X=1, Z=0))..... **M00**
 M11 (Group 11 Count (X=1, Z=1))..... **M00**
 δ (Interaction = $(\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00})$)..... **0.4**
 σ (Standard Deviation)..... **1**
 ρ (Intraclass Correlation, ICC) **0.1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for an Interaction in a 2x2 Factorial

Solve For: **K (Level-2 Unit Count)**

Power	Total Sample Size N	Number of Level-2 Units K	Number of Level-1 Units per Level-2 Unit				Interaction Difference δ	Standard Deviation σ	Intraclass Correlation ρ	Alpha
			Group 00 M00	Group 01 M01	Group 10 M10	Group 11 M11				
0.8074	720	18	10	10	10	10	0.4	1	0.1	0.05

PASS also calculates the value of K to be 18 and the power at 0.8074.