Chapter 548

Tests for Multiple Proportions in a One-Way Design

Introduction

This module calculates the power and sample size for binary data from a one-way design for testing for differences among response proportions of those groups. This is analogous to the classical one-way ANOVA design that is used to analyze means. In the case of only two groups, this procedure produces power and sample size estimates that match those from the two independent-group proportions procedure.

The methods used in this chapter come from Mai and Zhang (2017). They proposed using a likelihood ratio test that has similar power characteristics to logistic regression.

Technical Details

Test Statistic

Technical details are given in Mai and Zhang (2017).

Suppose we have N_g (g = 1, ..., G) subjects in each of G groups for a total of N subjects. Let Y_{gj} be the binary response (0/1) of subject j in group g. The response is modeled by the model $Y_{gj} \sim Bernoulli(P_g)$.

The log-likelihood ratio test statistic

$$\hat{D} = -2 \sum_{g=1}^{G} N_g \left\{ \bar{y}_g \left[\ln(\bar{y}) - \ln(\bar{y}_g) + (1 - \bar{y}_g) [\ln(1 - \bar{y}) - \ln(1 - \bar{y}_g)] \right] \right\}$$

where $\bar{y}$ is the overall mean of Y and $\bar{y}_g$ is the mean of Y in group g.

Under the null hypothesis of equal proportions, this test statistic follows the chi-squared distribution with G – 1 degrees of freedom.

Effect Size

Cramer's V is used as a standardized effect size measure. It gives the association between the binary outcome variable and the group variable as a proportion of the maximum possible variation.

The definition of the effect size is

$$V^2 = -2\sum_{g=1}^{G}\frac{N_g}{N(G-1)}\{\mu_g[\ln(\mu_0) - \ln(\mu_g)] + (1-\mu_g)[\ln(1-\mu_0) - \ln(1-\mu_g)]\}$$

where $\mu_g = P_g$ is the response proportion for group g, and μ_0 is the average response of all groups.

The sample effect size is estimated by

$$\hat{V} = \sqrt{\frac{\hat{D}}{N(G-1)}}$$

Power

The power of this test is given by

$$\begin{aligned}\text{power} &= Pr\{D \geq C|H_1\} \\ &= Pr\{\chi^2(df,\delta) \geq C\} \\ &= 1 - \Psi_{df,\delta}(C) \\ &= 1 - \Psi_{G-1,N(G-1)V^2}[\chi^2_{1-\alpha}(G-1)]\end{aligned}$$

where $\Psi_{df,\delta}$ is a cumulative distribution function of a noncentral chi-squared random variable with *df* degrees of freedom and noncentrality δ, and *C* is the critical value from a central chi-squared distribution with degrees of freedom *G* – 1 and significance level $1-\alpha$.

Sample Size

The above power formula can be used in a binary search to obtain an appropriate value for the sample size.

Example 1 – Determining Power

Researchers are planning a study comparing three heart-rate medications: a standard drug and two experimental drugs. The experimental drugs are expected to have about the same impact on heart rate.

Each subject will receive an application of only one drug. The response that will be analyzed is whether the heart rate is above 60 bpm. With the standard drug, this proportion has been about 0.40. The researchers want a sample size large enough to detect a difference of 0.2 in the response rate. They will use the response proportions of 0.40, 0.20, and 0.20 to represent this difference.

The test will be conducted at the 0.05 significance level. The subjects will be randomly split equally among the three groups.

What powers are achieved by group sample sizes between 20 and 100?

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alpha.. **0.05**
G (Number of Groups) **3**
Group Allocation Input Type .... **Equal (N1 = N2 = ··· = NG)**
Ni (Subjects Per Group)................................ **20 40 60 80 100**
Pi's Input Type .... **P1, P2, ..., PG**
P1, P2, ..., PG... **0.4 0.2 0.2**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Power
Number of Groups: 3

Power	Total Number of Subjects N	Subjects per Group Ni	Group Proportions Set Pi	Effect Size Cramer's V	Alpha
0.2867	60	20	Pi(1)	0.1482	0.05
0.5266	120	40	Pi(1)	0.1482	0.05
0.7124	180	60	Pi(1)	0.1482	0.05
0.8367	240	80	Pi(1)	0.1482	0.05
0.9121	300	100	Pi(1)	0.1482	0.05

Item	Values
Pi(1)	0.4, 0.2, 0.2

Power The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N The total number of subjects in the study.
Ni The number of subjects per group.
Pi Group Proportions Set. Gives the name and number of the set containing the proportion response for each group.
Cramer's V A measure of the effect size. It is the association between the outcome variable and the grouping variable. It is the proportion of the maximum variation in the outcome variable accounted for by the grouping variable.
Alpha The probability of rejecting a true null hypothesis.

Summary Statements

A one-way design with 3 groups will be used to test whether there is a difference among the 3 group proportions. The comparison will be made using a likelihood ratio chi-square test with 2 degrees of freedom, with a Type I error rate (α) of 0.05. To detect the proportions 0.4, 0.2, 0.2 (effect size = 0.1482), with group sample sizes of 20, 20, 20 (for a total of 60 subjects), the power is 0.2867.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	60	75	15
20%	120	150	30
20%	180	225	45
20%	240	300	60
20%	300	375	75

Dropout Rate The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N The evaluable sample size at which power is computed (as entered by the user). If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated power.
N' The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D The expected number of dropouts. D = N' - N.

Dropout Summary Statements

Anticipating a 20% dropout rate, 75 subjects should be enrolled to obtain a final sample size of 60 subjects.

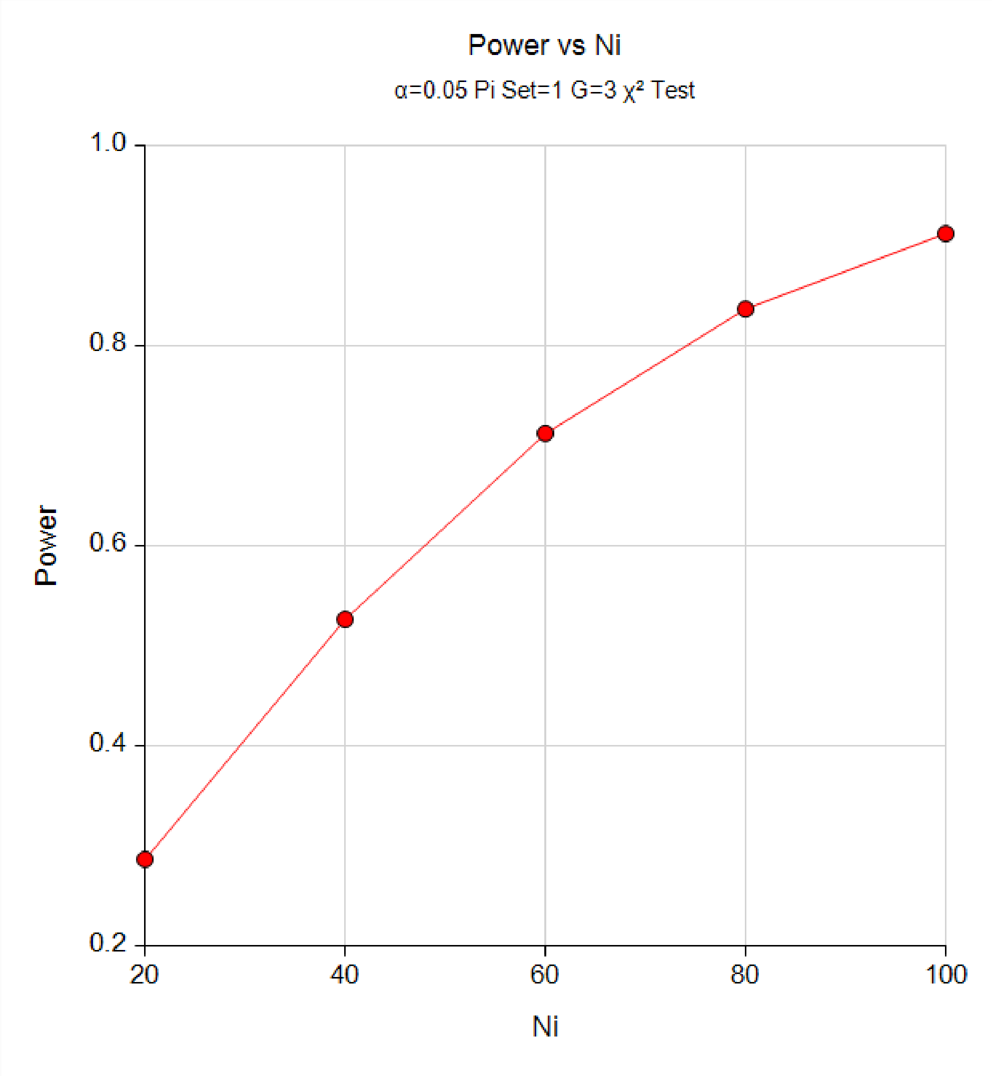
References

Y. Mai and Z.Zhang. 2017. Statistical power analysis for comparing means with binary or count data based on analogous ANOVA. In L. A. van der Ark, M. Wiberg, S. A. Culpepper, J. A. Douglas, & W.-C. Wang (Eds.), Quantitative Psychology - The 81st Annual Meeting of the Psychometric Society, Asheville, North Carolina, 2016: Springer.
Z. Zhang and K.H. Yuan. 2018. Practical Statistical Power Analysis: Using WebPower and R. ISDSA Press. Granger, IN.

This report gives the sample size for each value of the other parameters.

Plots Section

Plots



This shows the relationship among the design parameters.

Example 2 – Finding the Sample Size

Continuing with Example 1, the researchers want to determine the sample size corresponding to powers of 0.8 and 0.9.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Sample Size**
Power... **0.8 0.9**
Alpha.. **0.05**
G (Number of Groups) **3**
Group Allocation Input Type .... **Equal (N1 = N2 = ⋯ = NG)**
Pi's Input Type .... **P1, P2, ..., PG**
P1, P2, ..., PG... **0.4 0.2 0.2**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Number of Groups: 3

Power	Total Number of Subjects N	Group Subject Allocation Set ri	Group Proportions Set Pi	Effect Size Cramer's V	Alpha
0.8053	222	ri(1)	Pi(1)	0.1482	0.05
0.9001	288	ri(1)	Pi(1)	0.1482	0.05

Item	Values
ri(1)	0.333, 0.333, 0.333
Pi(1)	0.4, 0.2, 0.2

The sample sizes are 222 for a power of 0.8 and 288 for a power of 0.9. Note that the search routine only searches designs that have equal group sizes (i.e., that are divisible by 3 since there are 3 groups).

Example 3 – Comparing Various Effect Sizes

Continuing with Examples 1 and 2, the researchers want to compare the sample size of various sets of group response probabilities. To do this, they will compare the four sets of probabilities shown in the following table. The first row is the response rate for the standard medication. The second and third rows give the anticipated response rates of the experimental medications.

The values in this table must be loaded into the spreadsheet.

Table of Four Sets of Response Probabilities

C1	C2	C3	C4
0.40	0.40	0.40	0.40
0.10	0.20	0.30	0.30
0.10	0.20	0.30	0.10

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Sample Size**
Power.. **0.9**
Alpha... **0.05**
G (Number of Groups) **3**
Group Allocation Input Type .... **Equal (N1 = N2 = ⋯ = NG)**
Pi's Input Type .... **Columns containing sets of Pi's**
Columns Containing Sets of Pi's.................... **C1-C4**

Input Spreadsheet Data

Row	C1	C2	C3	C4
1	0.4	0.4	0.4	0.4
2	0.1	0.2	0.3	0.3
3	0.1	0.2	0.3	0.1

Output

Click the Calculate button to perform the calculations and generate the following output.

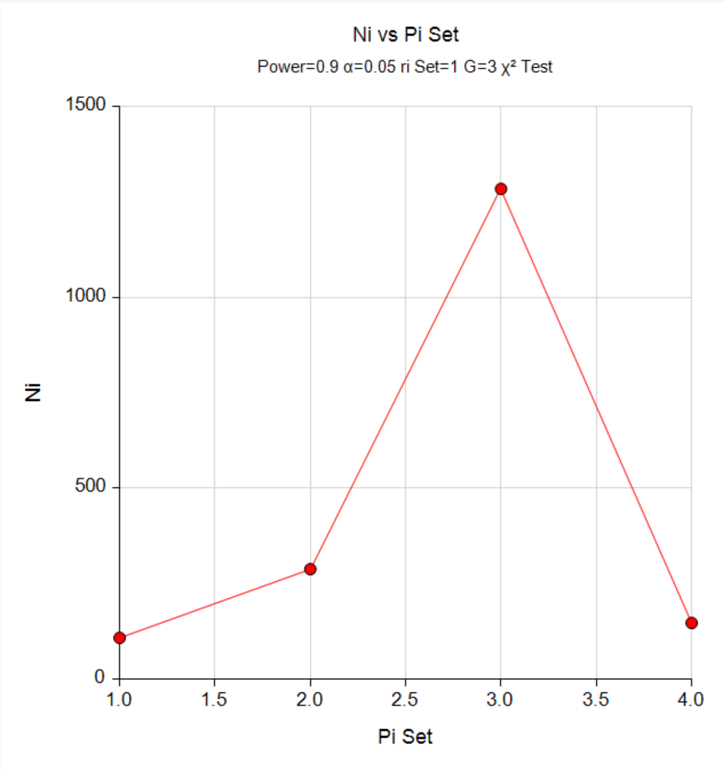
Numeric Results

Solve For: Sample Size
Number of Groups: 3

Power	Total Number of Subjects N	Group Subject Allocation Set ri	Group Proportions Set Pi	Effect Size Cramer's V	Alpha
0.9039	108	ri(1)	C1(1)	0.2436	0.05
0.9001	288	ri(1)	C2(2)	0.1482	0.05
0.9004	1284	ri(1)	C3(3)	0.0702	0.05
0.9038	147	ri(1)	C4(4)	0.2088	0.05

Item	Values
ri(1)	0.333, 0.333, 0.333
C1(1)	0.4, 0.1, 0.1
C2(2)	0.4, 0.2, 0.2
C3(3)	0.4, 0.3, 0.3
C4(4)	0.4, 0.3, 0.1

Plots



Note that the horizontal axis gives the index of the probability set footnote. Thus, '1.0' is column C1(1), '2.0' is column C2(2), and so on.

Example 4 – Validation of Power Calculation using Mai and Zhang (2017)

Mai and Zhang (2017) page 390 present an example in which G = 4, V = 0.15 α = 0.05, and Ni = 25. They obtained a power of 0.572.

Their example did not provide the individual group response rates. Through trial and error, we found the P1 = 0.475, P2 = P3 = P4 = 0.2 results in V = 0.15, so we will use those values in the validation.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alpha.. **0.05**
G (Number of Groups) **4**
Group Allocation Input Type .... **Equal (N1 = N2 = ··· = NG)**
Ni (Subjects Per Group)................................ **25**
Pi's Input Type .... **P1, P2, ..., PG**
P1, P2, ..., PG... **0.475 0.2 0.2 0.2**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Power
Number of Groups: 4

Power	Total Number of Subjects N	Subjects per Group Ni	Group Proportions Set Pi	Effect Size Cramer's V	Alpha
0.5721	100	25	Pi(1)	0.15	0.05

Item	Values
Pi(1)	0.475, 0.2, 0.2, 0.2

The power of 0.5721 matches the published power exactly, so the procedure is validated.