

Chapter 367

Mixed Models Tests for Interaction in a 2×2 Factorial 3-Level Hierarchical Design (Level-1 Randomization)

Introduction

This procedure calculates power and sample size for a three-level hierarchical mixed model which is randomized at the **first** level. The cross-sectional study uses a 2-by-2 factorial design with two binary factors, each with two possible values (0 and 1). This results in four treatment arms. The goal of the study is to test the significance of the two-way interaction between the two factors.

In this three-level hierarchical design, the level-1 units are nested in the level-2 units, which are nested in the level-3 units. For example, students might be nested in classes which are in turn nested within school.

This procedure is for cross-sectional (non-longitudinal) studies in which each subject is measured only one time.

In this case of level-1 randomization, each level-1 unit is randomly assigned to one of the four treatments combinations. The level-1 units within a specific level-2 unit usually receive different interventions (treatment). For example, the students within a class are randomly assigned a treatment, so students within the same class receive different treatments.

Technical Details

Our formulation comes from Ahn, Heo, and Zhang (2015), chapter 6, section 6.5.3, pages 216-219. The hierarchical mixed model that is adopted is

$$Y_{ijk} = \beta_0 + \delta_{X(1)}X_{ijk} + \delta_{Z(1)}Z_{ijk} + \delta_{XZ(1)}X_{ijk}Z_{ijk} + u_i + u_{j(i)} + e_{ijk}$$

where

- Y_{ijk} is the continuous response of the k^{th} level-1 unit, within the j^{th} level-2 unit, within the i^{th} level-3 unit.
- X_{ijk} is an indicator variable that is equal to "1" if the j^{th} level-2 unit is assigned to receive intervention X and "0" otherwise. Thus, $X_{ijk} = X_j$ for all i and k .
- Z_{ijk} is an indicator variable that is equal to "1" if the j^{th} level-2 unit is assigned to receive intervention Z and "0" otherwise. Thus, $Z_{ijk} = Z_j$ for all i and k .
- β_0 is the fixed intercept.
- $\delta_{X(1)}$ is the treatment effect of factor X.
- $\delta_{Z(1)}$ is the treatment effect of factor Z.

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$\delta_{XZ(1)}$ is the interaction effect of factors X and Z. In terms of the four group means, this effect is equal to $(\mu_{1,1} - \mu_{1,0}) - (\mu_{0,1} - \mu_{0,0})$. This is the term that is tested for this procedure.

u_i is the level-3 random intercept effect for the i^{th} level-3 unit. It is distributed as $N(0, \sigma_3^2)$.

$u_{j(i)}$ is the level-2 random intercept effect for the $j(i)^{\text{th}}$ level-2 unit. It is distributed as $N(0, \sigma_2^2)$.

e_{ijk} is the level-1 random intercept effect that is distributed as $N(0, \sigma_e^2)$.

σ^2 is the variance of Y, where $\sigma^2 = \sigma_3^2 + \sigma_2^2 + \sigma_e^2$.

ρ_1 is the correlation among level-1 units which are in a particular level-2 unit.

C is the number of level-3 units in the study.

K is the average number of level-2 units per level-3 unit.

$M_{0,0}$ is the number of level-1 units per level-2 unit for which X = 0 and Z = 0.

$M_{0,1}$ is the number of level-1 units per level-2 unit for which X = 0 and Z = 1.

$M_{1,0}$ is the number of level-1 units per level-2 unit for which X = 1 and Z = 0.

$M_{1,1}$ is the number of level-1 units per level-2 unit for which X = 1 and Z = 1.

The test of significance of the product $X_{ij}Z_{ij}$ term in the mixed model analysis is the test statistic of interest. It tests whether the difference between the two levels of one factor at the high level of the other factor is equal to the corresponding difference at the low level of the second factor.

Assume that $\delta_{XZ(2)}$ is to be tested using a Wald test. The statistical hypotheses are $H_0: \delta_{XZ(2)} = 0$ vs.

$H_a: \delta_{XZ(2)} \neq 0$.

The power is calculated using

$$Power = \Phi \left\{ \frac{\delta_{XZ(1)}}{\sigma} \sqrt{\frac{M}{f \left(\frac{1}{M_{0,0}} + \frac{1}{M_{1,1}} + \frac{1}{M_{1,0}} + \frac{1}{KM_{0,1}} \right)}} - \Phi^{-1}(1 - \alpha/2) \right\}$$

where $f = 1 - \rho_1$.

Example 1 – Calculating Power

Suppose that a three-level hierarchical design is planned in which there will be two interventions. Each intervention will be whether one of two drugs is administered. There will be only one measurement per subject and the four treatments will be applied to level-1 units. Five level-3 units and 5 or 10 level-2 units are planned.

The analysis will be a mixed model of continuous data using the model given earlier in this chapter. The following parameter settings are to be used for the power analysis: $\delta = 3$; $\sigma = 9.8$; $\rho_1 = 0.1$; $C = 5$; $K = 5$ or 10 ; $\alpha = 0.05$; and $M00 = M01 = M10 = M11 = 5$ to 20 by 5 . Find the power of each combination of parameter settings.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	Power
Alpha.....	0.05
C (Level 3 Units).....	5
K (Level-2 Units Per Level-2 Unit)	5 10
M00 (Group 00 Count (X=0, Z=0))	5 10 15 20
M01 (Group 01 Count (X=0, Z=1))	M00
M10 (Group 10 Count (X=1, Z=0))	M00
M11 (Group 11 Count (X=1, Z=1))	M00
δ (Interaction = $(\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00})$)	3
σ (Standard Deviation).....	9.8
ρ_1 (Correlation Among Level-1 Units).....	0.1

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: **Power**

Power	Total Sample Size N	Number of Level-3 Units C	Number of Level-2 Units per Level-3 Unit K	Number of Level-1 Units per Level-2 Unit				Interaction δ	Standard Deviation σ	Correlation Among Level-1 Units ρ_1		Alpha
				Group 00 M00	Group 01 M01	Group 10 M10	Group 11 M11					
0.4380	500	5	5	5	5	5	5	3	9.8	0.1	0.05	
0.7228	1000	5	5	10	10	10	10	3	9.8	0.1	0.05	
0.8779	1500	5	5	15	15	15	15	3	9.8	0.1	0.05	
0.9503	2000	5	5	20	20	20	20	3	9.8	0.1	0.05	
0.7228	1000	5	10	5	5	5	5	3	9.8	0.1	0.05	
0.9503	2000	5	10	10	10	10	10	3	9.8	0.1	0.05	
0.9930	3000	5	10	15	15	15	15	3	9.8	0.1	0.05	
0.9992	4000	5	10	20	20	20	20	3	9.8	0.1	0.05	

Power The probability of rejecting a false null hypothesis when the alternative hypothesis is true.

N The total number of Level-1 units in the study.

C The number of Level-3 units in the study.

K The number of Level-2 units per Level-3 unit.

M00 The number of Level-1 units per Level-2 unit in Group (0,0) (the group in which $X = 0$ and $Z = 0$).

M01 The number of Level-1 units per Level-2 unit in Group (0,1) (the group in which $X = 0$ and $Z = 1$).

M10 The number of Level-1 units per Level-2 unit in Group (1,0) (the group in which $X = 1$ and $Z = 0$).

M11 The number of Level-1 units per Level-2 unit in Group (1,1) (the group in which $X = 1$ and $Z = 1$).

δ The interaction difference at which the power is calculated. $\delta = (\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00})$.

σ The standard deviation of the subject responses.

ρ_1 The intraclass correlation among Level-1 units within a single Level-2 unit.

Alpha The probability of rejecting a true null hypothesis.

Summary Statements

A 2x2 factorial ($X = 0, 1$ and $Z = 0, 1$) 3-level hierarchical design will have level-1 units (e.g., students, subjects, or patients) in level-2 units (e.g., classes, clinics, or hospitals) in level-3 units (e.g., schools, regions, or networks) with random assignment of level-1 units to each of the 4 treatment arms (Groups 00, 01, 10, and 11) (level-1 randomization). This design will be used to test the two-way interaction term ($X \times Z$) of the linear mixed-effects model, with a Type I error rate (α) of 0.05. This interaction is formed from the four group means using the following formula: $\delta = (\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00})$. The standard deviation of responses (σ) is assumed to be 9.8 (where σ^2 equals the sum of the error term variance, the level-2 random intercept variance, and the level-3 random intercept variance). The intraclass correlation coefficient of level-1 units is assumed to be 0.1. To detect a two-way interaction effect (δ) of at least 3, with 5 level-3 units, with 5 level-2 units in each level-3 unit, and within each level-2 unit, 5 level-1 units in Group 00 ($X = 0, Z = 0$), 5 level-1 units in Group 01 ($X = 0, Z = 1$), 5 level-1 units in Group 10 ($X = 1, Z = 0$), and 5 level-1 units in Group 11 ($X = 1, Z = 1$) (for a grand total of 500 level-1 units), the power is 0.438.

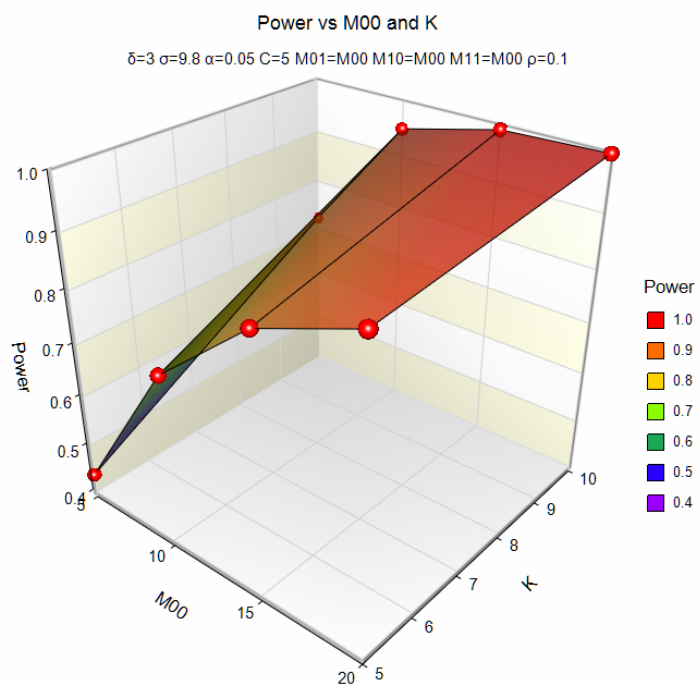
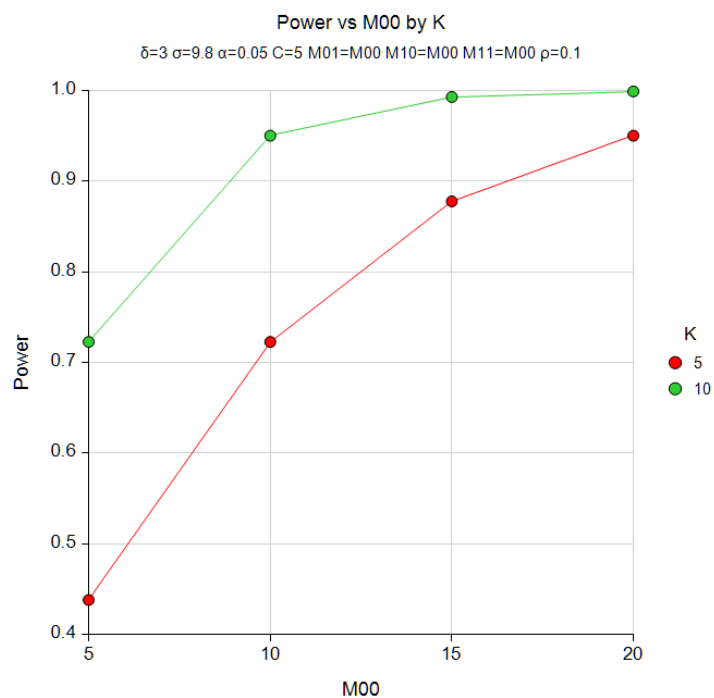
References

Ahn, C., Heo, M., and Zhang, S. 2015. Sample Size Calculations for Clustered and Longitudinal Outcomes in Clinical Research. CRC Press. New York.

This report shows the power for each of the scenarios.

Plots Section

Plots



These plots show the power for the various parameter settings.

Example 2 – Calculating Sample Size (M00)

Continuing with the last example, suppose the researchers want to determine the value of M00 needed to achieve 90% power for both values of K.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **M00 (Group 00 Count of Level-1 Units)**
 Power..... **0.90**
 Alpha..... **0.05**
 C (Level 3 Units)..... **5**
 K (Level-2 Units Per Level-2 Unit) **5 10**
 M01 (Group 01 Count (X=0, Z=1))..... **M00**
 M10 (Group 10 Count (X=1, Z=0))..... **M00**
 M11 (Group 11 Count (X=1, Z=1))..... **M00**
 δ (Interaction = $(\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00})$)..... **3**
 σ (Standard Deviation)..... **9.8**
 ρ_1 (Correlation Among Level-1 Units)..... **0.1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: **M00 (Group 00 Count of Level-1 Units)**

Power	Total Sample Size N	Number of Level-3 Units C	Number of Level-2 Units per Level-3 Unit K	Number of Level-1 Units per Level-2 Unit				Interaction δ	Standard Deviation σ	Correlation Among Level-1 Units ρ_1	Alpha
				Group 00 M00	Group 01 M01	Group 10 M10	Group 11 M11				
0.9141	1700	5	5	17	17	17	17	3	9.8	0.1	0.05
0.9282	1800	5	10	9	9	9	9	3	9.8	0.1	0.05

This report shows the M00 for each of the values of K.

Example 3 – Validation using Ahn, Heo, and Zhang (2015)

Ahn, Heo, and Zhang (2015) page 219 provide a table in which several scenarios are reported. We will validate this procedure by duplicating the top entry. The following parameter settings are used for the power analysis: Power = 0.80; $\delta = 0.3$; $\sigma = 1$; $\rho_1 = 0.1$; $C = 16$; $K = 4$; and $\alpha = 0.05$. The value of M00, M01, M10, M11 is 5 and the resulting power is 0.807.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **Power**
 Alpha..... **0.05**
 C (Level 3 Units)..... **16**
 K (Level-2 Units Per Level-2 Unit) **4**
 M00 (Group 00 Count (X=0, Z=0)) **5**
 M01 (Group 01 Count (X=0, Z=1)) **M00**
 M10 (Group 10 Count (X=1, Z=0)) **M00**
 M11 (Group 11 Count (X=1, Z=1)) **M00**
 δ (Interaction = $(\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00})$) **0.3**
 σ (Standard Deviation)..... **1**
 ρ_1 (Correlation Among Level-1 Units)..... **0.1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: **Power**

	Total Sample Size N	Number of Level-3 Units C	Number of Level-2 Units per Level-3 Unit K	Number of Level-1 Units per Level-2 Unit				Interaction δ	Standard Deviation σ	Correlation Among Level-1 Units ρ_1	Alpha
				Group 00 M00	Group 01 M01	Group 10 M10	Group 11 M11				
Power											
0.8074	1280	16	4	5	5	5	5	0.3	1	0.1	0.05

PASS also calculates the power at 0.807.