Chapter 810

Tests for Intraclass Correlation

Introduction

The intraclass correlation coefficient is often used as an index of reliability in a measurement study. In these studies, there are *K* observations made on each of *N* individuals. These individuals represent a factor observed at random. This design arises when *N* subjects are each rated by *K* raters.

The intraclass correlation coefficient may be thought of as the correlation between any two observations made on the same subject. When this correlation is high, the observations on a subject tend to match, and the measurement reliability is 'high.'

Technical Details

Our formulation comes from Walter, Eliasziw, and Donner (1998) and Winer (1991). However, to be consistent within **PASS**, we have switched N and K. Denote response *j* of subject *i* by Y_{ij}, where *i* = 1, 2, ..., *N* and *j* = 1, 2, ..., *K*. The model for this situation is

$$Y_{ij} = \mu + a_i + e_{ij}$$

where the random subject effects a_i are normally distributed with mean 0 and variance σ_a^2 and the measurement errors, e_{ij} are normally distributed with mean 0 and variance σ_e^2. We assume that the subject effects and the measurement errors are independent. The intraclass correlation is then defined as

$$\rho = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}$$

The hypothesis test is stated formally as

$$H_0: \rho = \rho_0$$

$$H_1: \rho = \rho_1 > \rho_0$$

This hypothesis is tested from the data of a one-way analysis of variance table using the value: $\frac{MS_a}{MS_e}$. The critical value for the test statistic is

$$C(F_{1-\alpha,df1,df2})$$

where

$$C = 1 + \frac{K\rho_0}{1-\rho_0}$$

$$df1 = N - 1$$

$$df2 = N(K-1)$$

The power of this test is given by

$$Power = 1 - P(F \geq C_0 F_{1-\alpha,df1,df2})$$

where

$$C_0 = \frac{1 + K\rho_0/(1-\rho_0)}{1 + K\rho_1/(1-\rho_1)}$$

Example 1 – Calculating Power

Suppose that a study is to be conducted in which ρ0 = 0.2; ρ1 = 0.3; N = 50 to 250 by 100; alpha = 0.05; and K = 2 to 5 by 1; and power is to be calculated.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Power**
Alpha	**0.05**
N (Number of Subjects)	**50 to 250 by 100**
K (Observations per Subject)	**2 to 5 by 1**
ρ0 (Intraclass Correlation 0)	**0.2**
ρ1 (Intraclass Correlation 1) > ρ0	**0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Power

			Intraclass Correlation		
Power	Number of Subjects N	Observations Per Subject K	Null ρ0	Alternative ρ1	Alpha
0.18333	50	2	0.2	0.3	0.05
0.29534	50	3	0.2	0.3	0.05
0.38528	50	4	0.2	0.3	0.05
0.45522	50	5	0.2	0.3	0.05
0.36558	150	2	0.2	0.3	0.05
0.60094	150	3	0.2	0.3	0.05
0.74538	150	4	0.2	0.3	0.05
0.83005	150	5	0.2	0.3	0.05
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

Power	The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N	The number of subjects.
K	The number of observations per subject in the sample.
ρ0	The intraclass correlation assuming the null hypothesis.
ρ1	The intraclass correlation assuming the alternative hypothesis.
Alpha	The probability of rejecting a true null hypothesis.

Summary Statements

A single-group intraclass correlation design (e.g., subjects and raters) will be used to test whether the intraclass correlation is greater than 0.2 (H0: ρ ≤ 0.2 versus H1: ρ > 0.2). The comparison will be made using an intraclass correlation F-test, with a Type I error rate (α) of 0.05. To detect an intraclass correlation of 0.3 with 50 subjects and 2 observations per subject, the power is 0.18333.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	50	63	13
20%	150	188	38
20%	250	313	63

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N	The evaluable sample size at which power is computed (as entered by the user). If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated power.
N'	The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D	The expected number of dropouts. D = N' - N.

Dropout Summary Statements

Anticipating a 20% dropout rate, 63 subjects should be enrolled to obtain a final sample size of 50 subjects.

References

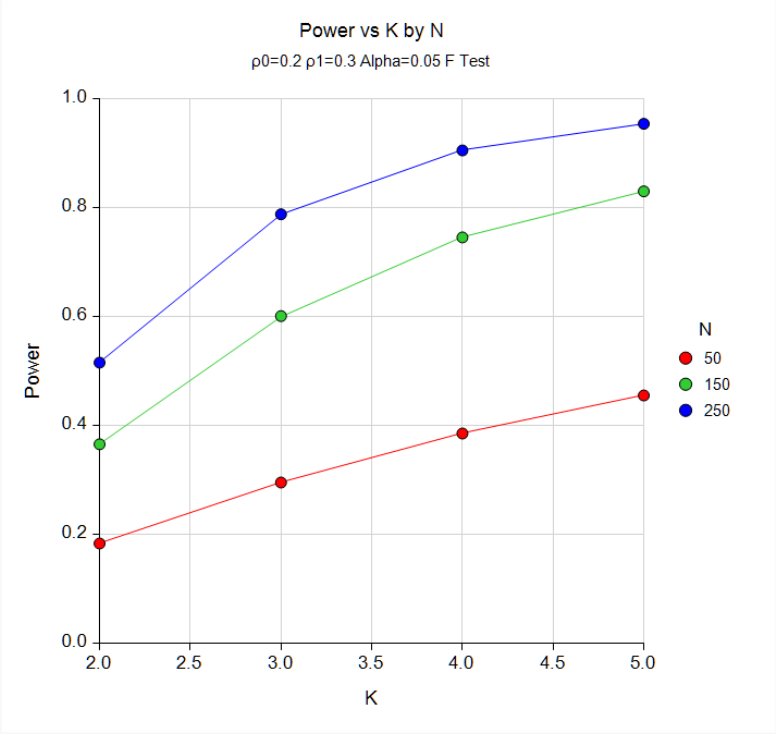
Walter, S.D., Eliasziw, M., and Donner, A. 1998. 'Sample Size and Optimal Designs For Reliability Studies.' Statistics in Medicine, 17, 101-110.

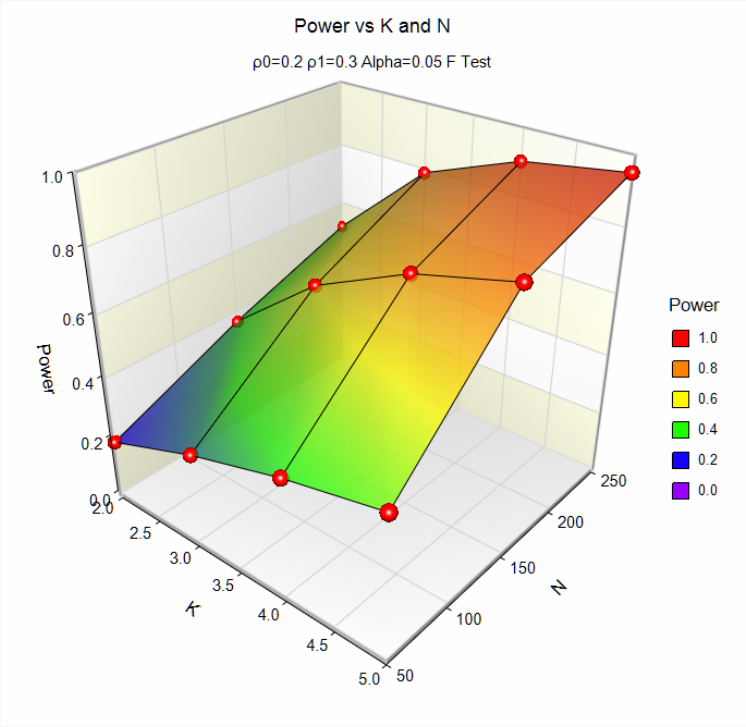
Winer, B.J. 1991. Statistical Principles in Experimental Design (Third Edition). McGraw-Hill. New York, NY.

This report shows the power for each of the scenarios.

Plots Section

Plots





These plots show the relation between power, number of subjects, and observations per subject.

Example 2 – Validation using Walter et al. (1998)

Walter *et al.* (1998) page 106 give a table of sample sizes. When ρ0 is 0.2, ρ1 is 0.3, power is 0.80, K is 2, and alpha is 0.05, the N is found to be 544.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**N (Number of Subjects)**
Power	**0.80**
Alpha	**0.05**
K (Observations Per Subject)	**2**
ρ0 (Intraclass Correlation 0)	**0.2**
ρ1 (Intraclass Correlation 1) > ρ0	**0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: N (Number of Subjects)

Power	Number of Subjects N	Observations Per Subject K	Intraclass Correlation		Alpha
			Null ρ0	Alternative ρ1	
0.80033	544	2	0.2	0.3	0.05

PASS has also calculated the power as 0.80.