Chapter 387

Mixed Models Tests for Two Means in a 2-Level Hierarchical Design (Level-1 Randomization)

Introduction

This procedure calculates power and sample size for a two-level hierarchical mixed model in which clusters (groups, classes, hospitals, etc.) of subjects are measured one time (cross-sectional) on a continuous variable. The goal of the study is to compare the two group means.

In this design, the subjects are the level one units and the clusters are the level two units. Each subject in a particular cluster (level two unit) is randomized individually to one of two possible interventions. Note that a companion procedure power analyzes the other case in which the randomization occurs for the level two units (the clusters).

Technical Details

Our formulation comes from Ahn, Heo, and Zhang (2015), chapter 5, section 5.3.2, pages 154-157. The hierarchical mixed model that is adopted is

$$Y_{ij} = \beta_0 + \delta X_{ij} + u_i + e_{ij}$$

where

Y_{ij} is the continuous response of the jth subject in the ith cluster.

β_0 is the fixed intercept.

δ is the treatment (intervention) effect of interest. It is the difference between the two treatment means.

X_{ij} is an indicator variable that is set to 1 if subject j in cluster i receives intervention 1 and 0 otherwise.

u_i is a random effect (subject-specific intercept) term for the ith cluster that is distributed as $N(0, \sigma_u^2)$.

e_{ij} is a random effect for the jth subject in the ith cluster that is distributed as $N(0, \sigma_e^2)$.

σ_u^2 is variance of the level two (cluster) random effects.

σ_e^2 is variance of the level one (subject) random effects.

σ^2 is the variance of Y, where $\sigma^2 = \sigma_e^2/(1-\rho)$.

ρ is the intraclass correlation (ICC), where $\rho = Corr(Y_{ij}, Y_{ij'})$

The test of significance of the X_{ij} term in the mixed model analysis is the test statistic of interest. It tests the difference of the two treatment means. Since these are asymptotic results, the specific type of mixed model is not stated.

Assume that $\delta = \mu_1 - \mu_2$ is to be tested using a z-test (large sample). The statistical hypotheses are $H_0: \delta = 0$ vs. $H_a: \delta \neq 0$. When $M_1 = M_2$, the test statistic is

$$z = \frac{(\bar{Y}_1 - \bar{Y}_2)\sqrt{KM_1}}{\sigma\sqrt{2(1-\rho)}}$$

where

$$\bar{Y}_g = \frac{1}{KM_1} \sum_{i=1}^{K} \sum_{j=1}^{M_1} Y_{ij} \, , g = 1,2$$

K is the number of clusters in the study.

$M1$ is the average number of subjects per cluster in group 1.

$M2$ is the average number of subjects per cluster in group 2.

This test statistic has an approximate normal distribution.

The power is calculated using

$$Power = \Phi\left\{\frac{\delta}{\sigma}\sqrt{KM_1M_2/[(1-\rho)(M_1 + M_2)]} - \Phi^{-1}(1 - \alpha/2)\right\}$$

Example 1 – Calculating Power

Suppose that a two-level hierarchical design is planned in which there will be only one measurement per subject and treatments will be selected randomly for each subject (level-one unit). The analysis will be a mixed model of continuous data. The following parameter settings are to be used for the power analysis: δ = 0.5; σ = 1; ρ = 0.01; $M1$= 5 or 10; ICC = 0.01; α = 0.05; and K = 5 to 20 by 5.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Power**
Alpha...**0.05**
K (Number of Clusters) **5 10 15 20**
M1 (Group 1 Subjects Per Cluster)................**5 10**
M2 (Group 2 Subjects Per Cluster)................**M1**
δ (Mean Difference = μ1 − μ2) **0.5**
σ (Standard Deviation)...................................**1**
ρ (Intracluster Correlation, ICC) **0.01**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results for a Two-Level Hierarchical Mixed Model
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Solve For: Power
Groups: 1 = Treatment, 2 = Control
Difference: $\delta = \mu_1 - \mu_2$
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	Number of Subjects			Number of Clusters	Number of Subjects per Cluster		Mean Difference	Standard Deviation	Intracluster Correlation	
Power	Group 1 N1	Group 2 N2	Total N	K	Group 1 M1	Group 2 M2	δ	σ	ρ	Alpha
0.42728	25	25	50	5	5	5	0.5	1	0.01	0.05
0.70974	50	50	100	5	10	10	0.5	1	0.01	0.05
0.70974	50	50	100	10	5	5	0.5	1	0.01	0.05
0.94446	100	100	200	10	10	10	0.5	1	0.01	0.05
0.86807	75	75	150	15	5	5	0.5	1	0.01	0.05
0.99162	150	150	300	15	10	10	0.5	1	0.01	0.05
0.94446	100	100	200	20	5	5	0.5	1	0.01	0.05
0.99891	200	200	400	20	10	10	0.5	1	0.01	0.05

Power	The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N1, N2, and N	The number of subjects in groups 1, 2, and both, respectively.
K	The number of clusters.
M1	The average number of subjects (items) per cluster in group 1, which is designated as the treatment group.
M2	The average number of subjects (items) per cluster in group 2, which is designated as the control group.
δ	The mean difference in the response at which the power is calculated. $\delta = \mu_1 - \mu_2$.
σ	The standard deviation of the subject responses.
ρ	The intracluster correlation (ICC). The correlation between the any two subjects within a cluster.
Alpha	The probability of rejecting a true null hypothesis.

Summary Statements
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A 2-group 2-level hierarchical design will have level-1 units (e.g., students, subjects, or patients) in level-2 units (e.g., classes, clinics, hospitals, or clusters) with random assignment of level-1 units to each of the 2 groups (level-1 randomization). This design will be used to test the difference of two means ($\mu_1 - \mu_2$), using the appropriate term of the hierarchical mixed-effects model, with a Type I error rate (α) of 0.05. The standard deviation of level-1 units (subjects) is assumed to be 1. The correlation of level-1 units within a level-2 unit (intracluster correlation) is assumed to be 0.01. To detect a mean difference ($\mu_1 - \mu_2$) of 0.5, with 5 level-2 units (clusters), and within each level-2 unit, 5 level-1 units (subjects) in Group 1 and 5 level-1 units in Group 2 (for a grand total of 50 level-1 units), the power is 0.42728.
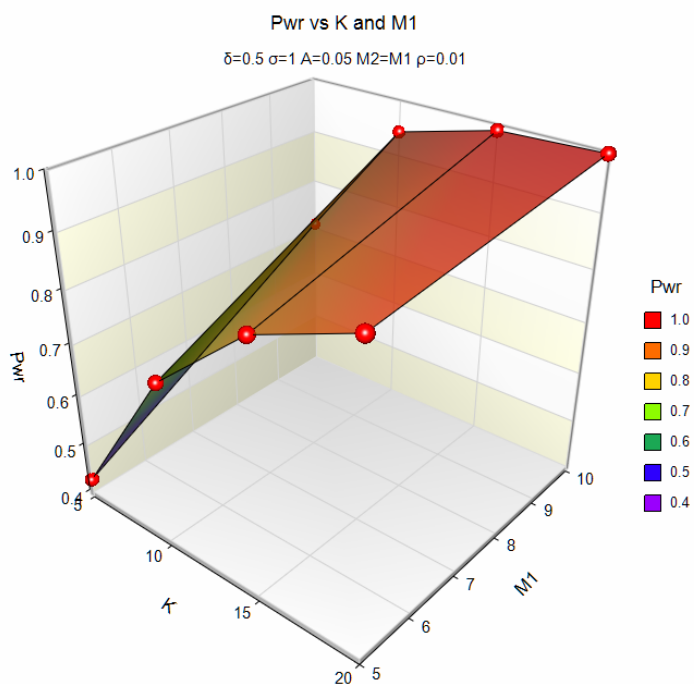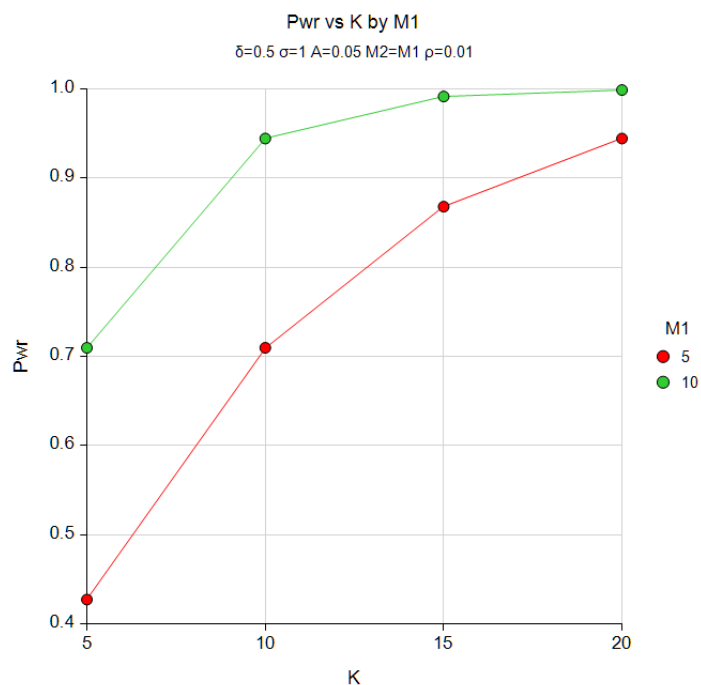_____

References
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Ahn, C., Heo, M., and Zhang, S. 2015. Sample Size Calculations for Clustered and Longitudinal Outcomes in Clinical Research. CRC Press. New York.
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This report shows the power for each of the scenarios.

Mixed Models Tests for Two Means in a 2-Level Hierarchical Design (Level-1 Randomization)

Plots Section

Plots
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These plots show the power versus the cluster count for the two cluster size values.

Example 2 – Calculating Sample Size (Number of Clusters)

Continuing with the last example, suppose the researchers want to determine the number of clusters needed to achieve 90% power for both values of M1.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**K (Number of Clusters)**
Power...**0.90**
Alpha..**0.05**
M1 (Group 1 Subjects Per Cluster)................**5 10**
M2 (Group 2 Subjects Per Cluster)...............**M1**
δ (Mean Difference = μ1 – μ2).......................**0.5**
σ (Standard Deviation)..................................**1**
ρ (Intracluster Correlation, ICC)....................**0.01**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for a Two-Level Hierarchical Mixed Model

Solve For: K (Number of Clusters)
Groups: 1 = Treatment, 2 = Control
Difference: δ = μ1 - μ2

| | Number of Subjects | | | | Number of Subjects per Cluster | | | | |
| | Group 1 | Group 2 | Total | Number of Clusters | Group 1 | Group 2 | Mean Difference | Standard Deviation | Intracluster Correlation | |
Power	N1	N2	N	K	M1	M2	δ	σ	ρ	Alpha
0.9059	85	85	170	17	5	5	0.5	1	0.01	0.05
0.9209	90	90	180	9	10	10	0.5	1	0.01	0.05

This report shows the number of clusters needed for each scenario.

Example 3 – Calculating Sample Size (Number of Subjects per Cluster)

Continuing with the last example, suppose the researchers want to determine the number of subjects needed to achieve 90% power for all values of K.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**M1 (Group 1 Subjects Per Cluster)**
Power	**0.90**
Alpha	**0.05**
K (Number of Clusters)	**5 10 15 20**
M2 (Group 2 Subjects Per Cluster)	**M1**
δ (Mean Difference = μ1 − μ2)	**0.5**
σ (Standard Deviation)	**1**
ρ (Intracluster Correlation, ICC)	**0.01**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for a Two-Level Hierarchical Mixed Model

Solve For: M1 (Group 1 Subjects Per Cluster)
Groups: 1 = Treatment, 2 = Control
Difference: δ = μ1 - μ2

	Number of Subjects				Number of Subjects per Cluster					
Power	Group 1 N1	Group 2 N2	Total N	Number of Clusters K	Group 1 M1	Group 2 M2	Mean Difference δ	Standard Deviation σ	Intracluster Correlation ρ	Alpha
0.9059	85	85	170	5	17	17	0.5	1	0.01	0.05
0.9209	90	90	180	10	9	9	0.5	1	0.01	0.05
0.9209	90	90	180	15	6	6	0.5	1	0.01	0.05
0.9445	100	100	200	20	5	5	0.5	1	0.01	0.05

This report shows the values of M1 and M2 needed for each value of K.

Example 4 – Validation using Ahn, Heo, and Zhang (2015)

Ahn, Heo, and Zhang (2015) page 157 provide a table in which several scenarios are reported. We will validate this procedure by duplicating two of the table entries. The following parameter settings are used for the power analysis: δ = 0.4; σ = 1; ρ = 0.1; K = 10; $M1=M2$ = 10; ICC = 0.1; and α = 0.05. The reported value of the power is 0.846.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Power**
Alpha...**0.05**
K (Number of Clusters) **10**
M1 (Group 1 Subjects Per Cluster)................**10**
δ (Mean Difference = $\mu1 - \mu2$) **0.4**
σ (Standard Deviation)..................................**1**
ρ (Intracluster Correlation, ICC) **0.1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for a Two-Level Hierarchical Mixed Model

Solve For: Power
Groups: 1 = Treatment, 2 = Control
Difference: $\delta = \mu1 - \mu2$

| | Number of Subjects | | | Number of Clusters | Number of Subjects per Cluster | | Mean Difference | Standard Deviation | Intracluster Correlation | |
Power	Group 1 N1	Group 2 N2	Total N	K	Group 1 M1	Group 2 M2	δ	σ	ρ	Alpha
0.8465	100	100	200	10	10	10	0.4	1	0.1	0.05

PASS also calculates the power at 0.846.