Chapter 439

Tests for the Difference Between Two Poisson Rates in a Cluster-Randomized Design

Introduction

Cluster-randomized designs are those in which whole clusters of subjects (classes, hospitals, communities, etc.) are sampled, rather than individual subjects. The difference between the event rates of two groups, each consisting of K_i clusters of M_{ij} individuals each, is tested using a two-sample t-test.

The formulas used here are based on Hayes and Bennett (1999) as quoted by Campbell and Walters (2014). These results are also available in Hayes and Moulton (2009).

Technical Details

Our formulation comes from Hayes and Bennett (1999). Let *K1* and *K2* represent the number of clusters in groups 1 (control) and 2 (treatment), respectively. Assume that *K1* = *K2* = *Ki*. Let *M* represent the number of person-years of observation in each cluster. Let λ_{ij} represent the true event rate in the j^{th} cluster of the i^{th} group and r_{ij} represent the corresponding observed rate. Let $\bar{r}_i$ represent the means of the two cluster rates. Assume that $E\left(r_{ij}\right) = \lambda_i$ and $V\left(r_{ij}\right) = \sigma_B^2$. Let the coefficient of variation in the i^{th} group be $CV_i = \sigma_{Bi}/\lambda_i$. Let s_i^2 be the sample variances computed from the r_{ij}.

The inequality of the λ_1 and λ_2 can be tested by the following two-sample t-test

$$t_{K1+K2-2} = \frac{\bar{r}_2 - \bar{r}_1}{\sqrt{\frac{s_1^2}{K_1} + \frac{s_2^2}{K_2}}}$$

The formula for the power, given by Hayes and Moulton (2009) for a two-sided significance test of level α to detect an event rate difference is given by

$$Power = \Phi\left[\sqrt{\frac{(K_1 - 1)(\lambda_2 - \lambda_1)^2}{\frac{(\lambda_1 + \lambda_2)}{M} + (CV_1\lambda_1)^2 + (CV_2\lambda_2)^2}} - z_{1-\alpha/2}\right]$$

where $z_x = \Phi(x)$ is the standard normal distribution function.

Example 1 – Calculating Power

Suppose that a cluster randomized study is to be conducted in which λ1 = 0.50; λ2=0.6; CV1 = CV2 = 0.25; M = 20, 40, 60, or 80; Ki = 20, 40, 60, 80, or 100; and alpha = 0.05. The power is to be calculated for a two-sided test.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Two-Sided**
Alpha.. **0.05**
Ki (Number of Clusters per Group) **20 40 60 80 100**
M (Person-Years per Cluster) **20 40 60 80**
λ1 (Event Rate of Group 1) **0.5**
Enter λ2, Diff, or Ratio for Group 2 .... **λ2 (Event Rate of Group 2)**
λ2 (Event Rate of Group 2) **0.6**
CV1 (COV of Rates in Group 1)..................... **0.25**
CV2 (COV of Rates in Group 2)..................... **CV1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Power
Groups: 1 = Control, 2 = Treatment
Test Type: T-Test of Event Rate Difference
Alternative Hypothesis: Two-Sided

	Number of Clusters		Person-Years			Event Rate				Coefficient of Variation		
Power	Group Ki	Total K	Cluster M	Group Ni	Total N	λ1	λ2	Difference Diff	Ratio RR	CV1	CV2	Alpha
0.29751	20	40	20	400	800	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.39804	20	40	40	800	1600	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.45007	20	40	60	1200	2400	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.48159	20	40	80	1600	3200	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.53446	40	80	20	800	1600	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.68362	40	80	40	1600	3200	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.74803	40	80	60	2400	4800	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.78288	40	80	80	3200	6400	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.71127	60	120	20	1200	2400	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.85047	60	120	40	2400	4800	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.89844	60	120	60	3600	7200	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.92110	60	120	80	4800	9600	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.82961	80	160	20	1600	3200	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.93443	80	160	40	3200	6400	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.96252	80	160	60	4800	9600	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.97396	80	160	80	6400	12800	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.90329	100	200	20	2000	4000	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.97283	100	200	40	4000	8000	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.98704	100	200	60	6000	12000	0.5	0.6	0.1	1.2	0.25	0.25	0.05
0.99199	100	200	80	8000	16000	0.5	0.6	0.1	1.2	0.25	0.25	0.05

Power The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
Ki Represents K1 and K2, the number of clusters in each group. This formulation assumes K1 = K2.
K The total number of clusters in the design. K = K1 + K2.
M The average number of person-years per cluster in all clusters.
Ni Represents N1 and N2, the number of person-years in each group. This formulation assumes N1 = N2.
N The total number of person-years in the design. N = N1 + N2.
λ1 The event (or incidence) rate of the control group. This is the baseline rate.
λ2 The event (or incidence) rate of the treatment group.
Diff The difference between the treatment event rate and the control event rate. Diff = λ2 - λ1.
RR The ratio of the treatment event rate and the control event rate. RR = λ2 / λ1.
CV1 The coefficient of variation of the cluster event rates in the control group.
CV2 The coefficient of variation of the cluster event rates in the treatment group.
Alpha The probability of rejecting a true null hypothesis.

Summary Statements

A parallel two-group cluster-randomized design will be used to test the Group 2 (treatment) Poisson event rate (λ2) against the Group 1 (control) Poisson event rate (λ1). The comparison will be made using a two-sided t-test based on the event rate difference, with a Type I error rate (α) of 0.05. The control group event rate (λ1) is assumed to be 0.5. The coefficient of variation of the cluster event rates in the control group (Group 1) is assumed to be 0.25 and the coefficient of variation of the cluster event rates in the treatment group (Group 2) is assumed to be 0.25. To detect a rate difference (λ2 - λ1) of 0.1 (or λ2 of 0.6), with 20 clusters per group and an average of 20 person-years per cluster (for a total of 800 person-years), the power is 0.29751.

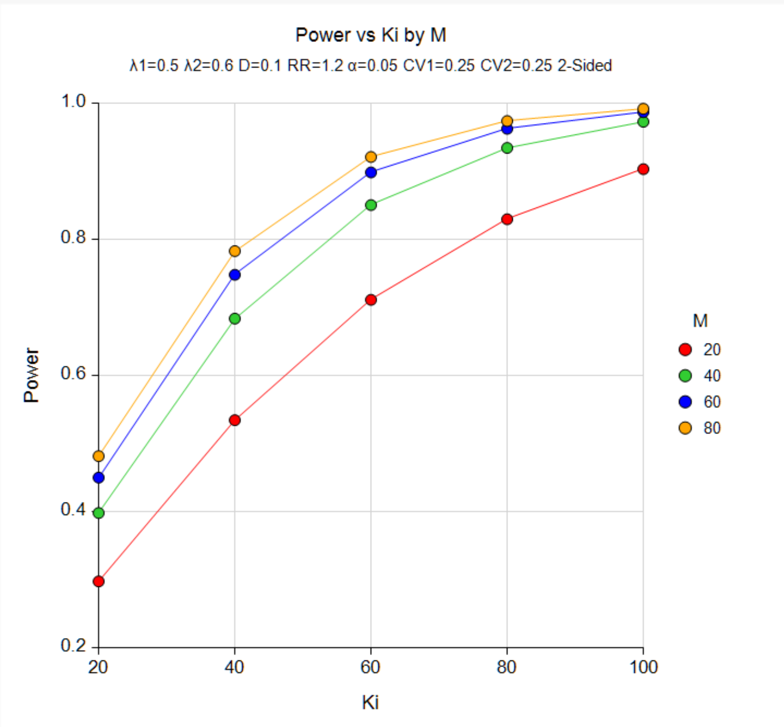
References

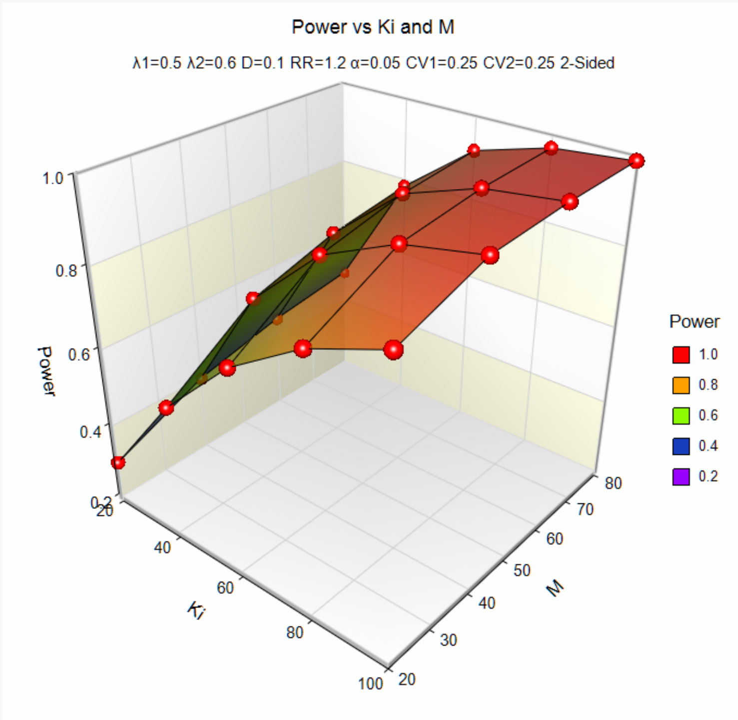
Hayes, R.J. and Bennett, S. 1999. 'Simple sample size calculation for cluster-randomized trials'. International Journal of Epidemiology. Vol 28, pages 319-326.
Hayes, R.J. and Moulton, L.H. 2009. Cluster Randomised Trials. CRC Press. New York.
Campbell, M.J. and Walters, S.J. 2014. How to Design, Analyse and Report Cluster Randomised Trials in Medicine and Health Related Research. Wiley. New York.

This report shows the power for each of the scenarios.

Plots Section

Plots





These plots show the power versus the cluster size for the two alpha values.

Example 2 – Validation using Hayes and Moulton (2009)

Hayes and Moulton (2009) on page 109 present a power calculation for this test. For the values *λ1* = 0.0148; *λ2=0.0104*; *CV1* = *CV2* =*0.29*; *M* = 424; *alpha* = 0.05; and *K1* = *K2* = 28. The resulting power value is 0.69.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Two-Sided**
Alpha... **0.05**
Ki (Number of Clusters per Group) **28**
M (Person-Years per Cluster) **424**
λ1 (Event Rate of Group 1) **0.0148**
Enter λ2, Diff, or Ratio for Group 2 .... **λ2 (Event Rate of Group 2)**
λ2 (Event Rate of Group 2) **0.0104**
CV1 (COV of Rates in Group 1)..................... **0.29**
CV2 (COV of Rates in Group 2)..................... **CV1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Power
Groups: 1 = Control, 2 = Treatment
Test Type: T-Test of Event Rate Difference
Alternative Hypothesis: Two-Sided

	Number of Clusters		Person-Years			Event Rate				Coefficient of Variation		
Power	Group Ki	Total K	Cluster M	Group Ni	Total N	λ1	λ2	Difference Diff	Ratio RR	CV1	CV2	Alpha
0.6886	28	56	424	11872	23744	0.0148	0.0104	-0.0044	0.7	0.29	0.29	0.05

PASS calculates the same power.