

Chapter 600

Appraisal Ratio Studies

Introduction

Ratio studies in mass appraisal are regularly used to evaluate the quality and accuracy of appraisal. Ratio studies provide a set of statistics describing the distribution of the ratios (such as central tendency and spread), as well as summaries of uniformity (horizontal and vertical equity).

The basic measure in ratio studies is the ratio of the appraised value to the sale price. This procedure provides a variety of statistics to describe the set of ratios under evaluation, including the median, mean, weighted mean, interquartile range (IQR), coefficient of dispersion (COD), coefficient of variation (COV), coefficient of concentration (COC), price-related differential (PRD), and coefficient of price-related bias (PRB), among many others. Confidence intervals are available for the median, mean, weighted mean, and PRB. Normality assumption tests and plots are also included in this procedure.

Often there are natural groupings of the appraised values that are compared for equity in appraisal. Examples of such groupings are neighborhood (or market area), property type (or use), size, age, condition, quality rating, and style. Several tools are available in this procedure to evaluate horizontal equity among such groups. These tools include comparative box plots, comparative density plots (and several other distribution comparison plots), assumption tests, and horizontal equity tests such as the Kruskal-Wallis Rank test, the ANOVA F-test, and Welch's test.

Another area of interest in ratio studies is vertical equity (whether ratios are similar across the whole range of appraisal and sale price values). In addition to the PRD and PRB indices of vertical equity, this procedure provides several scatter plots and tests for evaluating ratios across sale price values and appraisal values. Both slope / Pearson correlation and nonparametric Spearman rank tests are available.

Two excellent sources for greater detail on the topic of appraisal ratio study analysis are Gloudemans and Almy (2011) and the Standard on Ratio Studies put forth by the International Association of Assessing Officers (available through iaao.org at the time of this writing).

Data Structure

In the Appraisal Ratio Studies procedure, the ratio data may be specified in any of three ways.

- One appraisal column, one sale price column
- Group of appraisal columns, group of sale price columns
- Ratio column containing the already computed (and adjusted, if applicable) ratios

One appraisal column, one sale price column

The most common data structure for ratio studies is a single column containing the appraisal values, and a single column containing the sale price values. Ratios are computed as

Ratio = Appraisal Value / Sale Price Value

Appraisal	Sale_Price
62009	64000
21685	23000
41663	39500
52576	54000
32231	26500
90417	96000
.	.
.	.
.	.

Group of appraisal columns, group of sale price columns

In cases where the appraisal and/or the sale price is divided, multiple columns may be used to specify the components of the appraisal or sale. In these cases, the ratios are computed as

Ratio = Sum of Appraisal Values / Sum of Sale Price Values

App_Land	App_Bldg	Price_Land	Price_Bldg
325000	350900	316700	317000
218800	321900	212300	327500
287200	231400	281400	242600
176200	219300	150200	214800
336400	275600	366700	274900
216400	247700	220400	259700
.	.	.	.
.	.	.	.
.	.	.	.

Ratio column containing the already computed (and adjusted, if applicable) ratios

For the case where the ratios have already been produced, perhaps in the NCSS spreadsheet or in another program, the ratio values may be contained in a single column. However, statistics and reports that require appraisal values and sale price values cannot be generated in this case.

Ratio
1.232
1.094
0.973
1.049
0.964
0.982
.
.
.

Three optional column entries may also be used to enhance the appraisal study.

- Break columns for horizontal grouping
- Date column for filtering by date or for adjusting the sale price to a specific date
- Filter column for including a specific subset of the dataset

Break columns for horizontal grouping

In many mass appraisal ratio studies, investigators would like to compare ratios across horizontal groups (e.g., neighborhood or property type). In this procedure each set of groups is defined by a break column, a column with various group values. Any number of break columns may be used in this procedure. A separate analysis is performed for each break column entered.

Appraisal	Sale_Price	Neighborhood
62009	64000	Oak Hills
21685	23000	Green Point
41663	39500	Oak Hills
52576	54000	Oak Hills
32231	26500	Oak Hills
90417	96000	West Flats
20815	24400	Green Point
67843	62100	Green Point
28649	31000	Oak Hills
94812	95500	West Flats
55823	54000	West Flats
.	.	.
.	.	.
.	.	.

Date column for filtering by date or for adjusting the sale price to a specific date

In this procedure there are two potential uses for a column of sale dates. One available option is to remove the use of rows that fall outside a given date range. The second option compares the dates of this column to a basis date to make adjustments to the sale price.

Appraisal	Sale_Price	Neighborhood	Sale_Date
62009	64000	Oak Hills	4/18/2016
21685	23000	Green Point	3/2/2016
41663	39500	Oak Hills	1/23/2016
52576	54000	Oak Hills	4/14/2016
32231	26500	Oak Hills	7/3/2016
90417	96000	West Flats	2/19/2016
20815	24400	Green Point	5/12/2016
67843	62100	Green Point	4/25/2016
28649	31000	Oak Hills	1/20/2016
94812	95500	West Flats	9/6/2016
55823	54000	West Flats	3/13/2016
.	.	.	.
.	.	.	.
.	.	.	.

Filter column for including a specific subset of the dataset

A column may be used to define a specific subset of the rows that are desired to be included in the ratio study. For example, in the dataset below the user may wish to filter for use of only residential properties in the ratio study. The filtering can be done through the filter system of the spreadsheet, or the column and desired value(s) may be used directly in the Appraisal Ratio Studies procedure.

Appraisal	Sale_Price	Neighborhood	Type
62009	64000	Oak Hills	Residential
21685	23000	Green Point	Residential
41663	39500	Oak Hills	Commercial
52576	54000	Oak Hills	Commercial
32231	26500	Oak Hills	Residential
90417	96000	West Flats	Residential
20815	24400	Green Point	Commercial
67843	62100	Green Point	Residential
28649	31000	Oak Hills	Commercial
94812	95500	West Flats	Residential
55823	54000	West Flats	Commercial
.	.	.	.
.	.	.	.
.	.	.	.

Technical Details

A considerable number of statistics, confidence intervals and statistical tests are available in this procedure. The technical details of many of these are given below. For those that are not given, the reader is pointed to another section of the documentation where the details can be viewed.

Definitions and Ratios

Let A indicate the appraised value, and let S indicate the sale price. Let R designate the ratio as

$$R = \frac{A}{S}$$

Since there are a number of ratios in a ratio study, let each ratio be defined as

$$R_i = \frac{A_i}{S_i}$$

where i is the row or sequence number. Thus, the series of ratios is R_1, R_2, R_3 , etc.

Median Ratio

The median ratio is one of several measures of the center of the ratios. It is less influenced by outliers than the mean. The median ratio is the middle ratio of the ordered ratios if the number of ratios is odd, or it is the mean of the two middle ratios if the number of ratios is even. Symbolically, the median is \tilde{R} .

Mean Ratio

The mean (or average) ratio of n ratios is

$$\bar{R} = \frac{\sum_{i=1}^n R_i}{n}$$

Weighted Mean

The weighted mean is the ratio of the total appraised values for the entire sample and the total sales prices of the entire sample

$$WM = \frac{\bar{A}}{\bar{S}} = \frac{\sum_{i=1}^n A_i}{\sum_{i=1}^n S_i}$$

The weighted mean weights each ratio by the sales price. Hence, high priced properties carry a larger weight than low priced properties.

Interquartile Range (IQR)

The interquartile range is the difference between the 75th percentile of the ratios and the 25th percentile. In other words, it is the range of middle 50% of the ratios. It is commonly used as a measure of spread when the median is used as the measure of the center.

Standard Deviation (SD)

The standard deviation is roughly the average distance of the ratios from the ratio mean. For more details, see the Descriptive Statistics chapter of the documentation.

Coefficient of Dispersion (COD)

The coefficient of dispersion is a widely used measure of appraisal uniformity. It expresses as a percentage the average deviation of the ratios from the median.

$$COD = \frac{\frac{100}{n} \sum_{i=1}^n |R_i - \tilde{R}|}{\tilde{R}}$$

Coefficient of Variation (COV)

The coefficient of variation is defined as follows:

$$COV = \frac{100 * SD}{\bar{R}}$$

Price-Related Differential (PRD)

The price-related differential measures the regressivity or progressivity of the assessments. Regressive appraisals occur when high-value properties are under-appraised relative to low-value properties. Progressive appraisals occur when the opposite pattern occurs.

$$PRD = \frac{\bar{R}}{\bar{A}/\bar{S}} = \frac{\bar{R}}{WM}$$

Coefficient of Price-Related Bias (PRB)

The coefficient of price-related bias is used to analyze vertical equity (whether ratios are similar across the whole range of appraisal and sale price values). Further details of this index are given in the Vertical Equity section of this chapter.

Coefficient of Concentration (COC)

The coefficient of concentration is the percentage of ratios within a given percentage of the median. High COCs for low percentages indicate high performance.

Weighted Coefficient of Dispersion (Weighted COD)

The weighted COD can be compared to the COD to determine if high-value properties are being assessed with more or less variability than low-value properties.

$$COD_w = \frac{100}{\tilde{R}} \times \left(\frac{\sum_{i=1}^n (S_i/\bar{S}) |R_i - \tilde{R}|}{n} \right)$$

Weighted Coefficient of Variation (Weighted COV)

Similar to the weighted COD, the weighted COV can be compared to the COV to determine if high-value properties are being assessed with more or less variability than low-value properties.

$$COV_w = \frac{100}{\bar{R}} \times \sqrt{\frac{\sum_{i=1}^n (S_i/\bar{S})(R_i - \bar{R})^2}{n}}$$

Median Absolute Deviation from the Median (MADM)

Comparable to the standard deviation, this value is the median of the absolute values of all the distances from the ratios to the median.

Median Absolute Percent Deviation from the Median (MAPDM)

This value is the median of the absolute values of all the percent differences of the ratios from the median.

Trimmed Mean

The trimmed mean is the mean of the ratios after removing a desired percentage of the extreme ratios of both sides of the mean.

Confidence Interval for the Median Ratio

The details of obtaining the confidence interval for the median are described in the Percentile section of Descriptive Statistics chapter.

Confidence Interval for the Mean Ratio

The lower and upper confidence limits of the mean ratio are given by

$$\bar{R} \pm t_{\frac{\alpha}{2}, n-1} \frac{SD}{\sqrt{n}}$$

where t is the appropriate value of the t distribution.

Confidence Interval for the Weighted Mean Ratio

The confidence interval for the weighted mean ratio is calculated as described in Cochran (1971) and Gloudemans and Almy (2011). The lower and upper confidence limits of the weighted mean ratio are given by

$$\bar{A}/\bar{S} \pm t_{\frac{\alpha}{2}, n-1} \frac{\sqrt{\sum_{i=1}^n A_i^2 - 2\bar{A}/\bar{S} \sum_{i=1}^n A_i S_i + (\bar{A}/\bar{S})^2 \sum_{i=1}^n S_i^2}}{\bar{S} \sqrt{n(n-1)}}$$

where t is the appropriate value of the t distribution.

Skewness

The skewness measures the direction and degree of asymmetry. A value of zero indicates a symmetrical distribution. A positive value indicates skewness (long-tailedness) to the right while a negative value indicates skewness to the left. The method of calculation of skewness in this procedure is Fisher's g_1 . For more details, see the Descriptive Statistics chapter of the documentation.

Kurtosis

This statistic measures the heaviness of the tails of a distribution. The usual reference point in kurtosis is the Normal distribution. Unimodal distributions that have kurtosis greater than three have heavier or thicker tails than the Normal distribution. These same distributions also tend to have higher peaks in the center of the distribution (leptokurtic). Unimodal distributions whose tails are lighter than the Normal distribution tend to have a kurtosis that is less than three. In this case, the peak of the distribution tends to be broader than the Normal distribution (platykurtic). The method of calculation of kurtosis in this procedure is Fisher's g_2 . For more details, see the Descriptive Statistics chapter of the documentation.

Normality Tests

Seven normality tests are produced in this procedure. Each Normality test has its strengths. Typically, distributions of ratios that are strongly non-Normal will result in a rejection of Normality for most or all these tests. For technical details of these seven tests, go to the Normality Tests chapter of the documentation.

The seven Normality tests given in this procedure are

- Shapiro-Wilk Test
- Anderson-Darling Test
- Martinez-Iglewicz Test
- Kolmogorov-Smirnov (Lilliefors' adjusted) Test
- D'Agostino Skewness Test
- D'Agostino Kurtosis Test
- D'Agostino Omnibus (Skewness and Kurtosis) Test

Assumptions for Tests of Horizontal Equity – Equal Variance

There are four tests given in this procedure for testing equal variance among groups. For technical details of these four tests, go to the One-Way Analysis of Variance chapter of the documentation.

The four equal variance tests given in this procedure are

- Brown-Forsythe (Data - Medians)
- Levene (Data - Means)
- Conover (Ranks of Deviations)
- Bartlett (Likelihood Ratio)

Tests of Horizontal Equity

The Tests of Horizontal Equity are used to determine whether the mean or median ratio differs across horizontal groups.

The three tests given in this procedure are

- Kruskal-Wallis Rank Test
- ANOVA F-Test
- Welch's Test

The ANOVA F-test is recommended when Normality and equal variance can be assumed. Welch's test is recommended when Normality can be assumed, but equal variance cannot. The Kruskal-Wallis test is recommended when the assumption of Normality is under question. For technical details of these three tests, go to the One-Way Analysis of Variance chapter of the documentation.

Vertical Equity

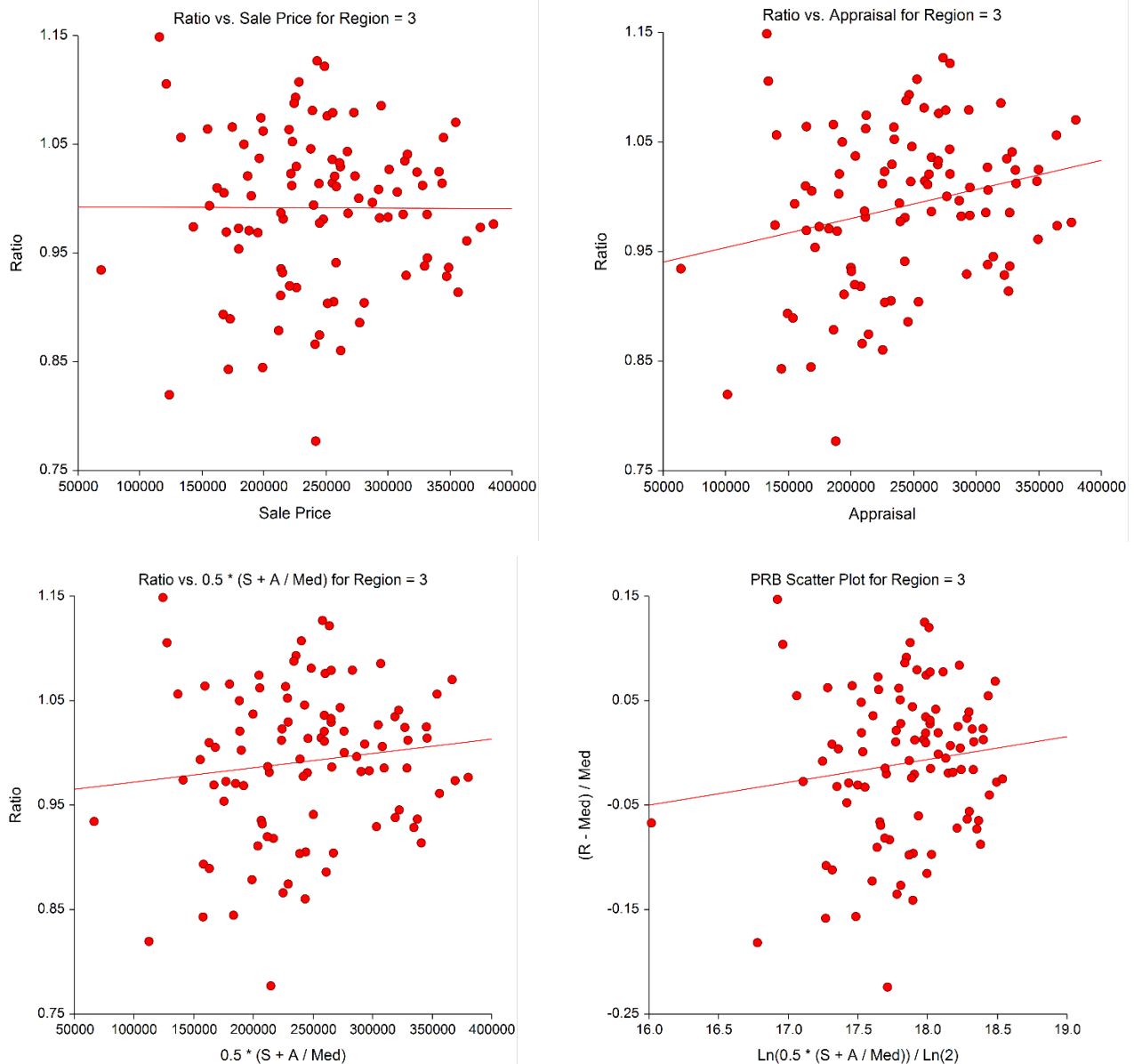
It is often of importance in ratio studies to determine whether ratios behave similarly for low-value, medium-value, and high-value properties. A variety of scatter plots, correlation values, and statistical tests may be used to assist in this determination.

Four similar analyses can be used in the Appraisal Ratio Studies procedure to assess vertical equity. For three of these, the y values are the ratios, and the x values are a measures of property value: sale price, appraisal, and adjusted average of sale price and appraisal. In the fourth case, the y values are based on the ratios and the x values are based on the sale price and appraisal values in such a way that the slope of the fitted line has a meaningful interpretation. The slope in this last case is called the coefficient of price-related bias (PRB). For the PRB method, the y and x values are computed as

$$y_i = \frac{R_i - \bar{R}}{\bar{R}}$$

$$x_i = \frac{\ln\left(0.5\left(S_i + \frac{A_i}{\bar{R}}\right)\right)}{\ln(2)}$$

Appraisal Ratio Studies



For each of these scenarios, the correlation (slope) can be tested to determine whether there is evidence of inequity across property value. Details for the calculation of the correlation, slope, testing correlation (slope) significance, and confidence intervals are given the Linear Regression and Correlation chapter of the documentation. Details of the Spearman (rank) correlation and corresponding test are also given in that chapter. The Spearman rank correlation is sometimes used when the Y-X relationship appears to be curved rather than linear.

The slope in the PRB analysis has the following interpretation: when property values double, ratios rise (or fall) by $100 \times \text{PRB} \%$. For example, if PRB is 0.1418, ratios increase by 14.2% when sale prices go from \$100,000 to \$200,000. If PRB is -0.2358, ratios decrease by 23.6% when sale prices go from \$100,000 to \$200,000.

Adjustment to Sale Price based on Sale Date

In some cases, it is of use to adjust sale prices of properties to a specific date, particularly when property values increase or decrease significantly during a short period of time. One simplistic approach to make these adjustments is available in this procedure. The date of interest (basis date) is compared to the date of each sale and the difference in months between the two dates is calculated. A monthly price adjustment factor (A) is used to produce sale prices adjusted to the basis date as follows

$$S_{i,adj} = (1 + A \times M)S_i$$

where $S_{i,adj}$ is the adjusted sale price, S_i is the original sale price, A is the adjustment factor, and M is the difference in months from the basis date. Positive A values indicate increasing sale prices, while negative A values indicate decreasing sale prices. Dates previous to the basis date result in positive M values, while dates after the basis date result in negative M values.

For example, suppose the basis date is January 1. Further suppose that a property is sold 4 months after the basis date for \$100,000. Also suppose that properties values in general increased about 6% during the year following the basis date. Increasing 6% per year, or 0.06, is comparable to 0.005 per month (if not compounding). The value for A in this example would be 0.005. The adjusted sale price used in the ratio analysis would be

$$S_{i,adj} = (1 + 0.005 \times (-4))\$100,000 = \$98,000$$

This is the estimated sale price for the property if the property had been sold 4 months earlier on January 1.

Example 1 – Appraisal Ratio Study

This example presents a broad ratio study based on the Appraisals dataset. Only the first three columns will be used in this example. Later examples show the use of outlier removal, multiple (horizontal grouping) break columns, and sale date adjustment. Summary statistics, assumptions, plots, horizontal equity, and vertical equity will all be examined in this example.

Appraisals Dataset

Appraisal	Price	Region	Type	Date
310900	296000	Fox Knoll	Residential	10/26/2015
264800	256300	Fox Knoll	Commercial	10/18/2015
204600	199700	Fox Knoll	Residential	1/2/2015
290600	264000	Fox Knoll	Commercial	3/11/2015
287900	281100	Fox Knoll	Residential	2/17/2015
292600	267400	Fox Knoll	Residential	5/12/2015
244000	253200	Fox Knoll	Commercial	4/12/2015
270100	270000	Fox Knoll	Residential	8/24/2015
210500	191200	Fox Knoll	Residential	11/8/2015
304700	302000	Fox Knoll	Commercial	6/6/2015
248200	228400	Fox Knoll	Residential	6/16/2015
.
.
.

Setup

To run this example, complete the following steps:

1 Open the Appraisals example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Appraisals** and click **OK**.

2 Specify the Appraisal Ratio Studies procedure options

- Find and open the **Appraisal Ratio Studies** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Columns Tab

Ratio Specification	Appraisal Column, Sale Price Column
Appraisal Column	Appraisal
Sale Price Column*	Price
Break Column(s)	Region

Appraisal Ratio Studies

Report Options (*in the Toolbar*)

Variable Labels.....Column Labels

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Ratio Summary Statistics**Ratio Summary Statistics**

Region	Count	Median	Mean		IQR	SD	COD	COV	PRD	PRB
			Value	Weighted						
Fox Knoll	117	1.0035	1.0025	1.0033	0.0672	0.0565	4.258	5.6357	0.9993	0.0415
Freedom Manor	184	0.9767	0.9697	0.9773	0.1075	0.0984	7.087	10.1452	0.9922	0.0812
Hastings Heights	98	1.0015	0.9918	0.9917	0.1038	0.0728	5.732	7.3422	1.0001	0.0218
Puente Real	206	1.0043	1.0049	1.0052	0.0553	0.0525	3.808	5.2233	0.9997	0.0103
Sussex Vale	143	1.0208	1.0209	1.0190	0.0450	0.0403	2.884	3.9449	1.0019	-0.0082
Combined	748	1.0035	0.9972	0.9994	0.0678	0.0680	4.901	6.8158	0.9979	0.0250

IQR: Interquartile Range

COD: Coefficient of Dispersion

COV: Coefficient of Variation

PRD: Price-Related Differential

PRB: Coefficient of Price-Related Bias

This report gives a commonly used set of ratio summary statistics for each value of the break column (Region), as well as for all regions combined. The technical details for each statistic are given earlier in the chapter in the Technical Details section. This report gives a numeric snapshot of the ratios in this study. The definitions given at the end can be hidden by unchecking Show Definitions on the Report Options tab.

Additional Ratio Summary Statistics

Additional Ratio Summary Statistics

Region	Count	Min	Max	Range	15% COC	Weighted		Absolute Deviation from the Median		5% Trimmed Mean
						COD	COV	MADM	MAPDM	
Fox Knoll	117	0.8160	1.1731	0.3571	98.291	4.040	5.2918	0.0299	2.978	1.0032
Freedom Manor	184	0.4714	1.3214	0.8501	91.304	6.174	8.6532	0.0517	5.296	0.9744
Hastings Heights	98	0.7770	1.1488	0.3718	95.918	5.450	6.9538	0.0481	4.806	0.9936
Puente Real	206	0.7481	1.1631	0.4149	97.573	3.203	4.2831	0.0273	2.716	1.0047
Sussex Vale	143	0.9056	1.1584	0.2528	100.000	2.400	3.2671	0.0214	2.097	1.0201
Combined	748	0.4714	1.3214	0.8501	96.257	4.249	6.0337	0.0340	3.387	1.0000
15% COC: Coefficient of Concentration (Percentage of Ratios within 15% of the Median)										
Weighted COD: Weighted Coefficient of Dispersion										
Weighted COV: Weighted Coefficient of Variation										
MADM: Median Absolute Deviation from the Median										
MAPDM: Median Absolute Percent Deviation from the Median										
5% Trimmed Mean: The mean after the 5% most extreme Ratios of each side are removed)										

This report gives (perhaps less commonly used) additional summary statistics for each region and the combined regions. The technical details for each statistic are given earlier in the chapter in the Technical Details section.

Appraisal and Sales Price Summary Statistics

Appraisal and Sale Price Summary Statistics

Region	Count	Mean		Median	
		Appraisal	Sale Price	Appraisal	Sale Price
Fox Knoll	117	251792.3	250976.1	254700	256300
Freedom Manor	184	273625.0	279974.5	263550	274500
Hastings Heights	98	243087.8	245115.3	243550	244500
Puente Real	206	321050.5	319381.1	321850	319200
Sussex Vale	143	298388.1	292823.8	292300	290800
Combined	748	284004.3	284180.6	275600	273900

Summary statistics of the original appraisal and sale price values are given here.

Confidence Intervals

Confidence Intervals

Region	Count	Median Ratio			Mean Ratio			Weighted Mean Ratio		
		Value	95% C.I. Limits		Value	95% C.I. Limits		Value	95% C.I. Limits	
			Lower	Upper		Lower	Upper		Lower	Upper
Fox Knoll	117	1.0035	0.9944	1.0136	1.0025	0.9922	1.0129	1.0033	0.9937	1.0128
Freedom Manor	184	0.9767	0.9629	0.9873	0.9697	0.9554	0.9840	0.9773	0.9662	0.9884
Hastings Heights	98	1.0015	0.9814	1.0143	0.9918	0.9772	1.0064	0.9917	0.9780	1.0054
Puente Real	206	1.0043	0.9954	1.0116	1.0049	0.9977	1.0121	1.0052	0.9997	1.0107
Sussex Vale	143	1.0208	1.0148	1.0260	1.0209	1.0143	1.0276	1.0190	1.0140	1.0240
Combined	748	1.0035	0.9969	1.0086	0.9972	0.9924	1.0021	0.9994	0.9954	1.0034

This report gives confidence intervals for the median, mean, and weighted mean. The technical details for each confidence interval are given earlier in the chapter in the Technical Details section.

Normality Assumption

Normality Assumption

Region	Count	Skewness	Kurtosis	P-Values (< 0.1 rejects Normality)						
				S-W	A-D	M-I	K-S	D-S	D-K	D-O
Fox Knoll	117	-0.0901	0.8100	0.3083	0.2056	< 0.1	> 0.1	0.6788	0.1037	0.2442
Freedom Manor	184	-0.9729	4.6534	0.0000	0.0000	< 0.1	< 0.1	0.0000	0.0000	0.0000
Hastings Heights	98	-0.4356	0.0173	0.3375	0.2245	> 0.1	> 0.1	0.0743	0.8216	0.1983
Puente Real	206	-0.2937	3.2823	0.0000	0.0006	< 0.1	< 0.1	0.0830	0.0000	0.0000
Sussex Vale	143	0.3837	1.6873	0.0036	0.0080	< 0.1	< 0.1	0.0594	0.0052	0.0034
Combined	748	-1.2057	6.9241	0.0000	0.0000	< 0.1	< 0.1	0.0000	0.0000	0.0000

Skewness: Negative values indicate left skewness; positive values indicate right skewness; values close to 0 indicate little skewness.

Kurtosis: Negative values indicate fatter tails; positive values indicate a strong peak and lighter tails.

S-W: Shapiro-Wilk W Test

A-D: Anderson-Darling Test

M-I: Martinez-Iglewicz Test

K-S: Kolmogorov-Smirnov (Lilliefors' adjusted) Test

D-S: D'Agostino Skewness Test

D-K: D'Agostino Kurtosis Test

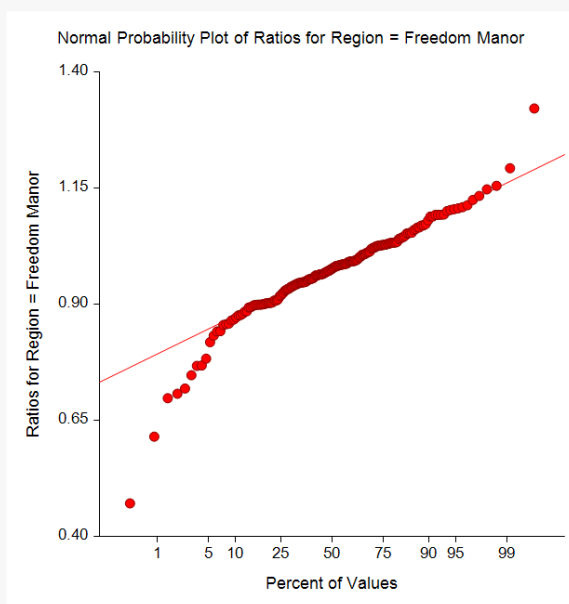
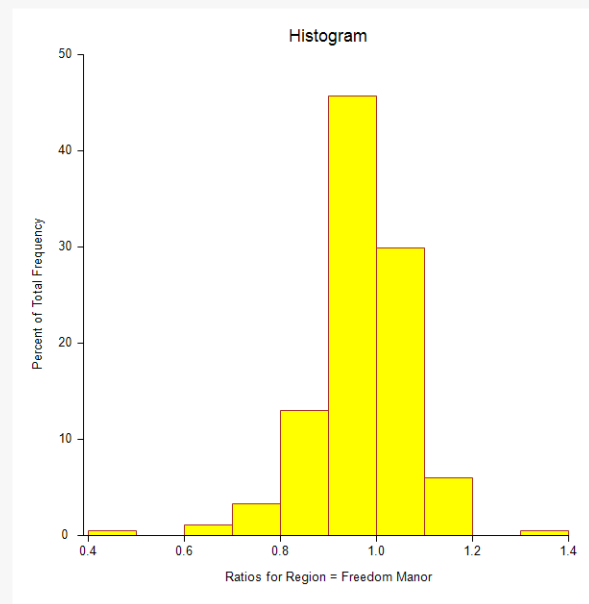
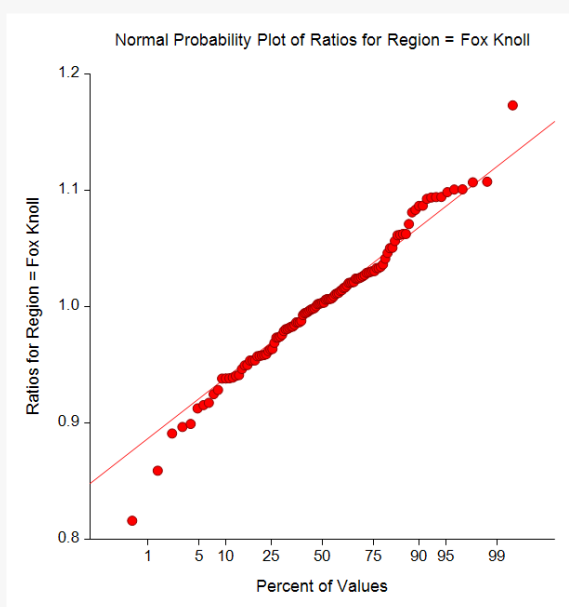
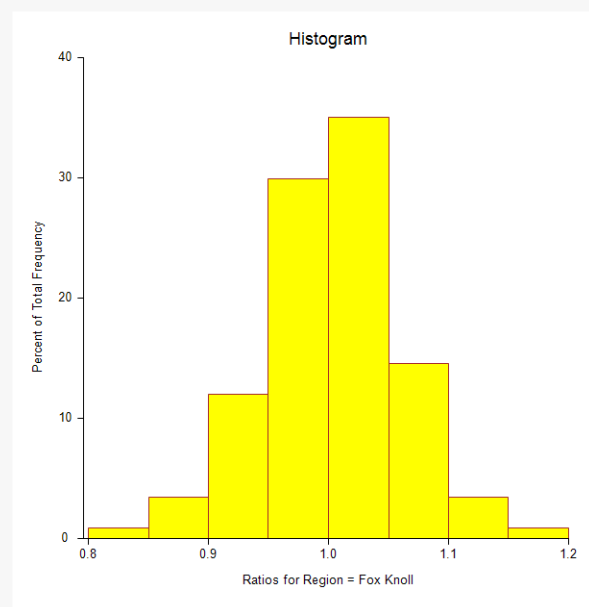
D-O: D'Agostino Omnibus (Skewness and Kurtosis) Test

Tests for which the Normality assumption is rejected are highlighted in red.

The skewness and kurtosis statistics, along with the seven Normality tests, give indication of whether the assumption of Normality is reasonable for each group. This report shows that Normality should not be assumed for the Freedom Manor, Puente Real, and Sussex Vale regions. For formulas and details, the user is pointed to the Descriptive Statistics and Normality Tests chapters in the documentation.

Normality Assumption Plots

Normality Assumption Plots

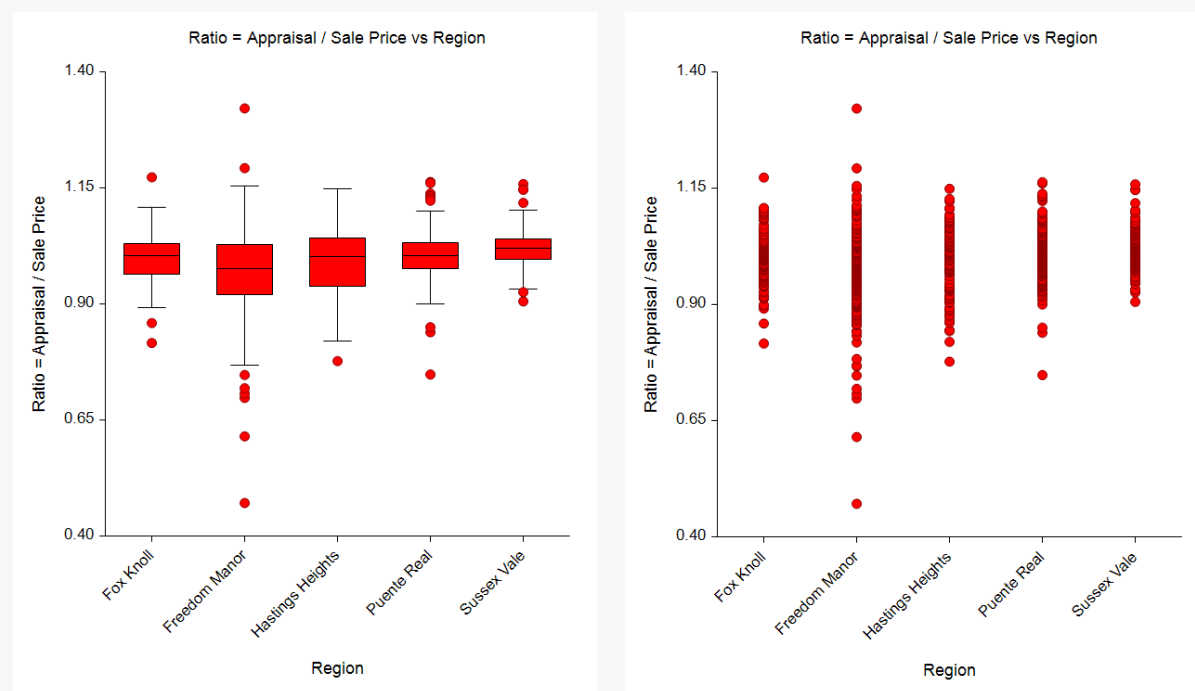


Report continues with more Normality Assumption Plots...

The histograms and Normal probability plots give a visual representation of whether the assumption of Normality is reasonable. For the histograms, a bell-shaped appearance indicates Normality, while for the Normal probability plot, points that roughly follow the straight line indicate Normality. In this example the ratios for the Fox Knoll region appear to be Normally distributed, while those in the Freedom Manor region seem to exhibit a departure from Normality.

Comparative (Horizontal Equity) Plots

Comparative (Horizontal Equity) Plots



The comparative plots are used to compare the distribution of the values of the break column. In the box plot, the line across the near-middle of the box is the median line. The box itself shows the interquartile range (IQR), or the region containing the middle 50% of the ratios. The dot plot shows each ratio individually. Some quick takeaways from these plots are the lower median for the Freedom Manor region, and greater variability in the Freedom Manor and Hastings Heights ratios. These characteristics can be confirmed by examining the ratio summary statistics. For statistical comparisons of the means or medians, use the tests of horizontal equity.

Horizontal Equity Test Assumption: Normality

Horizontal Equity Test Assumption: Normality

Normality Attributes	Test Statistic	P-Value	Reject Normality at $\alpha = 0.1$?
Skewness Test	-8.1115	0.0000	Yes
Kurtosis Test	10.4382	0.0000	Yes
Skewness and Kurtosis (Omnibus)	174.7531	0.0000	Yes

These Normality tests, based in the residuals from the group means, are used to determine whether the Normality assumption is reasonable in the test of horizontal equity. The tests here show strong evidence that the residuals should not be assumed to be Normal, and therefore, a nonparametric test of horizontal equity should be used (Kruskal-Wallis rank test).

Horizontal Equity Test Assumption: Equal Variance

Horizontal Equity Test Assumption: Equal Variance

Test Name	Test Statistic	P-Value	Reject Equal Variances at $\alpha = 0.1$?
Brown-Forsythe (Data - Medians)	18.6639	0.0000	Yes
Levene (Data - Means)	19.2449	0.0000	Yes
Conover (Ranks of Deviations)	72.6317	0.0000	Yes
Bartlett (Likelihood Ratio)	152.2970	0.0000	Yes

These tests compare the spread of the ratios across the groups. When the ratios are Normally distributed, these tests can help determine whether the ANOVA F-test or Welch's test should be used for comparing the groups. These tests may also be used to evaluate whether the spread of ratios is significantly different across groups. In this example, the spread of the five groups is clearly not equal.

Tests of Horizontal Equity - Region

Tests of Horizontal Equity - Region

Test Name	Test Statistic	P-Value	Conclusion at $\alpha = 0.05$
Kruskal-Wallis Rank Test*	45.8080	0.0000	Reject Equality of Ratio Medians
ANOVA F-Test**	12.8003	0.0000	Reject Equality of Ratio Means
Welch's Test***	12.1189	0.0000	Reject Equality of Ratio Means

* The Kruskal-Wallis Test is recommended when the Normality assumption is not met. The test statistic is the Z-Value.

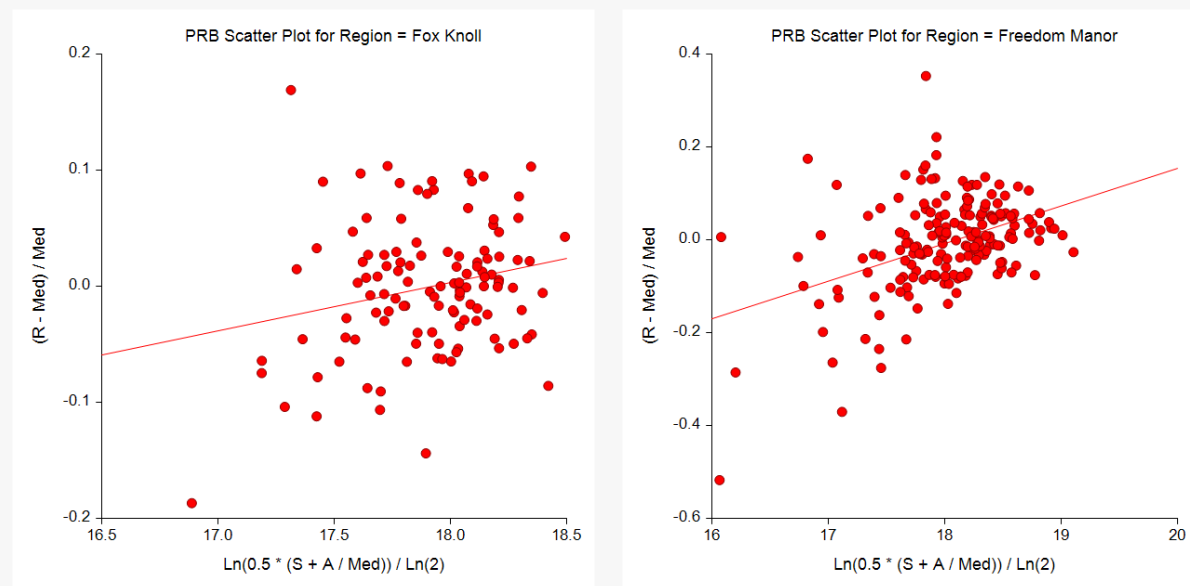
** The ANOVA F-Test assumes the variances are similar across groups, and that the distribution of residuals is approximately Normal. The test statistic is the F-Value.

*** Welch's Test is recommended when the data can be assumed to be approximately Normally distributed, but the variances are unequal. The test statistic is the F-Value.

These tests are used to compare the center (medians or means) of the groups. Since the assumption of Normality is not reasonable (based on previous Normality tests), the Kruskal-Wallis test should be used here. However, all three tests show strong evidence that there is a difference among the centers of the five groups. If the user wishes to find out specifically which groups are different from the others, the ratios could be output to the spreadsheet and multiple comparison tests could be conducted using the One-Way Analysis of Variance procedure (use the Tukey-Kramer Test for Normally distributed data, or the Kruskal-Wallis Z Test (Dunn's Test) when the Normality assumption is not met).

Price-Related Bias (PRB) Scatter Plots

Price-Related Bias (PRB) Scatter Plots



Report continues with more PRB scatter plots...

These plots give a visual representation of the vertical bias. The greater the slope, the greater the indication of vertical bias.

Price-Related Bias (PRB) Details

Price-Related Bias (PRB) Details

$Y = (\text{Ratio} - \text{Median Ratio}) / \text{Median Ratio}$

$X = \text{Ln}(0.5 * (\text{Sale Price} + \text{Appraisal} / \text{Median Ratio})) / \text{Ln}(2)$

Region	Count	Pearson Correlation	PRB (Slope)	Standard Error	Test of H0: PRB (Slope) = 0		Reject Vertical Equity at $\alpha = 0.05$?	95% Confidence Interval Limits for PRB (Slope)	
					T	P-Value		Lower	Upper
Fox Knoll	117	0.2207	0.0415	0.0171	2.4262	0.0168	Yes	0.0076	0.0753
Freedom Manor	184	0.4203	0.0812	0.0130	6.2487	0.0000	Yes	0.0556	0.1068
Hastings Heights	98	0.1283	0.0218	0.0172	1.2679	0.2079	No	-0.0123	0.0558
Puente Real	206	0.1191	0.0103	0.0060	1.7137	0.0881	No	-0.0016	0.0222
Sussex Vale	143	-0.1337	-0.0082	0.0051	-1.6025	0.1113	No	-0.0184	0.0019
Combined	748	0.1943	0.0250	0.0046	5.4103	0.0000	Yes	0.0160	0.0341

Pearson Correlation:
PRB (Slope):

The common measure of the linear relationship of two variables.
Coefficient of Price-Related Bias. This is the slope of the regression line (for example, a PRB of 0.032 indicates a 3.2% increase in assessment ratio for every 100% increase in value).

Standard Error:

The standard error of the estimate of PRB.

T:

The test statistic for testing whether PRB is statistically different from 0.

P-Value:

This P-Value indicates the likelihood that the slope is flat (PRB = 0) given the sale price and appraisal values in question.

Confidence Interval Limits:

The lower and upper confidence interval limits for the value of PRB.

Appraisal Ratio Studies

The PRB report gives tests of whether the slope (PRB) is significantly different from zero. For slopes significantly different from zero there is a meaningful interpretation of the slope. For example, for the Fox Knoll region, PRB is 0.0415 and the test indicates the slope is significantly different from zero. The interpretation may be made that for a doubling in property value, there is approximately a 4.15% increase in ratio. Higher valued properties are valued relatively higher than lower valued properties by this amount. Some technical details are given in the Vertical Equity sub-section of the Technical Details of this chapter. For the technical details of regression analysis, the reader is referred to the Linear Regression and Correlation chapter.

Example 2 – Removing Outliers with the IQR Method

In this example, each row is examined to determine whether it should be considered an outlier based on the IQR method. An IQR multiplier of 3 (and later 1.5) will be used in this example. In the IQR method, IQR is the range of the middle 50% of the ratios. A ratio is deemed an outlier if it is greater than 3 times the (group) IQR from the (group) first and third quartiles. Only the first three columns will be used in this example.

Appraisals Dataset

Appraisal	Price	Region	Type	Date
310900	296000	Fox Knoll	Residential	10/26/2015
264800	256300	Fox Knoll	Commercial	10/18/2015
204600	199700	Fox Knoll	Residential	1/2/2015
290600	264000	Fox Knoll	Commercial	3/11/2015
287900	281100	Fox Knoll	Residential	2/17/2015
292600	267400	Fox Knoll	Residential	5/12/2015
244000	253200	Fox Knoll	Commercial	4/12/2015
270100	270000	Fox Knoll	Residential	8/24/2015
210500	191200	Fox Knoll	Residential	11/8/2015
304700	302000	Fox Knoll	Commercial	6/6/2015
248200	228400	Fox Knoll	Residential	6/16/2015
.
.
.

Setup

To run this example, complete the following steps:

1 Open the Appraisals example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Appraisals** and click **OK**.

2 Specify the Appraisal Ratio Studies procedure options

- Find and open the **Appraisal Ratio Studies** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Columns Tab

Appraisal Column **Appraisal**
 Sale Price Column* **Price**
 Break Column(s) **Region**
 Remove Outliers using IQR Method **Checked**
 IQR Multiplier **3.0**

Appraisal Ratio Studies

Summary Reports Tab

Sale Price Adjustment by Date Summary.....**Unchecked**Removed Ratios Report**Checked**Ratio Summary Statistics.....**Checked****3 Run the procedure**

- Click the **Run** button to perform the calculations and generate the output.

Removed Ratios**Removed Ratios**

Row removal options used in this analysis:

Remove rows with ratios $< (Q1 - 3 * IQR)$ or $> (Q3 + 3 * IQR)$

The following ratios were removed from the analysis for the reason(s) given.

Row #	Region	Removed Ratio	Reason
239	Freedom Manor	0.4714	$< 0.5974 (Q1 - 3 * IQR)$
547	Puente Real	0.7481	$< 0.8108 (Q1 - 3 * IQR)$

This report shows the ratios that were removed based on the IQR method. The IQR and quartiles (Q1 and Q3) are different for each region, so the boundary value for each region is different.

Ratio Summary Statistics**Ratio Summary Statistics**

Region	Count	Median	Mean		IQR	SD	COD	COV	PRD	PRB
			Value	Weighted						
Fox Knoll	117	1.0035	1.0025	1.0033	0.0672	0.0565	4.258	5.6357	0.9993	0.0415
Freedom Manor	183	0.9776	0.9724	0.9782	0.1050	0.0914	6.836	9.4023	0.9941	0.0661
Hastings Heights	98	1.0015	0.9918	0.9917	0.1038	0.0728	5.732	7.3422	1.0001	0.0218
Puente Real	205	1.0049	1.0062	1.0055	0.0544	0.0494	3.700	4.9133	1.0006	0.0034
Sussex Vale	143	1.0208	1.0209	1.0190	0.0450	0.0403	2.884	3.9449	1.0019	-0.0082
Combined	746	1.0035	0.9983	0.9997	0.0676	0.0649	4.809	6.4982	0.9986	0.0188

IQR: Interquartile Range

COD: Coefficient of Dispersion

COV: Coefficient of Variation

PRD: Price-Related Differential

PRB: Coefficient of Price-Related Bias

Some of the summary statistics are slightly different from Example 1 for the Freedom Manor, Puente Real, and Combined regions.

Removed Ratios using the IQR Multiplier 1.5

Generally, an IQR Multiplier of around 3 is recommended. If the IQR Multiplier is set to 1.5, considerably more ratios will be deemed outliers.

Removed Ratios

Row removal options used in this analysis:

Remove rows with ratios $< (Q1 - 1.5 * IQR)$ or $> (Q3 + 1.5 * IQR)$

The following ratios were removed from the analysis for the reason(s) given.

Row #	Region	Removed Ratio	Reason
45	Fox Knoll	1.1731	$> 1.1314 (Q3 + 1.5 * IQR)$
91	Fox Knoll	0.8160	$< 0.8628 (Q1 - 1.5 * IQR)$
96	Fox Knoll	0.8590	$< 0.8628 (Q1 - 1.5 * IQR)$
128	Freedom Manor	0.6149	$< 0.7587 (Q1 - 1.5 * IQR)$
149	Freedom Manor	1.1928	$> 1.1888 (Q3 + 1.5 * IQR)$
199	Freedom Manor	0.6977	$< 0.7587 (Q1 - 1.5 * IQR)$
234	Freedom Manor	0.7185	$< 0.7587 (Q1 - 1.5 * IQR)$
239	Freedom Manor	0.4714	$< 0.7587 (Q1 - 1.5 * IQR)$
259	Freedom Manor	1.3214	$> 1.1888 (Q3 + 1.5 * IQR)$
286	Freedom Manor	0.7074	$< 0.7587 (Q1 - 1.5 * IQR)$
300	Freedom Manor	0.7471	$< 0.7587 (Q1 - 1.5 * IQR)$
303	Hastings Heights	0.7770	$< 0.7821 (Q1 - 1.5 * IQR)$
433	Puente Real	1.1270	$> 1.1148 (Q3 + 1.5 * IQR)$
447	Puente Real	1.1631	$> 1.1148 (Q3 + 1.5 * IQR)$
487	Puente Real	1.1605	$> 1.1148 (Q3 + 1.5 * IQR)$
489	Puente Real	0.8500	$< 0.8937 (Q1 - 1.5 * IQR)$
506	Puente Real	1.1319	$> 1.1148 (Q3 + 1.5 * IQR)$
527	Puente Real	0.8392	$< 0.8937 (Q1 - 1.5 * IQR)$
547	Puente Real	0.7481	$< 0.8937 (Q1 - 1.5 * IQR)$
549	Puente Real	1.1333	$> 1.1148 (Q3 + 1.5 * IQR)$
571	Puente Real	1.1382	$> 1.1148 (Q3 + 1.5 * IQR)$
596	Puente Real	1.1225	$> 1.1148 (Q3 + 1.5 * IQR)$
643	Sussex Vale	1.1179	$> 1.1083 (Q3 + 1.5 * IQR)$
654	Sussex Vale	0.9056	$< 0.9285 (Q1 - 1.5 * IQR)$
723	Sussex Vale	1.1584	$> 1.1083 (Q3 + 1.5 * IQR)$
725	Sussex Vale	1.1466	$> 1.1083 (Q3 + 1.5 * IQR)$
727	Sussex Vale	1.1464	$> 1.1083 (Q3 + 1.5 * IQR)$
734	Sussex Vale	0.9256	$< 0.9285 (Q1 - 1.5 * IQR)$

These ratios are the same as the ratios outside the whiskers of the box plot of Example 1 since the end of the whisker of the box plot is defined the same way as the IQR method with 1.5 as the IQR multiplier.

Ratio Summary Statistics using the IQR Multiplier 1.5

Ratio Summary Statistics

Region	Count	Median	Mean		IQR	SD	COD	COV	PRD	PRB
			Value	Weighted						
Fox Knoll	114	1.0035	1.0039	1.0045	0.0630	0.0503	3.931	5.0076	0.9994	0.0377
Freedom Manor	176	0.9795	0.9770	0.9805	0.0968	0.0740	5.956	7.5751	0.9965	0.0428
Hastings Heights	97	1.0026	0.9940	0.9939	0.1025	0.0698	5.553	7.0223	1.0001	0.0202
Puente Real	196	1.0034	1.0031	1.0032	0.0504	0.0404	3.231	4.0286	0.9999	0.0050
Sussex Vale	137	1.0202	1.0189	1.0186	0.0441	0.0329	2.514	3.2326	1.0003	-0.0008
Combined	720	1.0035	0.9986	0.9999	0.0652	0.0550	4.323	5.5112	0.9988	0.0157

IQR: Interquartile Range
 COD: Coefficient of Dispersion
 COV: Coefficient of Variation
 PRD: Price-Related Differential
 PRB: Coefficient of Price-Related Bias

While the median is relatively unaffected by the removal of so many outliers, several of the other statistics are affected considerably.

Example 3 – More than One Break Column

Using the same dataset as Examples 1 and 2, this example shows the use of more than one break column (region as well as property type). When more than one break column is specified, a separate analysis is given for each column. In this example, only the ratio summary statistics and box plots will be produced.

Appraisals Dataset

Appraisal	Price	Region	Type	Date
310900	296000	Fox Knoll	Residential	10/26/2015
264800	256300	Fox Knoll	Commercial	10/18/2015
204600	199700	Fox Knoll	Residential	1/2/2015
290600	264000	Fox Knoll	Commercial	3/11/2015
287900	281100	Fox Knoll	Residential	2/17/2015
292600	267400	Fox Knoll	Residential	5/12/2015
244000	253200	Fox Knoll	Commercial	4/12/2015
270100	270000	Fox Knoll	Residential	8/24/2015
210500	191200	Fox Knoll	Residential	11/8/2015
304700	302000	Fox Knoll	Commercial	6/6/2015
248200	228400	Fox Knoll	Residential	6/16/2015
.
.
.

Setup

To run this example, complete the following steps:

1 Open the Appraisals example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Appraisals** and click **OK**.

2 Specify the Appraisal Ratio Studies procedure options

- Find and open the **Appraisal Ratio Studies** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Columns Tab

Ratio Specification **Appraisal Column, Sale Price Column**
 Appraisal Column **Appraisal**
 Sale Price Column* **Price**
 Break Column(s) **Region, Type**

Summary Reports Tab

Ratio Summary Statistics **Checked**

Appraisal Ratio Studies

Horizontal Equity Tab

Box Plot**Checked**Report Options (*in the Toolbar*)Variable Labels**Column Names****3 Run the procedure**

- Click the **Run** button to perform the calculations and generate the output.

Ratio Summary Statistics and Box Plot – Region**Ratio Summary Statistics**

Region	Count	Median	Mean		IQR	SD	COD	COV	PRD	PRB
			Value	Weighted						
Fox Knoll	117	1.0035	1.0025	1.0033	0.0672	0.0565	4.258	5.6357	0.9993	0.0415
Freedom Manor	184	0.9767	0.9697	0.9773	0.1075	0.0984	7.087	10.1452	0.9922	0.0812
Hastings Heights	98	1.0015	0.9918	0.9917	0.1038	0.0728	5.732	7.3422	1.0001	0.0218
Puente Real	206	1.0043	1.0049	1.0052	0.0553	0.0525	3.808	5.2233	0.9997	0.0103
Sussex Vale	143	1.0208	1.0209	1.0190	0.0450	0.0403	2.884	3.9449	1.0019	-0.0082
Combined	748	1.0035	0.9972	0.9994	0.0678	0.0680	4.901	6.8158	0.9979	0.0250

IQR: Interquartile Range

COD: Coefficient of Dispersion

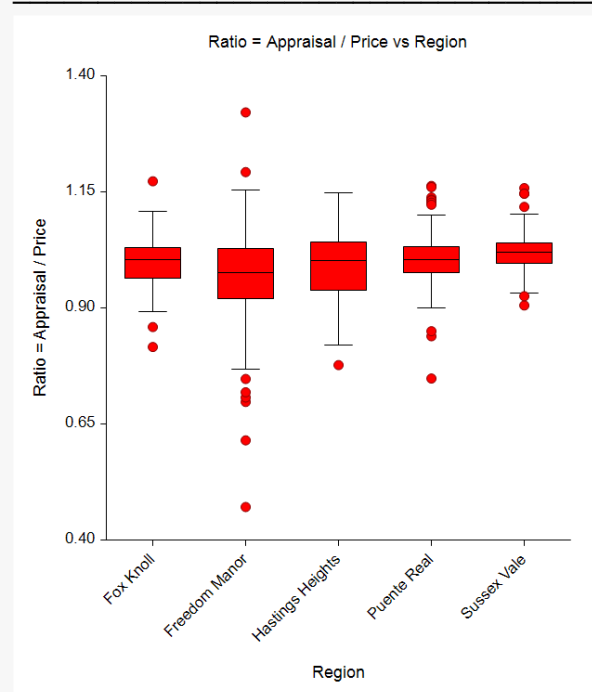
COV: Coefficient of Variation

PRD: Price-Related Differential

PRB: Coefficient of Price-Related Bias

Appraisal Ratio Studies

Comparative (Horizontal Equity) Plot



These results are the same as those of Example 1.

Ratio Summary Statistics and Box Plot – Type

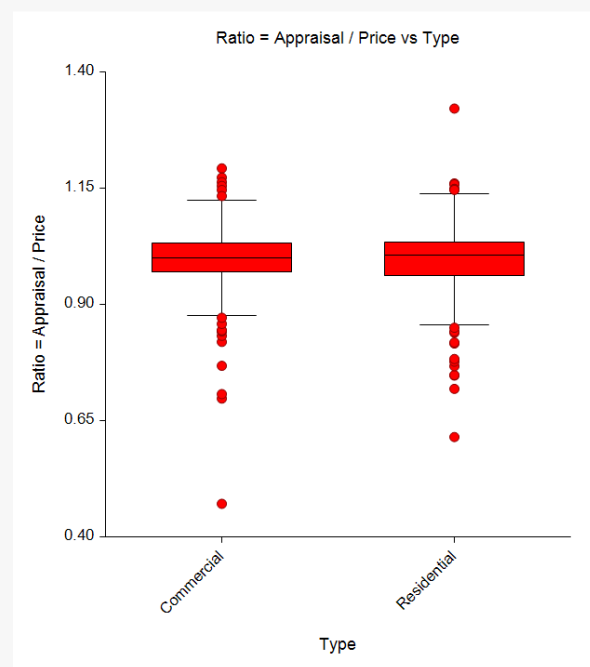
Ratio Summary Statistics

Type	Count	Median	Mean		IQR	SD	COD	COV	PRD	PRB
			Value	Weighted						
Commercial	305	1.0008	0.9976	1.0004	0.0632	0.0697	4.644	6.9843	0.9973	0.0330
Residential	443	1.0060	0.9970	0.9987	0.0722	0.0708	5.068	7.0972	0.9983	0.0201
Combined	748	1.0035	0.9972	0.9994	0.0678	0.0703	4.901	7.0467	0.9979	0.0250

IQR: Interquartile Range
 COD: Coefficient of Dispersion
 COV: Coefficient of Variation
 PRD: Price-Related Differential
 PRB: Coefficient of Price-Related Bias

Appraisal Ratio Studies

Comparative (Horizontal Equity) Plot



This report shows the results for comparing commercial to residential properties. Additional tests could be examined, but there appears to be very little difference in the statistics of these two groups.

Example 4 – Adjusting Sale Price based on Date of Sale

This example presents a similar analysis to that of Example 1, except that each sale price will be adjusted to the date January 1, 2015.

Suppose a mass appraisal process has produced all the appraised values based on the date January 1, 2015. Suppose further that in the area under examination, the overall increase in property values was about 12% over the course of the year 2015. The analyst would like to perform a ratio analysis based on sale prices that are adjusted back to an estimated sale price for January 1, 2015. (In some cases, the sale date may be taken into account as part of the appraisal process, in which case a sale price adjustment should likely not be used.)

Appraisals Dataset

Appraisal	Price	Region	Type	Date
310900	296000	Fox Knoll	Residential	10/26/2015
264800	256300	Fox Knoll	Commercial	10/18/2015
204600	199700	Fox Knoll	Residential	1/2/2015
290600	264000	Fox Knoll	Commercial	3/11/2015
287900	281100	Fox Knoll	Residential	2/17/2015
292600	267400	Fox Knoll	Residential	5/12/2015
244000	253200	Fox Knoll	Commercial	4/12/2015
270100	270000	Fox Knoll	Residential	8/24/2015
210500	191200	Fox Knoll	Residential	11/8/2015
304700	302000	Fox Knoll	Commercial	6/6/2015
248200	228400	Fox Knoll	Residential	6/16/2015
.
.
.

Setup

To run this example, complete the following steps:

1 Open the Appraisals example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Appraisals** and click **OK**.

2 Specify the Appraisal Ratio Studies procedure options

- Find and open the **Appraisal Ratio Studies** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Appraisal Ratio Studies

Columns Tab

Ratio Specification **Appraisal Column, Sale Price Column**
 Appraisal Column **Appraisal**
 Sale Price Column* **Price**
 Break Column(s) **Region**

Sale Date Tab

Sale Date Column **Date**
 Adjust Prices To This Date **Checked**
 Date **1/1/2015**
 Monthly Price Adjustment Factor **0.01**

Summary Reports Tab

Sale Price Adjustment by Date Summary **Checked**
 Removed Ratios Report **Unchecked**
 Ratio Summary Statistics **Checked**
 Additional Summary Statistics **Unchecked**
 Appraisal and Sale Price Summary **Unchecked**
 Confidence Intervals **Unchecked**
 Normality Assumptions **Unchecked**
 Histogram **Unchecked**
 Normal Probability Plot **Unchecked**

Storage Tab

Time-Adjusted Sale Price **Checked**
 Skip one or more columns before item storage **Checked**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Sale Price Adjustment by Date

Sale Price Adjustment by Date

Sale Date Column:	Sale Date
Ratios removed if before the date:	None Specified or no Sale Date Column
Ratios removed if after the date:	None Specified or no Sale Date Column
Basis Date:	January 1, 2015 (Prices are adjusted to be as if they were sold on this date.)
Monthly Price Adjustment Factor:	0.01
Price Adjustment Formula:	Adjusted Price = $(1 + 0.01 * M) * \text{Sale Price}$ M is the number of months from the Basis Date to the sale date. Dates previous to the Basis Date result in a positive M value, while dates after the Basis Date result in a negative M value.

Appraisal Ratio Studies

The first portion of this report shows that no date boundaries were set. The second portion shows the summary of how each sale price is adjusted to the basis date. Since each date is after the basis date, the *M* values used are negative (the sale prices are adjusted down). The final line indicates that the time-adjusted sale prices were stored in column 7. You can go to the Data Table to see the values listed there.

	Appraisal	Price	Region	Type	Date	C6	Adj_Sale_Price	C8
1	310900	296000	Fox Knoll	Residential	10/26/2015		267019.95893	
2	264800	256300	Fox Knoll	Commercial	10/18/2015		231880.45175	
3	204600	199700	Fox Knoll	Residential	1/2/2015		199634.39014	
4	290600	264000	Fox Knoll	Commercial	3/11/2015		258015.27721	
5	287900	281100	Fox Knoll	Residential	2/17/2015		276759.40041	
6	292600	267400	Fox Knoll	Residential	5/12/2015		255891.36756	
7	244000	253200	Fox Knoll	Commercial	4/12/2015		244798.12731	
8	270100	270000	Fox Knoll	Residential	8/24/2015		249154.00411	
9	210500	191200	Fox Knoll	Residential	11/8/2015		171663.83573	
10	304700	302000	Fox Knoll	Commercial	6/6/2015		286521.72485	
11	248200	228400	Fox Knoll	Residential	6/16/2015		215943.52361	
12	144900	154300	Fox Knoll	Commercial	3/1/2015		151309.05133	
13	328100	341100	Fox Knoll	Residential	3/13/2015		333143.3347	
14	232100	230400	Fox Knoll	Residential	7/7/2015		216244.82957	
15	215100	215800	Fox Knoll	Commercial	12/4/2015		191906.9076	
16	265600	274100	Fox Knoll	Residential	8/18/2015		253477.77413	

The sale prices shown in column 7 are those used in all statistics, plots, and tests of the report.

Ratio Summary Statistics

Ratio Summary Statistics

Region	Count	Median	Mean		IQR	SD	COD	COV	PRD	PRB
			Value	Weighted						
Fox Knoll	117	1.0687	1.0704	1.0684	0.1080	0.0747	5.672	6.9743	1.0018	0.0140
Freedom Manor	184	1.0307	1.0264	1.0324	0.1175	0.1113	7.753	10.8417	0.9941	0.0736
Hastings Heights	98	1.0635	1.0537	1.0539	0.1001	0.0827	5.879	7.8466	0.9998	0.0306
Puente Real	206	1.0694	1.0713	1.0687	0.0894	0.0718	5.149	6.6989	1.0024	0.0026
Sussex Vale	143	1.0775	1.0869	1.0802	0.0900	0.0624	4.624	5.7421	1.0061	-0.0203
Combined	748	1.0629	1.0608	1.0604	0.1028	0.0833	6.009	7.8535	1.0003	0.0170

IQR: Interquartile Range

COD: Coefficient of Dispersion

COV: Coefficient of Variation

PRD: Price-Related Differential

PRB: Coefficient of Price-Related Bias

Because the sale prices are all adjusted down in this scenario, the ratios are all increased, and this is reflected in the summary statistics (compare to the results of Example 1).