Chapter 543

One-Way Analysis of Variance Assuming Equal Variances (F-Tests)

Introduction

A common task in research is to compare the averages of two or more populations (groups). We might want to compare the income level of two regions, the nitrogen content of three lakes, or the effectiveness of four drugs. The one-way analysis of variance compares the means of two or more groups to determine if at least one mean is different from the others. The F test is used to determine statistical significance. F tests are non-directional in that the null hypothesis specifies that all means are equal, and the alternative hypothesis simply states that at least one mean is different.

The methods described here are usually applied to a one-way experimental design. This design is an extension of the design used for the two-sample t test. Instead of two groups, there are three or more groups.

Assumptions

Using the F test requires certain assumptions. One reason for the popularity of the F test is its robustness in the face of assumption violation. However, if an assumption is not even approximately met, the significance levels and the power of the F test are invalidated. Unfortunately, in practice it often happens that several assumptions are not met. This makes matters even worse. Hence, steps should be taken to check the assumptions before important decisions are made.

The assumptions of the one-way analysis of variance are:

1. The data are continuous (not discrete).

2. The data follow the normal probability distribution. Each group is normally distributed about the group mean.

3. The variances within the groups are equal.

4. The groups are independent. There is no relationship among the individuals in one group as compared to another.

5. Each group is a simple random sample from its population. Each individual in the population has an equal probability of being selected in the sample.

Technical Details for the One-Way ANOVA F-Test

Suppose G groups each have a normal distribution and with means $\mu 1_1, \mu 1_2, \ldots, \mu 1_G$ and common variance σ^2. Let $n_1, n_2, \ldots, n_G$ denote the sample size of each group and let N denote the total sample size of all groups. The F-test requires one to show that the means are sufficiently different from each other.

Let $\bar{\mu}_w$ denote the weighted mean of all groups. That is

$$\bar{\mu}_w = \sum_{i=1}^{G} \left(\frac{n_i}{N}\right) \mu 1_i$$

Let σ denote the common standard deviation of all groups.

Given the above terminology, the ratio of the mean square between groups to the mean square within groups follows a central F distribution with two parameters matching the degrees of freedom of the numerator mean square and the denominator mean square. When the null hypothesis of mean equality is rejected, the above ratio has a noncentral F distribution which also depends on the noncentrality parameter, λ. This parameter is calculated as

$$\lambda = N \frac{\sigma_{m1}^2}{\sigma^2}$$

where

$$\sigma_{m1}^2 = \sum_{i=1}^{G} \left(\frac{n_i}{N}\right) (\mu 1_i - \bar{\mu}_w)^2$$

Some authors use the symbol ϕ for the noncentrality parameter. The relationship between the two noncentrality parameters is

$$\phi = \sqrt{\frac{\lambda}{G}}$$

The process of planning an experiment should include the following steps:

1. Determine an estimate of the within group standard deviation, σ. This may be done from prior studies, from experimentation with the Standard Deviation Estimation module, from pilot studies, or from crude estimates based on the range of the data. See the chapter on estimating the standard deviation for more details.

2. Determine a set of means that represent the group differences that you want to detect.

3. Determine the appropriate group sample sizes that will ensure desired levels of α and β. Although it is tempting to set all group sample sizes equal, it is easy to show that putting more subjects in some groups than in others may have better power than keeping group sizes equal (see Example 4).

Specifying σ_m^2

The statistical hypotheses involve the variance of the means, σ_m^2. This quantity may be hard to interpret and thus difficult to come up with during the study planning phase. Cohen (1988) suggested using a different parameter that is easier to interpret. This parameter is the *range* given by

$$\delta = \text{Max}\{\mu_1, \mu_2, \dots, \mu_G\} - \text{Min}\{\mu_1, \mu_2, \dots, \mu_G\}$$

Note that the range of a set of means easier to understand than their variance or standard deviation. For example, suppose a set of means has the values {2, 3, 2, 1}. Most people could quickly grasp the meaning of the fact that the range of these means is 2 but would find it more difficult to understand that the standard deviation is 0.7071. So, even though the hypotheses are stated in terms of the standard deviations of the means, at the planning stage, it is much easier to agree on target values for the ranges under the null and alternative hypotheses.

Unfortunately, except for the case when G = 2, there is no specific relationship between δ and σ_m^2. Upon closer investigation, it has been found that there is a relationship between σ_m^2 and the range of possible values of δ. Specifically, when all group sample sizes are identical, Cohen (1988, chapter 8) gives this relationship

1. The minimum σ_m for a particular value of δ occurs when means of equal magnitude are placed at each end point and the remaining means are set to their average. This pattern of means, apart from a constant, is $\left\{\frac{-\delta}{2}, 0, \dots, 0, \frac{\delta}{2}\right\}$. In this case, $\sigma_{Min} = \delta/\sqrt{2G}$.

2. The maximum σ_m occurs when two means whose difference is δ are placed at the end points. When G is even, this pattern of means, apart from a constant, is $\left\{\frac{-\delta}{2}, \dots, \frac{-\delta}{2}, \frac{\delta}{2}, \dots, \frac{\delta}{2}\right\}$. This configuration gives $\sigma_{Max} = \delta/2$ if G is even and $\sigma_{Max} = \delta\sqrt{G^2 - 1}/2G$ if G is odd.

Examples of the Relationship between δ and σ_m

G	δ	Min Means	σ_{Min}	Max Means	σ_{Max}
4	2	{-1,0,0,1}	0.7071	{-1,-1,1,1}	1.0
5	2	{-1,0,0,0,1}	0.6325	{-1,-1,1,1,1}	0.9798
6	2	{-1,0,0,0,0,1}	0.5774	{-1,-1,-1,1,1,1}	1.0
7	2	{-1,0,0,0,0,0,1}	0.5345	{-1,-1,-1,1,1,1,1}	0.9897

Specifying a Reasonable Effect Size

Here are a reasonable set of steps to take when specifying a valid effect size for this procedure.

1. Find a reasonable value of σ. This gives you the scale.

2. Select a value for the minimally important range. This is the maximum range that you are willing to interpret as being negligible.

3. Find the minimum and maximum possible values of σ_m for the range in step 2.

4. Evaluate whether the mean pattern is closer to the *Min Means* or *Max Means* and then plan accordingly.

5. Use this information to set a range of possible σ_{m1} values.

Power Calculations for One-Way ANOVA

The calculation of the power of a particular test proceeds as follows:

1. Determine the critical value, $F_{G-1,N-G,\alpha}$ where α is the probability of a type-I error and G and N are defined above. Note that this is a two-tailed test as no direction is assigned in the alternative hypothesis.

2. From a set of $\mu_{1i}'s$, calculate the noncentrality parameter λ based on the values of $N, G, \sigma_{m1},$ and σ.

3. Compute the power as the probability of being greater than $F_{G-1,N-G,\alpha}$ on a noncentral-F distribution with noncentrality parameter λ.

Example 1 – Finding Power

An experiment is being designed to compare the means of four groups using an *F* test with a significance level of either 0.01 or 0.05. Previous studies have shown that the standard deviation is 18. Treatment means of 40, 10, 10, and 10 represent clinically important treatment differences. To better understand the relationship between power and sample size, the researcher wants to compute the power for several group sample sizes between 2 and 14. The sample sizes will be equal across all groups.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Power**
Alpha...**0.01 0.05**
G (Number of Groups) **4**
Group Allocation Input Type ....**Equal to ni (Sample Size per Group)**
ni (Sample Size per Group) **2 4 6 8 10 12 14**
μ1 Input Type...**Enter μ1 (Group Means|H1)**
μ1 (Group Means|H1) **40 10 10 10**
σ (Standard Deviation)..................................**18**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Power
Number of Groups: 4

Power	Total N	Group ni	Means μ1	SD of μ1 σm1	Standard Deviation σ	Effect Size σm1/σ	Alpha
0.04238	8	2	μ1(1)	12.99	18	0.722	0.01
0.23886	16	4	μ1(1)	12.99	18	0.722	0.01
0.50581	24	6	μ1(1)	12.99	18	0.722	0.01
0.72695	32	8	μ1(1)	12.99	18	0.722	0.01
0.86702	40	10	μ1(1)	12.99	18	0.722	0.01
0.94143	48	12	μ1(1)	12.99	18	0.722	0.01
0.97623	56	14	μ1(1)	12.99	18	0.722	0.01
0.17513	8	2	μ1(1)	12.99	18	0.722	0.05
0.52165	16	4	μ1(1)	12.99	18	0.722	0.05
0.77327	24	6	μ1(1)	12.99	18	0.722	0.05
0.90642	32	8	μ1(1)	12.99	18	0.722	0.05
0.96514	40	10	μ1(1)	12.99	18	0.722	0.05
0.98802	48	12	μ1(1)	12.99	18	0.722	0.05
0.99614	56	14	μ1(1)	12.99	18	0.722	0.05

Item	Values
μ1(1)	40, 10, 10, 10

Power	The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N	The total number of subjects in the study.
ni	The Sample Size per Group is the number of items sampled from each group in the study.
μ1	The Group Means|H1 is the set name and number of the group means under the alternative hypothesis. This is the set of means at which the power is calculated.
σm1	The SD of Group Means|H1 is the standard deviation of the group means assumed by the alternative hypothesis. Note that this value also depends on the group sample sizes.
σ	The common standard deviation of the responses within a group.
σm1/σ	The Effect Size. It is the ratio of σm1 and σ.
Alpha	The probability of rejecting a true null hypothesis.

Group Sample Size Details

n	N	Group Sample Sizes	Group Allocation Proportions
n(1)	8	2, 2, 2, 2	0.25, 0.25, 0.25, 0.25
n(2)	16	4, 4, 4, 4	0.25, 0.25, 0.25, 0.25
n(3)	24	6, 6, 6, 6	0.25, 0.25, 0.25, 0.25
n(4)	32	8, 8, 8, 8	0.25, 0.25, 0.25, 0.25
n(5)	40	10, 10, 10, 10	0.25, 0.25, 0.25, 0.25
n(6)	48	12, 12, 12, 12	0.25, 0.25, 0.25, 0.25
n(7)	56	14, 14, 14, 14	0.25, 0.25, 0.25, 0.25

Summary Statements

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A one-way ANOVA design with 4 groups will be used to test whether there is a difference among the 4 group means. The comparison will be made using an F-test with a Type I error rate (α) of 0.01. The common within-group standard deviation of responses for all groups is assumed to be 18. To detect the means 40, 10, 10, 10 (standard deviation of group means = 12.99, effect size = 0.722), with group sample sizes of 2, 2, 2, 2 (for a total of 8 subjects), the power is 0.04238.

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Dropout-Inflated Sample Size

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Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	8	10	2
20%	16	20	4
20%	24	30	6
20%	32	40	8
20%	40	50	10
20%	48	60	12
20%	56	70	14

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Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N	The evaluable sample size at which power is computed (as entered by the user). If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated power.
N'	The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D	The expected number of dropouts. D = N' - N.

Dropout Summary Statements

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Anticipating a 20% dropout rate, 10 subjects should be enrolled to obtain a final sample size of 8 subjects.

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References

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Desu, M. M. and Raghavarao, D. 1990. Sample Size Methodology. Academic Press. New York.
Fleiss, Joseph L. 1986. The Design and Analysis of Clinical Experiments. John Wiley & Sons. New York.
Kirk, Roger E. 1982. Experimental Design: Procedures for the Behavioral Sciences. Brooks/Cole. Pacific Grove, California.
Cohen, Jacob. 1988. Statistical Power Analysis for the Behavioral Sciences. Lawrence Erlbaum Associates. Hillsdale, New Jersey.

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This report shows the numeric results of this power study.

Plots Section
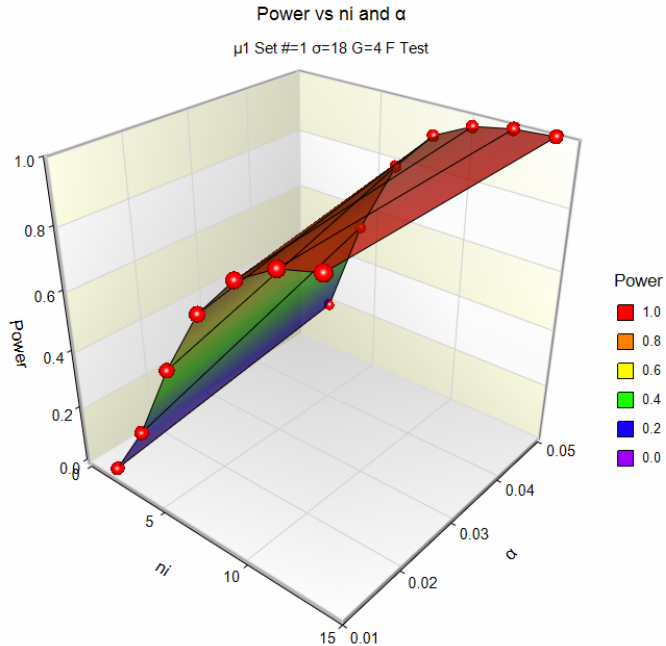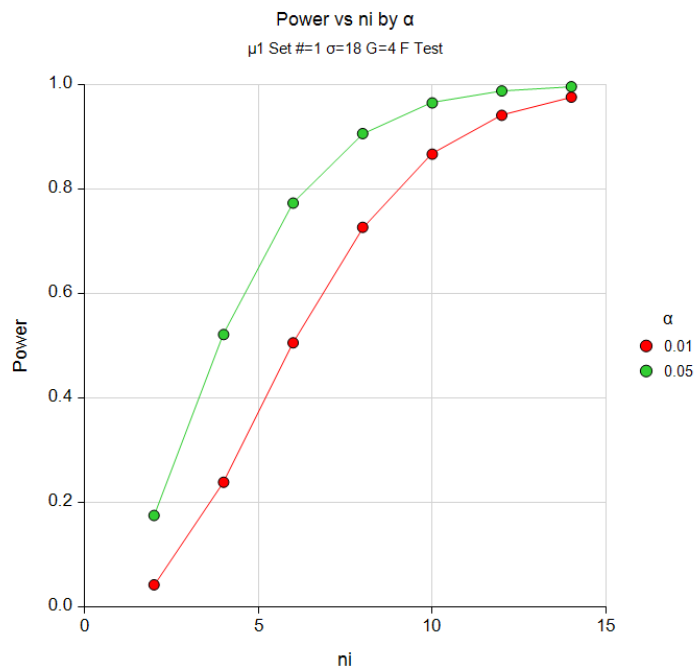
Plots
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These plots give a visual presentation to the results in the Numeric Report. We can quickly see the impact on the power of increasing the sample size and the increase in the significance level.

When you create one of these plots, it is important to use trial and error to find an appropriate range for the horizontal variable so that you have results with both low and high power.

Example 2 – Power After a Study

This example will cover the situation in which you are calculating the power of a one-way analysis of variance *F* test on data that have already been collected and analyzed.

An experiment included a control group and two treatment groups. Each group had seven individuals. A single response was measured for each individual and recorded in the following table.

Control	T1	T2
452	646	685
674	547	658
554	774	786
447	465	536
356	759	653
654	665	669
558	767	557

When analyzed using the one-way analysis of variance procedure in **NCSS**, the following results were obtained.

Analysis of Variance Table

Model Term	DF	Sum of Squares	Mean Square	F-Ratio	P-Value	Reject Equal Means at α = 0.05?
Between	2	75629.81	37814.91	3.2765	0.06117	No
Within (Error)	18	207743.4	11541.3			
Adjusted Total	20	283373.3				
Total	21					

Descriptive Statistics

Group	Count (ni)	Mean	Effect	Median	Standard Deviation	Standard Error √(MSE/ni)
All	21	612.4762	612.4762			
Control	7	527.8571	-84.61905	554	115.9661	40.60489
T1	7	660.4286	47.95238	665	119.2055	40.60489
T2	7	649.1429	36.66667	658	83.46143	40.60489

The significance level (P-Value) was 0.06117—not enough for statistical significance. The researcher had hoped to show that the treatment groups had higher response levels than the control group. He could see that the group means followed this pattern since the mean for *T1* was about 25% higher than the control mean and the mean for *T2* was about 23% higher than the control mean. He decided to calculate the power of the experiment using these values of the means. (We do not recommend this approach because the power should be calculated for the minimum difference among the means that is of interest, not at the values of the sample means.)

The data entry for this problem is simple. The only entry that is not straight forward is finding an appropriate value for the standard deviation. Since the standard deviation is estimated by the square root of the mean square error, it is calculated as $\sqrt{11541.3} = 107.4304$.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Power**
Alpha...**0.05**
G (Number of Groups) **3**
Group Allocation Input Type ....**Equal to ni (Sample Size per Group)**
ni (Sample Size per Group) **7**
µ1 Input Type..**Enter µ1 (Group Means|H1)**
µ1 (Group Means|H1) **527.8571 660.4286 649.1429**
σ (Standard Deviation)..................................**107.4304**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Power
Number of Groups: 3

Power	Sample Size		Group Means|H1		Standard Deviation σ	Effect Size σm1/σ	Alpha
	Total N	Group ni	Means µ1	SD of µ1 σm1			
0.54788	21	7	µ1(1)	60.012	107.4304	0.559	0.05

Item	Values
µ1(1)	527.8571, 660.4286, 649.1429

Group Sample Size Details

n	N	Group Sample Sizes	Group Allocation Proportions
n(1)	21	7, 7, 7	0.333, 0.333, 0.333

The power is only 0.55.

Example 3 – Finding the Sample Size Necessary to Reject

Continuing with the last example, we will determine how large the sample size would need to have been for alpha = 0.05 and power = 0.80.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Sample Size**
Power...	**0.8**
Alpha...	**0.05**
G (Number of Groups)	**3**
Group Allocation Input Type	**Equal (n1 = ··· = nG)**
μ1 Input Type...	**Enter μ1 (Group Means|H1)**
μ1 (Group Means|H1)	**527.8571 660.4286 649.1429**
σ (Standard Deviation)...................................	**107.4304**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Number of Groups: 3

	Sample Size		Group Means|H1				
Power	Total N	Group ni	Means μ1	SD of μ1 σm1	Standard Deviation σ	Effect Size σm1/σ	Alpha
0.82511	36	12	μ1(1)	60.012	107.4304	0.559	0.05

Item	Values
μ1(1)	527.8571, 660.4286, 649.1429

Group Sample Size Details

n	N	Group Sample Sizes	Group Allocation Proportions
n(1)	36	12, 12, 12	0.333, 0.333, 0.333

The required sample size is 36, or 12 per group.

Example 4 – Power using Unequal Sample Sizes

Continuing with the last example, consider the impact of allowing the group sample sizes to be unequal. Since the control group is being compared to two treatment groups, the mean of the control group is assumed to be different from those of the treatment groups. In this situation, adding extra subjects to the control group can increase power because the value of σm is increased.

In this example, we will evaluate to designs, both of which have a total of 33 subjects. In the first design, all three groups are assigned 11 subjects. In the second design, the sample size of the control group is set to 15 and to 9 in each of the treatment groups.

In order to carry out this comparison, we will enter the two sets of sample sizes in the spreadsheet. The resulting spreadsheet will appear as

C1	**C2**
11	15
11	9
11	9

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Power**
Alpha..**0.05**
G (Number of Groups) **3**
Group Allocation Input Type ....**Enter Columns of n's (Sample Sizes)**
Columns of n's (Sample Sizes)......................**1 2**
μ1 Input Type..**Enter μ1 (Group Means|H1)**
μ1 (Group Means|H1) **527.8571 660.4286 649.1429**
σ (Standard Deviation)..................................**107.4304**

Input Spreadsheet Data
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Row	C1	C2
1	11	15
2	11	9
3	11	9
_____

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Power
Number of Groups: 3

Power	Total N	Group n	Means μ1	SD of μ1 σm1	Standard Deviation σ	Effect Size σm1/σ	Alpha
	Sample Size		**Group Means|H1**				
0.78510	33	C1(1)	μ1(1)	60.012	107.4304	0.559	0.05
0.82967	33	C2(2)	μ1(1)	63.339	107.4304	0.590	0.05

Item	Values
C1(1)	11, 11, 11
C2(2)	15, 9, 9
μ1(1)	527.8571, 660.4286, 649.1429

Group Sample Size Details

n	N	Group Sample Sizes	Group Allocation Proportions
C1(1)	33	11, 11, 11	0.333, 0.333, 0.333
C2(2)	33	15, 9, 9	0.455, 0.273, 0.273

One-Way Analysis of Variance Assuming Equal Variances (F-Tests)

Plots
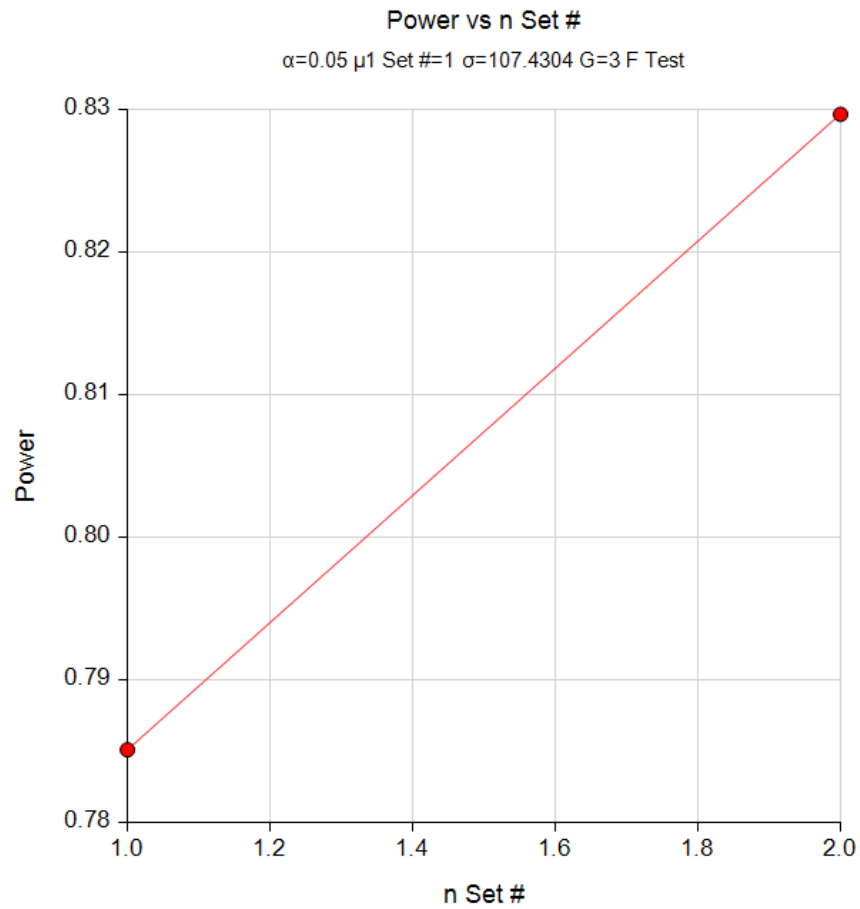
The power of 0.8297 achieved with the second design is slightly higher than the power of 0.7851 that was achieved by the first design. This occurs because the value of $\sigma m1$ was increased with the second design when the group sample sizes were unequal.

Example 5 – Minimum Detectable Difference

It may be useful to determine the minimum detectable difference among the means that can be found at the experimental conditions. This amounts to finding σm.

Continuing with the previous example, find σm for a range of sample sizes from 15 to 240 when alpha is 0.05 and power is 0.80 or 0.90.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Effect Size**
Power...**0.8 0.9**
Alpha..**0.05**
G (Number of Groups) **3**
Group Allocation Input Type ....**Equal to ni (Sample Size per Group)**
ni (Sample Size per Group) **5 10 15 20 40 60 80**
σ (Standard Deviation)..................................**107.4304**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results
───
Solve For: Effect Size
Number of Groups: 3
───

| | Sample Size | | Standard Deviation of Means | Standard Deviation | Effect Size | |
Power	Total N	Group ni	σm1	σ	σm1/σ	Alpha
0.8	15	5	98.081	107.4304	0.913	0.05
0.8	30	10	64.424	107.4304	0.600	0.05
0.8	45	15	51.537	107.4304	0.480	0.05
0.8	60	20	44.207	107.4304	0.411	0.05
0.8	120	40	30.835	107.4304	0.287	0.05
0.8	180	60	25.067	107.4304	0.233	0.05
0.8	240	80	21.662	107.4304	0.202	0.05
0.9	15	5	112.616	107.4304	1.048	0.05
0.9	30	10	73.859	107.4304	0.688	0.05
0.9	45	15	59.072	107.4304	0.550	0.05
0.9	60	20	50.666	107.4304	0.472	0.05
0.9	120	40	35.338	107.4304	0.329	0.05
0.9	180	60	28.727	107.4304	0.267	0.05
0.9	240	80	24.825	107.4304	0.231	0.05

Group Sample Size Details

n	N	Group Sample Sizes	Group Allocation Proportions
n(1)	15	5, 5, 5	0.333, 0.333, 0.333
n(2)	30	10, 10, 10	0.333, 0.333, 0.333
n(3)	45	15, 15, 15	0.333, 0.333, 0.333
n(4)	60	20, 20, 20	0.333, 0.333, 0.333
n(5)	120	40, 40, 40	0.333, 0.333, 0.333
n(6)	180	60, 60, 60	0.333, 0.333, 0.333
n(7)	240	80, 80, 80	0.333, 0.333, 0.333

Plots
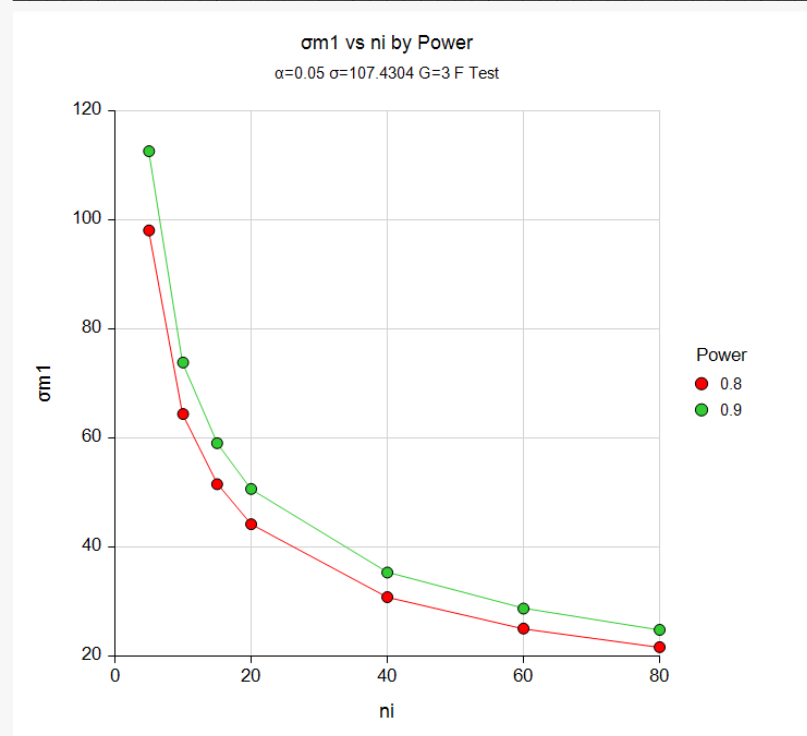
One-Way Analysis of Variance Assuming Equal Variances (F-Tests)
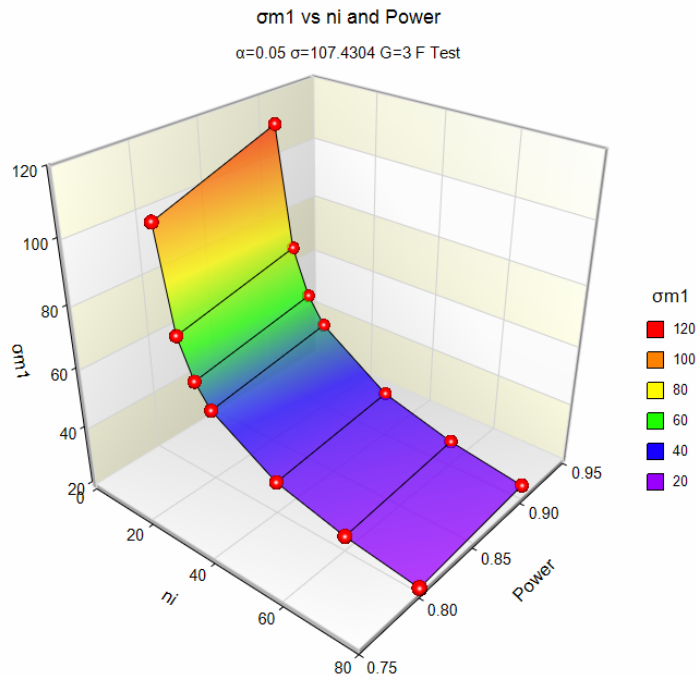
These plots show the relationships between power, sample size, and effect size. Several conclusions are possible, but the most impressive is the sharp elbow in the curve that occurs near ni = 10 when σm1 is about 60.

How do you to interpret a *σm1* of 60? One way is to find a set of means that have a standard deviation of 60.

To do this, press the σ button in the lower right corner of the panel to load the Standard Deviation Estimator module. Under the *Data* tab, enter the three values 0, 0, and 64 as a starting point. These values have a standard deviation of 30. Doubling the 64 to 128 results in an SD of 60. The difference between the minimum and the maximum of these three values is 128. Hence the minimum detectable difference is about 128 for a *σm1* of 60 when ni is 10 and the power is 80%.

Example 6 – Validation using Fleiss (1986)

Fleiss (1986) page 374 presents an example of determining a sample size in an experiment with 4 equal sized groups; means of 9.775, 12, 12, and 14.225; standard deviation of 3; alpha of 0.05, and beta of 0.20. He finds a sample size of 11 per group which amounts to a total sample size of 44.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 6** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Power...**0.8**
Alpha...**0.05**
G (Number of Groups) **4**
Group Allocation Input Type ....**Equal (n1 = ··· = nG)**
μ1 Input Type..**Enter μ1 (Group Means|H1)**
μ1 (Group Means|H1) **9.775 12 12 14.225**
σ (Standard Deviation)..................................**3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Number of Groups: 4

	Sample Size		Group Means|H1		Standard Deviation	Effect Size	
Power	Total N	Group ni	Means μ1	SD of μ1 σm1	σ	σm1/σ	Alpha
0.80273	44	11	μ1(1)	1.573	3	0.524	0.05

Item	Values
μ1(1)	9.775, 12, 12, 14.225

Group Sample Size Details

n	N	Group Sample Sizes	Group Allocation Proportions
n(1)	44	11, 11, 11, 11	0.25, 0.25, 0.25, 0.25

PASS also found N = 44. Note that Fleiss used calculations based on a normal approximation, but **PASS** uses exact calculations based on the non-central F distribution.

Example 7 – Validation using Desu (1990)

Desu (1990) page 48 presents an example of determining a sample size in an experiment with 3 groups; means of 0, -0.2553, and 0.2553; standard deviation of 1; alpha of 0.05, and beta of 0.10. He finds a sample size of 99 per group for a total of 297.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 7** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab
___

Solve For ....**Sample Size**
Power...**0.9**
Alpha..**0.05**
G (Number of Groups) **3**
Group Allocation Input Type ....**Equal (n1 = ··· = nG)**
μ1 Input Type..**Enter μ1 (Group Means|H1)**
μ1 (Group Means|H1) ....**0 -0.2553 0.2553**
σ (Standard Deviation)..................................**1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results
___

Solve For: Sample Size
Number of Groups: 3
___

| Power | Sample Size | | Group Means\|H1 | | Standard Deviation σ | Effect Size σm1/σ | Alpha |
	Total N	Group ni	Means μ1	SD of μ1 σm1			
0.90285	297	99	μ1(1)	0.208	1	0.208	0.05

___

Item	Values
μ1(1)	0, -0.2553, 0.2553

Group Sample Size Details
___

n	N	Group Sample Sizes	Group Allocation Proportions
n(1)	297	99, 99, 99	0.333, 0.333, 0.333

PASS also found N = 297.

Example 8 – Validation using Kirk (1982)

Kirk (1982) pages 140-144 presents an example of determining a sample size in an experiment with 4 groups; means of 2.75, 3.50, 6.25, and 9.0; standard deviation of 1.20995; alpha of 0.05, and beta of 0.05. He finds a sample size of 3 per group for a total sample size of 12.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 8** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Power..**0.95**
Alpha...**0.05**
G (Number of Groups) **4**
Group Allocation Input Type ....**Equal (n1 = ··· = nG)**
μ1 Input Type...**Enter μ1 (Group Means|H1)**
μ1 (Group Means|H1) **2.75 3.5 6.25 9**
σ (Standard Deviation)..................................**1.20995**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Number of Groups: 4

| | Sample Size | | Group Means|H1 | | | | |
Power	Total N	Group ni	Means μ1	SD of μ1 σm1	Standard Deviation σ	Effect Size σm1/σ	Alpha
0.99767	12	3	μ1(1)	2.465	1.20995	2.038	0.05

Item	Values
μ1(1)	2.75, 3.5, 6.25, 9

Group Sample Size Details

n	N	Group Sample Sizes	Group Allocation Proportions
n(1)	12	3, 3, 3, 3	0.25, 0.25, 0.25, 0.25

PASS also found N = 12.