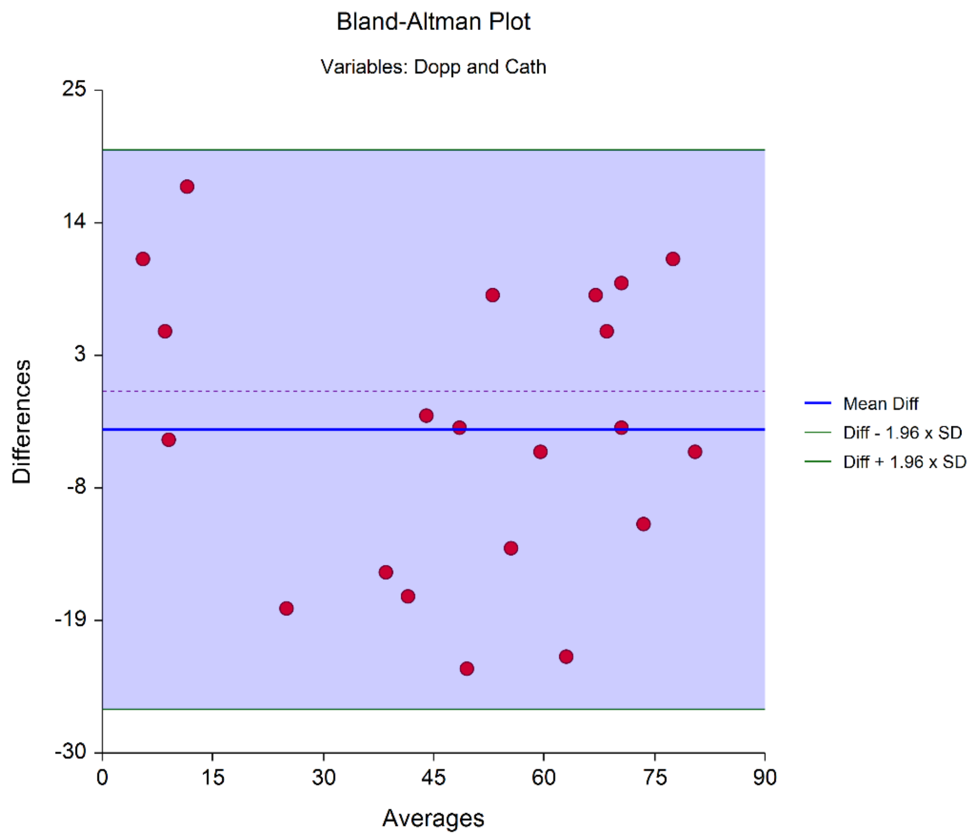
Chapter 204

Bland-Altman Plot and Analysis

Introduction

The Bland-Altman (mean-difference or limits of agreement) plot and analysis is used to compare two measurements of the same variable. That is, it is a method comparison technique. For example, an expensive measurement system might be compared with a less expensive one or an intrusive measurement system might be compared to one that is less intrusive. The technique is documented in a series of papers by J. Martin Bland and Douglas G. Altman (1983, 1986, and 1999).



Repeatability

An important part of method comparison is to understand how repeatable the measurement system is. This can only be understood by sampling each subject multiple times on the same method. This provides for the analysis of designs that include replicates.

Technical Details for Three Designs

There are three study designs that can be analyzed by this procedure. Each type has different input and different technical details and the output is not identical. The technical details of each design will be presented here.

Design 1: Exactly one data-pair per subject

This is the design that has been used for many years. In this design, each of the two measurement-methods is measured once on each subject at nearly the same point in time. A big drawback of this design is that no repeatability parameter can be computed.

Data Structure

For this design, the data are entered in two columns.

X1	X2
57	53
63	69
66	63
74	76
77	75
77	79
78	77
79	77
80	81
81	82
81	81
82	83
82	81
83	85
84	83
85	74
86	88
86	87
86	82
87	88

Suppose you want to evaluate the agreement between a continuous random variable X_1 and a second random variable X_2 which each measure the same variable, such as blood pressure. Assume that n paired observations (X_{1k}, X_{2k}), $k = 1, 2, \ldots, n$ are available. The Bland-Altman plot (1983) is formed by plotting the differences $X_1 - X_2$ on the vertical axis versus the averages $(X_1+X_2)/2$ on the horizontal axis. A horizontal line representing the *bias* is drawn at $\bar{d}$. Additional horizontal lines, known as *limits of agreement*, are added to the plot at $\bar{d} - 1.96\, S_d$ and $\bar{d} + 1.96\, S_d$. The d's are the differences formed as $d = X_1 - X_2$.

Sometimes the '1.96' is replaced with '2' or with another value. Of course, 1.96 represents the z-value used to form 95% limits for a unit-normal random variable.

Bias

The bias between the two tests is measured by the mean of the differences calculated in the usual fashion as

$$\bar{d} = \frac{1}{n}\sum_{k=1}^{n} \mathrm{d}_k$$

Limits of Agreement

Limits of agreement between the two tests are defined by a 95% prediction interval of a particular value of the difference which are computed as follows

$$\bar{d} \pm 1.96 S_d$$

where

$$S_d = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}\left(d_k - \bar{d}\right)^2}$$

Bland and Altman (1999) provide the following variances and confidence intervals for the bias and the limits of agreement, assuming that the differences are normally distributed.

Variances

$$\widehat{Var}\left(\bar{d}\right) = \frac{S_d^2}{n}$$

$$\widehat{Var}\left(\bar{d} \pm 1.96 S_d\right) = \left(\frac{1}{n} + \frac{1.96^2}{2(n-1)}\right) S_d^2$$

Confidence Intervals

Hence, 95% confidence intervals for the mean difference (the bias) are

$$\bar{d} \pm t_{1-\alpha/2,n-1}\sqrt{\widehat{Var}\left(\bar{d}\right)}$$

The 95% confidence intervals for the limits of agreement are

$$\left(\bar{d} - 1.96\, S_d\right) - t_{1-\alpha/2,n-1}\sqrt{\widehat{Var}\left(\bar{d} \pm 1.96\, S_d\right)}$$

and

$$\left(\bar{d} + 1.96\, S_d\right) + t_{1-\alpha/2,n-1}\sqrt{\widehat{Var}\left(\bar{d} \pm 1.96\, S_d\right)}$$

These confidence intervals provide a measure of the precision of these values that aids in the interpretation of the plot.

Design 2: Multiple replicates for each method, no pairing

In this Bland-Altman design, each subject is measured several times (usually in immediate succession) on one method and then measured several times on the other method. There is no natural pairing of the measures. In fact, the number of replicates does not have to be the same for each method. It is assumed that the overall response mean stays constant throughout the data gathering period.

Data Structure

For this design, all measurements for a specific subject are entered on one row. That is, each row represents a different subject. In the example below, there were two measurements of method X followed by three measurements of method Y.

X1	X2	Y1	Y2	Y3
57	53	56	55	51
63	69	62	64	66
66	63	67	64	65
74	76	76	77	77
77	75	74	76	73
77	79	75	76	78
78	77	73	78	74
79	77	78	79	76
80	81	82	84	83
81	82	80	84	85
81	85	84	83	80

Limits of Agreement Calculation

Suppose you want to evaluate the agreement between a continuous random variable *X* and a second random variable *Y* which each measure the same underlying variable, such as blood pressure. Assume that *n* subjects are available. Variables X and Y are measured repeatedly on each subject. Variable X is measured m_X times and variable Y is measured m_Y times. These measurements are not made in X, Y pairs. In fact, all measurements of one method are made in rapid succession followed by all measurements of the other method. The order in which the methods are measured is random for each subject.

The following details come from Zou (2013). Data values x_{ij} and y_{ij} are assumed to follow one-way random effects models

$$x_{ij} = \mu_x + a_{xi} + e_{xij}$$

and

$$y_{ij} = \mu_y + a_{yi} + e_{yij}$$

It is assumed that the quantities a_{xi}, a_{yi}, e_{xij}, and e_{yij} are normal variates with means 0 and variances σ_{xb}^2, σ_{yb}^2, σ_{xw}^2, and σ_{yw}^2, respectively.

Now, make the following computations.

Step 1. Compute individual subject means and variances.

$$\bar{x}_i = \frac{1}{m_{xi}}\sum_{j=1}^{m_{xi}} x_{ij}, \qquad \bar{y}_i = \frac{1}{m_{yi}}\sum_{j=1}^{m_{yi}} y_{ij}, \qquad \bar{d}_i = \bar{x}_i - \bar{y}_i, \qquad \bar{d} = \sum_{i=1}^{n}\frac{d_i}{n}$$

$$s_{xi}^2 = \sum_{j=1}^{m_{xi}} \frac{\left(x_{ij} - \bar{x}_i\right)^2}{m_{xi} - 1}, \qquad s_{yi}^2 = \sum_{j=1}^{m_{yi}} \frac{\left(y_{ij} - \bar{y}_i\right)^2}{m_{yi} - 1}, \qquad s_{\bar{d}}^2 = \sum_{i=1}^{n} \frac{\left(\bar{d}_i - \bar{d}\right)^2}{n - 1}$$

Step 2. Compute pooled estimates of the within subject random errors.

$$\bar{x}_i = \frac{1}{m_{xi}}\sum_{j=1}^{m_{xi}} x_{ij}, \qquad \bar{y}_i = \frac{1}{m_{yi}}\sum_{j=1}^{m_{yi}} y_{ij}$$

$$s_{xw}^2 = \sum_{i=1}^{n} \frac{m_{xi} - 1}{N_x - 1} s_{xi}^2, \qquad s_{yw}^2 = \sum_{i=1}^{n} \frac{m_{yi} - 1}{N_y - 1} s_{yi}^2$$

where

$$N_x = \sum_{i=1}^{n} m_{xi}, \quad N_y = \sum_{i=1}^{n} m_{yi}$$

Step 3. Compute the harmonic means of the replicate counts.

$$m_{xh} = \frac{n}{\sum_{i=1}^{n}\frac{1}{m_{xi}}}, \quad m_{yh} = \frac{n}{\sum_{i=1}^{n}\frac{1}{m_{yi}}}$$

Step 4. Compute the standard deviation of a difference.

$$s_d^2 = s_{\bar{d}}^2 + \left(1 - \frac{1}{m_{xh}}\right) s_{xw}^2 + \left(1 - \frac{1}{m_{yh}}\right) s_{yw}^2$$

Step 5. Finally, compute the limits of agreement.

$$LoA_{lower} = \bar{d} - z_{\beta/2} s_d$$

$$LoA_{upper} = \bar{d} + z_{\beta/2} s_d$$

where $z_{\beta/2}$ is the value from the standard normal distribution that puts *β/2* in each tail. Usually, $z_{\beta/2}$ is set to 1.96 or rounded-off to 2.

Confidence interval estimation for LoA based on the delta method

(1 - α)% confidence intervals can be calculated for the lower and upper LoA values using a variance based on the delta method. This variance is computed using

$$\widehat{Var}(LoA_{lower}) = \widehat{Var}(LoA_{upper}) = \frac{s_{\bar{d}}^2}{n} + \frac{z_{\beta/2}^2}{2s_d^2}\left[\frac{\left(s_{\bar{d}}^2\right)^2}{n-1} + \left(1 - \frac{1}{m_{xh}}\right)^2 \frac{\left(s_{xw}^2\right)^2}{N_x - n} + \left(1 - \frac{1}{m_{yh}}\right)^2 \frac{\left(s_{yw}^2\right)^2}{N_y - n}\right]$$

Confidence Intervals

Hence, (1 - α)% confidence intervals for the two *LoA* are

$$LoA_{lower} \pm |z_{\alpha/2}|\sqrt{\widehat{Var}(LoA_{lower})}$$

and

$$LoA_{upper} \pm |z_{\alpha/2}|\sqrt{\widehat{Var}(LoA_{upper})}$$

Confidence interval estimation for LoA based on the MOVER method

The above confidence intervals are symmetric and may not by accurate for typical sample sizes. Zou provides the following adjusted confidence interval which simulation studies show to be more accurate in small to moderate sample sizes.

Step 1. Compute *l* and *u* as follows.

$$l = s_d^2 - S_1, \quad u = s_d^2 + S_1$$

where

$$S_1 = \sqrt{\left[s_{\bar{d}}^2\left(1 - \frac{n-1}{\chi^2_{1-\frac{\alpha}{2},n-1}}\right)\right]^2 + \left[\left(1 - \frac{1}{m_{xh}}\right)\left(1 - \frac{N_x - n}{\chi^2_{1-\alpha/2,N_x-n}}\right)s_{xw}^2\right]^2 + \left[\left(1 - \frac{1}{m_{yh}}\right)\left(1 - \frac{N_y - n}{\chi^2_{1-\alpha/2,N_y-n}}\right)s_{yw}^2\right]^2}$$

Step 2. Compute *LME* and *RME* as follows.

$$LME = \sqrt{\frac{z_{\alpha/2}^2 s_{\bar{d}}^2}{n} + z_{\beta/2}^2\left(\sqrt{u} - \sqrt{s_d^2}\right)^2}$$

$$RME = \sqrt{\frac{z_{\alpha/2}^2 s_{\bar{d}}^2}{n} + z_{\beta/2}^2\left(\sqrt{l} - \sqrt{s_d^2}\right)^2}$$

Step 3. Compute the MOVER confidence intervals as follows.

The (1 - α)% MOVER confidence interval for the lower limit of agreement is

$$LoA_{lower} - LME, \;\; LoA_{lower} + RME$$

The (1 - α)% MOVER confidence interval for the upper limit of agreement is

$$LoA_{upper} - RME, \;\; LoA_{upper} + LME$$

Design 3: Multiple replicates for each method obtained as pairs

In this Bland-Altman design, each subject is measured by each method several times. At each measurement point, a value for each method is obtained in rapid succession. These are referred to as measurement pairs. It is assumed that the overall response mean varies during the data gathering period.

Data Structure

For this design, each measurement pair is reported on a single row. Three columns of data are required: one with the subject value, one with the measurement of method 1, and one with the measurement of method 2.

Subject	X	Y
1	53	56
1	69	62
2	63	67
2	76	76
2	75	74
3	79	75
3	77	73
4	77	78
4	81	82
4	82	80
4	85	84

Limits of Agreement Calculation

Suppose you want to evaluate the agreement between a continuous random variable X and a second random variable Y which each measure the same underlying variable, such as blood pressure. Assume that n subjects are available. Variables X and Y are measured together repeatedly on each subject m times. Data values $d_{ij} = x_{ij} - y_{ij}$ are assumed to follow a one-way random-effects model

$$d_{ij} = \mu_d + a_i + e_{ij}$$

It is assumed that the quantities a_i and e_{ij} are normal variates with means 0 and variances σ_b^2 and σ_w^2, respectively.

In this case, there are two ways two compute the mean difference, neither of which is always better than the other. These two calculation methods may be defined as using the mean of the means and using the mean of the individual differences. These methods require different calculations and will be defined separately.

Limits of Agreement Calculation – Subject Differences.

The following details come from Zou (2013). Now, make the following computations.

Step 1. Compute various means and variances.

$$\bar{d}_i = \sum_{j=1}^{m_i} \frac{d_{ij}}{m_i}, \quad \bar{d} = \sum_{i=1}^{n} \frac{d_i}{n}, \quad s_i^2 = \sum_{j=1}^{m_i} \frac{\left(d_{ij} - \bar{d}_i\right)^2}{m_i - 1}$$

Step 2. Compute pooled estimate of the within subject random error.

$$s_{dw}^2 = \sum_{i=1}^{n} \frac{m_i - 1}{N - n} s_i^2$$

where

$$N = \sum_{i=1}^{n} m_i$$

Step 3. Compute pooled estimate of the between subject random error.

$$s_{\bar{d}}^2 = \sum_{i=1}^{n} \frac{\left(\bar{d}_i - \bar{d}\right)^2}{n - 1}$$

Step 4. Compute the harmonic mean of the replicate counts.

$$m_h = \frac{n}{\sum_{i=1}^{n} \frac{1}{\mathrm{m}_i}}$$

Step 4. Compute the standard deviation of a difference.

$$s_d^2 = \mathrm{s}_{\bar{d}}^2 + \left(1 - \frac{1}{m_h}\right) s_{dw}^2$$

Step 5. Finally, compute the limits of agreement.

$$LoA_{lower} = \bar{d} - z_{\beta/2}\mathrm{s}_d$$

$$LoA_{upper} = \bar{d} + z_{\beta/2}\mathrm{s}_d$$

where $z_{\beta/2}$ is the value from the standard normal distribution that puts *β/2* in each tail. Usually, $z_{\beta/2}$ is set to 1.96 or rounded-off to 2.

Confidence interval estimation for LoA based on the delta method

(1 - α)% confidence intervals can be calculated for the lower and upper LoA values using a variance based on the delta method. This variance is computed using

$$\widehat{Var}(LoA_{lower}) = \widehat{Var}\left(LoA_{upper}\right) = \frac{\mathrm{s}_{\bar{d}}^2}{n} + \frac{z_{\beta/2}^2}{2s_d^2}\left[\frac{\left(\mathrm{s}_{\bar{d}}^2\right)^2}{n-1} + \left(1 - \frac{1}{m_h}\right)^2 \frac{\left(s_{dw}^2\right)^2}{N - n}\right]$$

Confidence Intervals

Hence, (1 - α)% confidence intervals for the two *LoA* are

$$LoA_{lower} \pm \left|z_{\alpha/2}\right| \sqrt{\widehat{Var}(LoA_{lower})}$$

and

$$LoA_{upper} \pm \left|z_{\alpha/2}\right| \sqrt{\widehat{Var}\left(LoA_{upper}\right)}$$

Confidence interval estimation for LoA based on the MOVER method

The above confidence intervals are symmetric and may not by accurate for typical sample sizes. Zou provides the following adjusted confidence interval which simulation studies show to be more accurate in small to moderate sample sizes.

Step 1. Compute *l* and *u* as follows.

$$l = s_d^2 - \sqrt{\left[s_{\bar{d}}^2\left(1-\frac{n-1}{\chi^2_{1-\frac{\alpha}{2},n-1}}\right)\right]^2 + \left[\left(1-\frac{1}{m_h}\right)\left(1-\frac{N-n}{\chi^2_{1-\alpha/2,N-n}}\right)s_{dw}^2\right]^2}$$

$$u = s_d^2 + \sqrt{\left[s_{\bar{d}}^2\left(1-\frac{n-1}{\chi^2_{1-\frac{\alpha}{2},n-1}}\right)\right]^2 + \left[\left(1-\frac{1}{m_h}\right)\left(1-\frac{N-n}{\chi^2_{1-\alpha/2,N-n}}\right)s_{dw}^2\right]^2}$$

Step 2. Compute *LME* and *RME* as follows.

$$LME = \sqrt{\frac{z_{\alpha/2}^2 s_{\bar{d}}^2}{n} + z_{\beta/2}^2\left(\sqrt{u} - \sqrt{s_d^2}\right)^2}$$

$$RME = \sqrt{\frac{z_{\alpha/2}^2 s_{\bar{d}}^2}{n} + z_{\beta/2}^2\left(\sqrt{l} - \sqrt{s_d^2}\right)^2}$$

Step 3. Compute the MOVER confidence intervals as follows.

The (1 - α)% MOVER confidence interval for the lower limit of agreement is

$$LoA_{lower} - LME,\ \ LoA_{lower} + RME$$

The (1 - α)% MOVER confidence interval for the upper limit of agreement is

$$LoA_{upper} - RME,\ \ LoA_{upper} + LME$$

Limits of Agreement Calculation – Individual Differences.

This approach is mentioned in Olofsen, Dahan, Borsboom, and Drummond (in press as of July, 2014). In this approach, the bias is measured by the grand mean of the individual differences rather than by the mean of the subject differences. The authors give situations in which this approach might be better. They outline the rather complicated calculations for this method. We refer you to their article for the details of our implementation of the method.

Procedure Options

This section describes the options available in this procedure.

Variables Tab

This option specifies the variables that will be used in the analysis.

Data Input Type

Specify the way the data are arranged on the database. Three arrangements are possible. Each data arrangement corresponds to a different study design.

1. One row per subject with only one replicate for each method.

This arrangement is suitable for the most common Bland-Altman design in which there is one measurement for each variable (method) per subject. It is assumed that the data are taken in pairs.

Note that Bland and Altman have commented in several articles that they recommend that more than one measurement be made of a method per subject so that repeatability may be studied.

Example of Data Input Type 1

X	Y
23	25
24	22
26	27
21	22
24	23
25	26

2. One row per subject with multiple replicates for each method.

In this Bland-Altman design, each subject is measured several times (usually in immediate succession) on one method and then measured several times on the other method. There is no natural pairing of the measures. In fact, the number of replicates does not have to be the same for each variable.

It is assumed that the overall response mean stays constant throughout the data gathering period.

Example of Data Input Type 2

X1	X2	X3	Y1	Y2
22	21	23	24	22
25	24	23	24	21
21	24	22	25	21
23	21	25	23	24
24	23	21	22	25

3. Multiple rows per subject with one replicate for each method per row.

In this Bland-Altman design, several pairs of measurements (one for each variable) are obtained per subject. Usually, each pair is obtained in rapid succession. The number of pairs per subject does not have to be identical for all subjects.

It is assumed that the overall response mean varies during the course of the study. For example, if you were studying heart-rate monitors, you would assume that an individual's actual heart-rate is constantly changing throughout the day.

Example of Data Input Type 3
Note that 'Id' represents a subject identification number.

Id X Y
11 23 24
11 22 21
11 21 22
11 22 24
A1 22 21
A1 23 24
A1 24 22
42 23 24
42 22 21
42 21 22
42 22 24
B4 22 21
B4 23 24
B4 24 22

Method Variables (Input Type = 1)

Method 1 Variable

Enter the first variable of each pair here. The second variable of each pair is entered in the "Second Variable(s)" box below. Differences are calculated as First - Second.

Select (or enter) the name or column number of the variable containing the measurement values of the first method. Only one variable may be entered. It is assumed that the two values on each row come from the same subject. Each row represents a different subject.

Method 2 Variable

Select (or enter) the name or column number of the variable containing the measurement values of the second method. Only one variable may be entered. It is assumed that the two values on each row come from the same subject. Each row represents a different subject.

Multiple Replicate Variables for each Method (Input Type = 2)

Method 1 Variables

Select the variable(s) containing the measurements of the first method. It is assumed that all the values on a row come from the same, unique subject. That is, there is only one row for each subject.

Note that the total number of variables for the Method 1 and Method 2 variables must be greater than two.

Method 2 Variables

Select the variable(s) containing the measurements of the second method. It is assumed that all the values on a row come from the same, unique subject. That is, there is only one row for each subject.

Note that the total number of variables for the Method 1 and Method 2 variables must be greater than two.

Subject Variable and Method Variables (Input Type = 3)

Subject Variable

Specify (or enter) the name or column number of the variable containing the subject id values. These values may be text or numeric. These values are used to identify which rows are associated with which subjects.

Method 1 Variable

Select (or enter) the name or column number of the variable containing the measurement values of the first method. Only one variable may be entered. It is assumed that the two values on each row come from the same subject. The database may contain several rows for each subject with one measurement pair per row.

Method 2 Variable

Select (or enter) the name or column number of the variable containing the measurement values of the second method. Only one variable may be entered. It is assumed that the two values on each row come from the same subject. The database may contain several rows for each subject with one measurement pair per row.

Variables – Calculation Options

These options specify which techniques and methods you want to use. Note that these options will only be displayed when they apply, depending on the setting of Data Input Type.

SD Multiplier for Limits of Agreement

The limits of agreement are formed using the mean difference plus and minus the standard deviation of the difference times this multiplier value. Commonly, '1.96' is used because, assuming a normal distribution, 95% of the data fall within these limits. Historically, this value was rounded to '2' for convenience in computing the limits of agreement by hand.

Confidence Level of Confidence Intervals

This confidence level is used for the confidence intervals of the means and agreement limits. Typical confidence levels are 90%, 95%, and 99%, with 95% being the most common.

Mean Difference (Bias) Estimation

If each measurement method produced accurate measurements, the only difference between the means would be due to random error. Any systematic difference is called the bias.

There are two methods available for estimating the mean difference between the measurement methods.

- **Subject Differences (Recommended)**
  Compute the mean difference for each subject, then compute the mean of these mean differences.

- **Individual Row Differences**
  Calculate the mean difference for each row, then compute the mean of these means.

The most recommended method is the Subject Differences method. However, in some cases, the Individual Row Differences method may be better.

Note that this option only is available for Data Input Type 3. For Data Input Types 1 and 2, the Subject Differences method is used.

Limits of Agreement Confidence Interval

There are two options available for computing a confidence interval for the limits of agreement (LoA): Delta Method and MOVER. This option specifies which of these methods to use.

- **MOVER (Recommended)**

  This method (Method Of Variance Estimates Recovery) has been shown by simulation studies to be more accurate than the delta method.

- **Delta Method**

  This is the classical method. This method produces symmetrical confidence limits. Simulation studies have shown this method to be inaccurate.

Reports Tab

The options on this panel specify which reports will be included in the output.

Select Reports

Descriptive Statistics

This section reports the count, mean, standard deviation, standard error, and mean for the specified variable.

Bland-Altman Analysis

This provides a numeric report of the statistics used in the Bland-Altman plot.

Variance and Standard Deviation Reports

This provides a numeric report of the statistics used in the Bland-Altman plot.

Test of Normality Assumption (Data Input Type = 1)

This section reports a Shapiro-Wilk normality test.

Assumption Alpha

This is the significance level of the Shapiro-Wilk normality test. A value of 0.05 is recommended. Typical values range from 0.001 to 0.200.

Report Options Tab

The options on this panel control the label and decimal options of the report.

Report Options

Variable Names

This option lets you select whether to display only variable names, variable labels, or both.

Decimal Places

Means, Differences, and Limits – Test Statistics

These options specify the number of decimal places used in the reports. If one of the Auto options is used, the ending zero digits are not shown. For example, if 'Significant Digits (Up to 7)' is chosen, 0.0500 is displayed as 0.05 and 1.314583689 is displayed as 1.314584.

The output formatting system is not designed to accommodate (Up to 13), and if chosen, this will likely lead to lines that run on to a second line. This option is included, however, for the rare case when a very large number of decimals is needed.

Plots Tab

The options on this panel control the inclusion and appearance of the plots. The plot format used depends on the setting of the Data Input Type setting

Select Plots

Bland-Altman Plot … Scatter Plot

Check the boxes to display the plot. Click the plot format button to change the plot settings.

Example 1 – Bland-Altman Plots and Reports

This section presents an example of how to generate a Bland-Altman plot. In this example, two measurements were made on each of 100 subjects. The first measurement was made by a lengthy, invasive method and the second measurement was made by a second, much less invasive method. The data are in the **Bland-Altman** dataset. The engineers wish to analyze whether the two methods agree.

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Bland-Altman Plot window.

1 Open the Bland-Altman dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Bland-Altman.NCSS**.
- Click **Open**.

2 Open the Bland-Altman Plot window.

- Using the Analysis or Graphics menu or the Procedure Navigator, find and select the **Bland-Altman Plot and Analysis** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

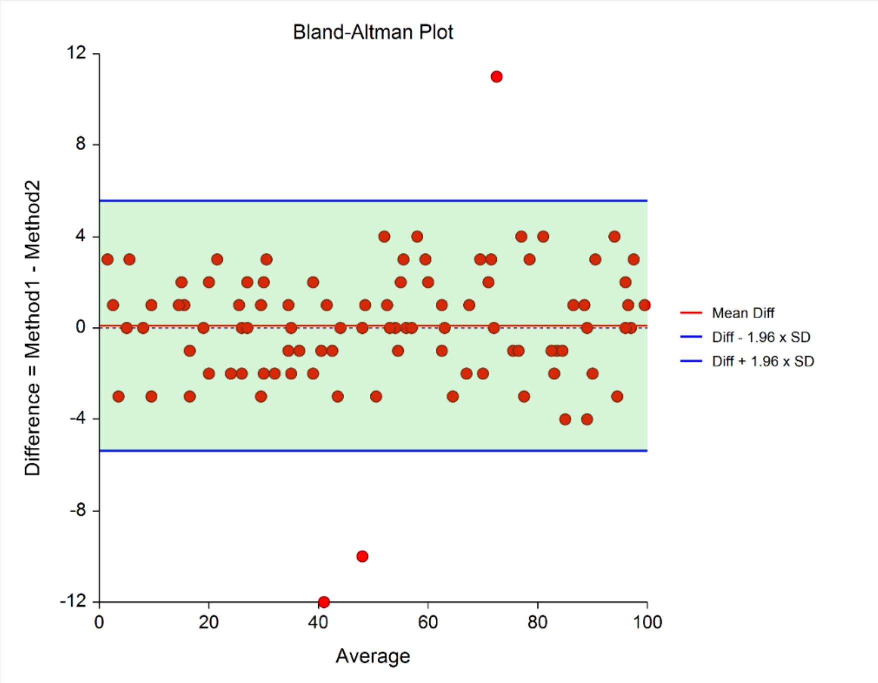
- Select the **Variables tab**. (This is the default.)
- Set the **Data Input Type** to “1. One row per subject …”
- Double-click in the **Method 1 Variable** box. This will bring up the variable selection window.
- Select **Method1** from the list of variables and then click **Ok**. “Method1” will appear in this box.
- Double-click in the **Method 2 Variable** box. This will bring up the variable selection window.
- Select **Method2** from the list of variables and then click **Ok**. “Method2” will appear this box.

4 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

The following reports and charts will be displayed in the Output window.

Bland-Altman Plot



This is an example of the Bland-Altman plot. The average of the two measurements is plotted along the horizontal axis and the difference between the two methods is plotted along the vertical axis.

Look for trends, patterns, and anomalies in this plot. A few outliers are to be expected. They do not need to be removed.

Descriptive Statistics

Variable	Count	Mean	Standard Deviation	95.0% LCL of Mean	95.0% UCL of Mean
Method1	100	50.72	28.30893	45.10289	56.3371
Method2	100	50.62	28.07701	45.04891	56.19109
Difference	100	0.1	2.787055	-0.4530122	0.6530122

Correlation Coefficient = 0.995147

This report provides basic descriptive statistics and confidence intervals for the two variables and their difference.

Bland-Altman Analysis

Bland-Altman Analysis: Bias and Limits of Agreement for Method1 and Method2
Limits of Agreement = Diff ± 1.96 x (Std Dev of Difference)

Parameter	Count	Value	Standard Deviation*	95.0% LCL of Value	95.0% UCL of Value
Bias (Difference)	100	0.1	2.787055	-0.4530122	0.6530122
Lower Limit of Agreement	100	-5.362628	0.4778968	-6.310879	-4.414377
Upper Limit of Agreement	100	5.562628	0.4778968	4.614377	6.510879

The report provides the bias (mean difference) and the limits of agreement in the Values column. Also included are 95% confidence intervals for each quantity.
*The standard deviations for the lower and upper limits of agreement are actually standard errors.

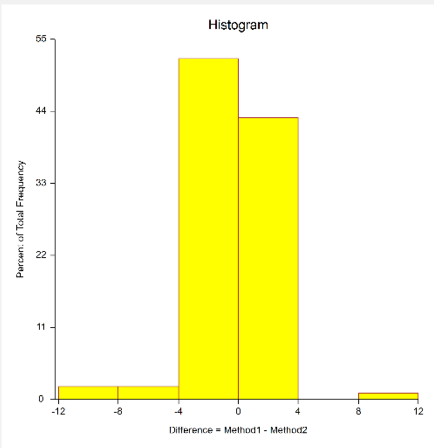
Test of Normality of Differences

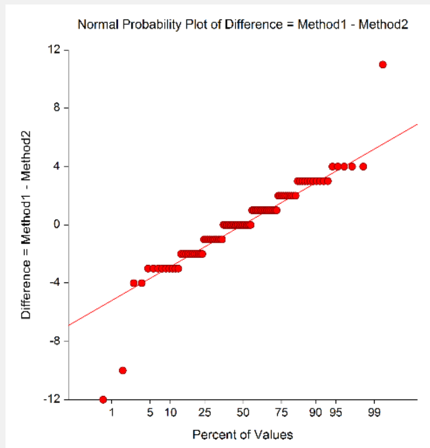
Tests of Assumptions Section

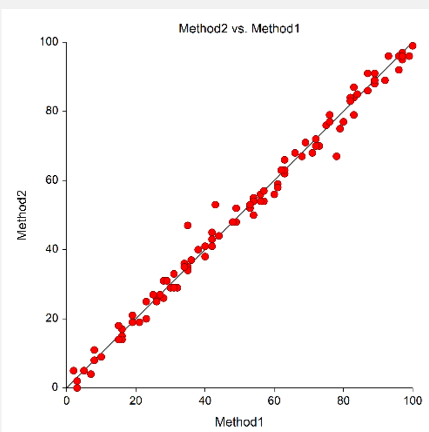
Assumption	Value	Prob Level	Decision (α = 0.050)
Shapiro-Wilk	0.893	0.0000	Reject normality

This section reports the results of a diagnostic test to determine if the differences are normal. In this case, they are not, probably because of the outliers that were found.

Evaluation of Assumptions Plots







Histogram

The histogram provides a general idea of the distribution of the differences.

Normal Probability Plot

If the observations fall along a straight line, this indicates the data follow a normal distribution.

Method1 vs Method2 Plot

This plot lets you see the relationship between the two methods. The 45-degree diagonal line is a reference line that shows perfect equality between the two variables.

Example 2 – Bland-Altman Plot with Confidence Intervals

This section shows the result of embellishing the standard Bland-Altman plot with confidence intervals for the three horizontal lines. The data are in the **Bland-Altman** dataset. You may follow along here by making the appropriate entries or load the completed template **Example 2** by clicking on Open Example Template from the File menu of the Bland-Altman Plot window.

1 Open the Bland-Altman dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Bland-Altman.NCSS**.
- Click **Open**.

2 Open the Bland-Altman Plot window.

- Using the Analysis or Graphics menu or the Procedure Navigator, find and select the **Bland-Altman Plot and Analysis** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- Select the **Variables tab**. (This is the default.)
- Set the **Data Input Type** to "1. One row per subject …"
- Double-click in the **Method 1 Variable** box. This will bring up the variable selection window.
- Select **Method1** from the list of variables and then click **Ok**. "Method1" will appear this box.
- Double-click in the **Method 2 Variable** box. This will bring up the variable selection window.
- Select **Method2** from the list of variables and then click **Ok**. "Method2" will appear in this box.

4 Change the Plot Options.

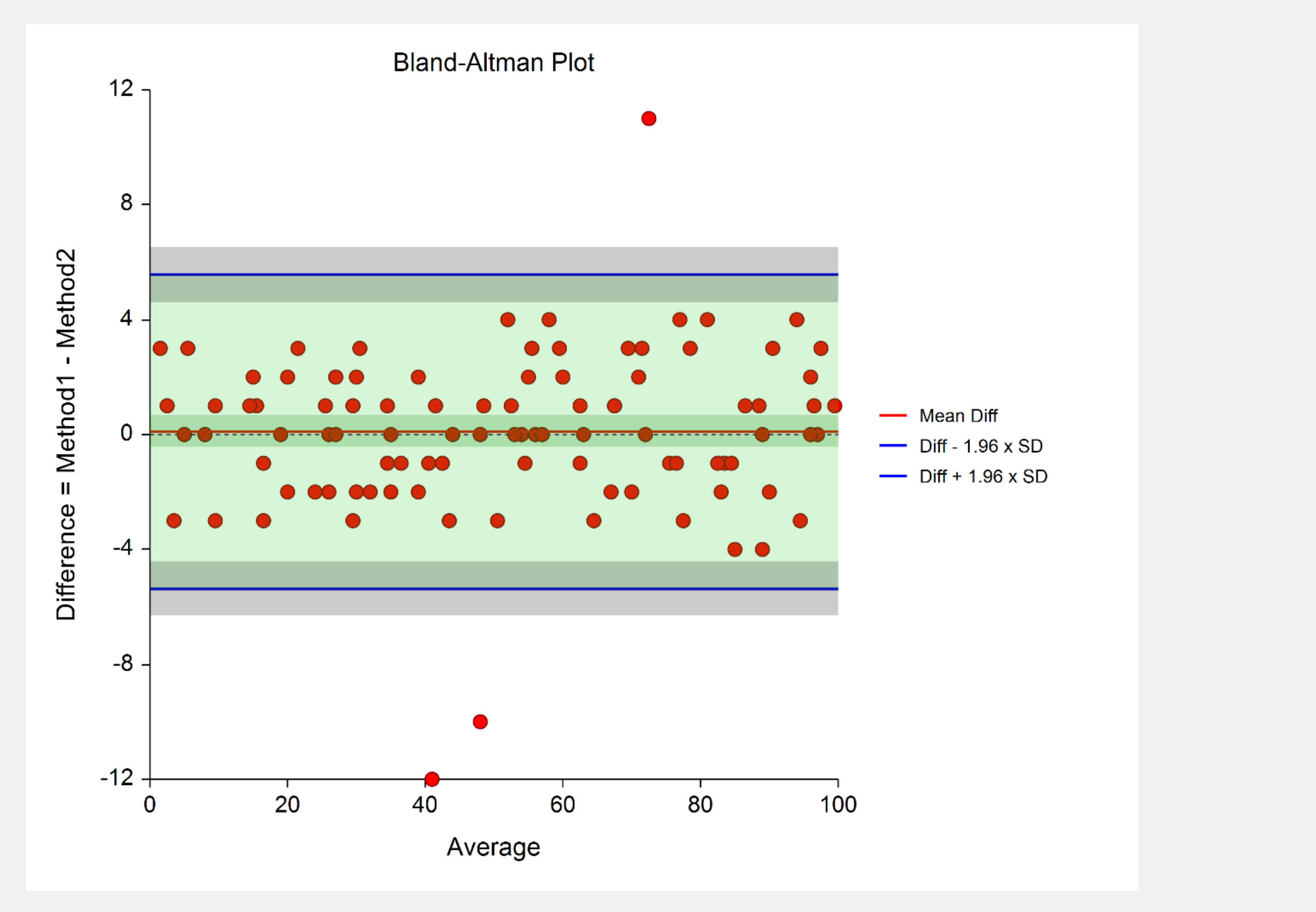
- Select the **Plots tab**.
- Click the **Bland-Altman Plot** button. This will bring up the Bland-Altman Plot Format window.
- Check the **Fill** box next to **Mean Difference** row.
- Check the **Fill** box on the **Lower Limit** row.
- Check the **Fill** box on the **Upper Limit** row.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

The following reports and charts will be displayed in the Output window.

Bland-Altman Plot



This version of the Bland-Altman plot adds confidence intervals for the mean difference (darker green horizontal bar) and the agreement limits (gray bars). These give you a visual impression of the precision of these lines.

Example 3 – Bland-Altman Plot with Data Input Type = 2

This section presents an example of how to generate a Bland-Altman plot for Data Input Type 2. Suppose that 12 subjects are measured up to six times using each of two measurement devices: RV and IC. Although the data are entered on the database as RV1 – RV6 followed by IC1 – IC6, they were not always obtained in this order. Sometimes the RV measurements were made first, while other times the IC measurements were made first, depending on the flip of a coin. The data are in the **Bland-Altman – Data Input Type 2** dataset. The engineers wish to analyze whether the two methods are in complete agreement.

You may follow along here by making the appropriate entries or load the completed template **Example 3** by clicking on Open Example Template from the File menu of the Bland-Altman Plot window.

1 Open the Bland-Altman2 dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Bland-Altman2.NCSS**.
- Click **Open**.

2 Open the Bland-Altman Plot window.

- Using the Analysis or Graphics menu or the Procedure Navigator, find and select the **Bland-Altman Plot and Analysis** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

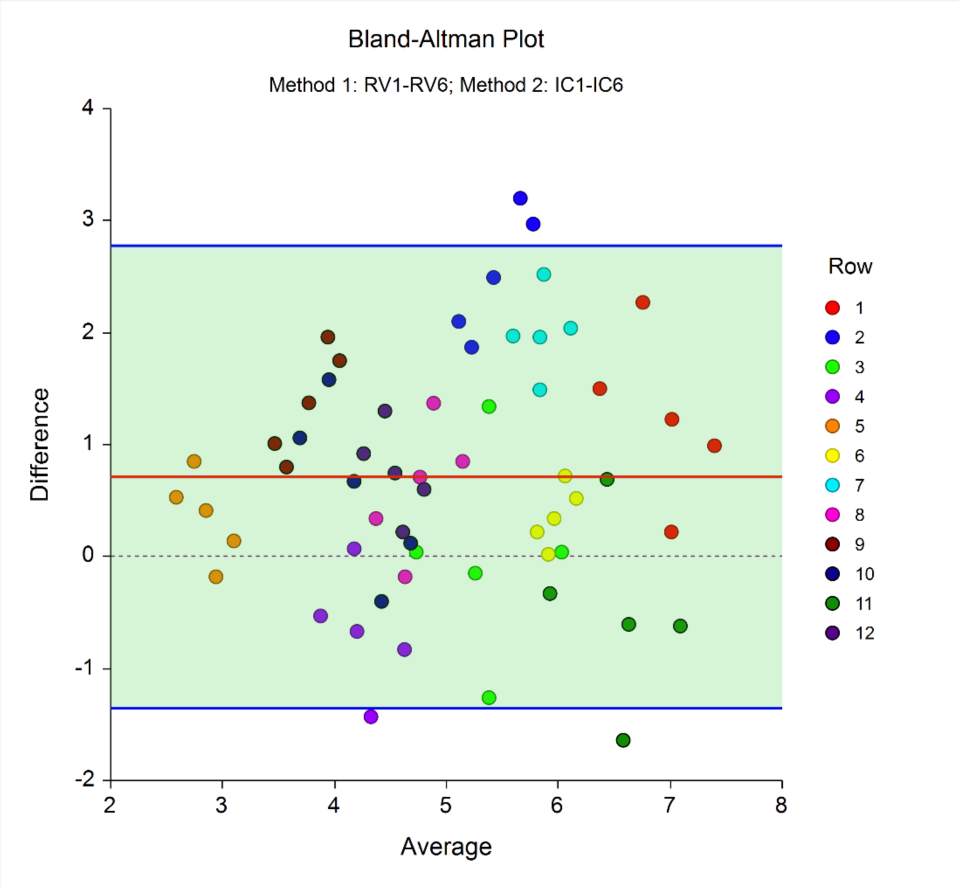
- Select the **Variables tab**. (This is the default.)
- Set the **Data Input Type** to “2. One row per subject with multiple replicates for each method.”
- Double-click in the **Method 1 Variables** box. This will bring up the variable selection window.
- Select **RV1-RV6** from the list of variables and then click **Ok**. “RV1-RV6” will appear in this box.
- Double-click in the **Method 2 Variables** box. This will bring up the variable selection window.
- Select **IC1-IC6** from the list of variables and then click **Ok**. “IC1-IC6” will appear this box.

4 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

The following reports and charts will be displayed in the Output window.

Bland-Altman Plot



Variables Analyzed

Method	Variables
1	RV1-RV6
2	IC1-IC6

This is an example of the Bland-Altman plot for the case when pairing does not matter. In this case, each subject is plotted using a different colored symbol. Since there measurements are not paired, it is somewhat arbitrary which values are plotted. We chose to plot five points per subject, no matter how many possible combinations of points where possible. The five points were generated as follows:

1. Determine the minimum, maximum, and average for the method 1 variables.
2. Determine the minimum, maximum, and average for the method 2 variables.
3. Compute four pairs using all combinations of the minimum and maximum values of each method. For example, pair min RV with max IC.
4. Compute one additional pair using the average RV and then average IC.
5. Compute the difference and average values of these five pairs and plot them.

You can scan the plot to learn what you can about the patterns in the data.

Descriptive Statistics

Method	Subject Count n	Observation Count N	Mean	Mean-Square Between-Subjects MSB = s^2	Mean-Square Within-Subjects MSE = $s^2[w]$
1	12	60	5.3895	1.805107	0.1072278
2	12	60	4.680264	1.612857	0.1378741
Difference	12	120	0.7092361	0.9126912	

Correlation between Methods 1 and 2 Subject Means = 0.734134

This report provides the means and mean squares of each set of variables. The mean-square between and within subjects were computed from a one-way analysis of variance on each variable.

Mean Difference between Methods

Parameter	Subject Count	Value	Standard Deviation	95.0% LCL of Mean	95.0% UCL of Mean
Mean Difference	12	0.7092361	0.9553487	0.1022365	1.316236

This report provides the mean of the subject differences, their standard deviations, and confidence limits for the mean.

Limits of Agreement with Confidence Intervals using MOVER

Limit of Agreement	Subject Count	Value	Standard Deviation	95.0% LCL of Value	95.0% UCL of Value
Lower	12	-1.352391	0.4563031	-2.699204	-0.6283661
Upper	12	2.770863	0.4563031	2.046838	4.117676

This report provides the limits of agreement (LoA) along with their confidence limits.

Example 4 – Bland-Altman Plot with Data Input Type = 3

This section presents an example of how to generate a Bland-Altman plot for Data Input Type 3. Suppose that 12 subjects are measured a variable number of times on each of two measurement devices: RV and IC. Each measurement pair is recorded on a row of the database. The data are in the **Bland-Altman – Data Input Type 3** dataset. The engineers wish to analyze whether the two methods are in complete agreement.

You may follow along here by making the appropriate entries or load the completed template **Example 4** by clicking on Open Example Template from the File menu of the Bland-Altman Plot window.

1 Open the Bland-Altman3 dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Bland-Altman3.NCSS**.
- Click **Open**.

2 Open the Bland-Altman Plot window.

- Using the Analysis (or Graphics) menu or the Procedure Navigator, find and select the **Bland-Altman Plot** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

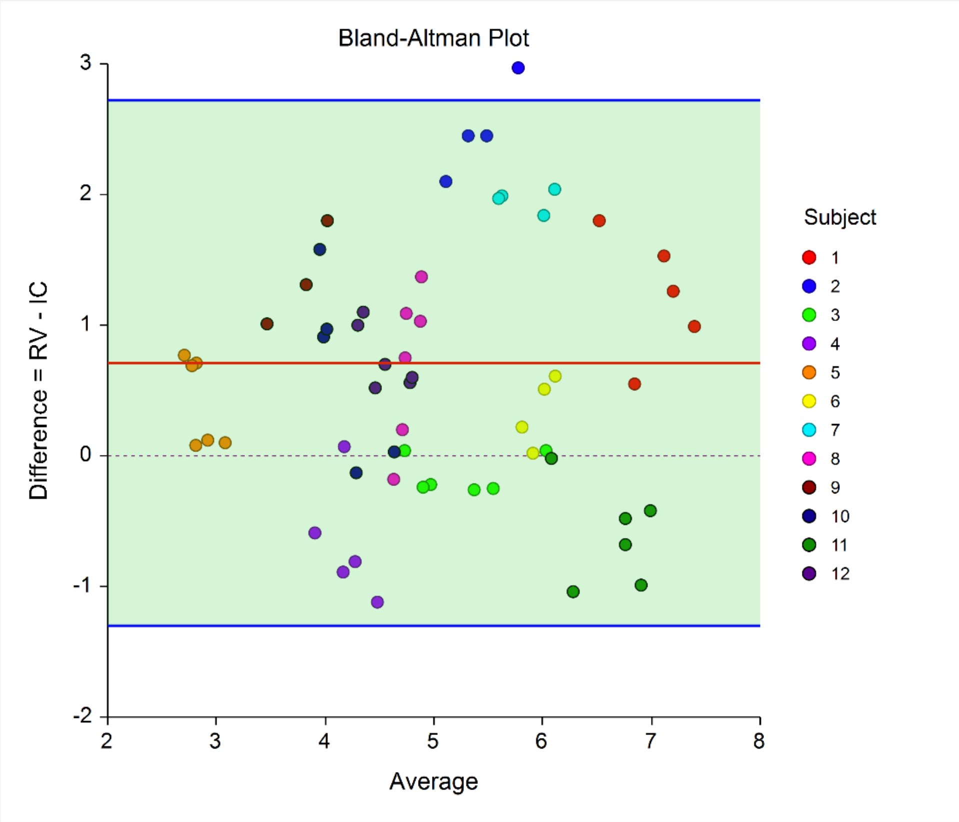
- Select the **Variables tab**. (This is the default.)
- Set the **Data Input Type** to “3. Multiple rows per subject...”
- Double-click in the **Subject Variable** box. This will bring up the variable selection window.
- Select **Subject** from the list of variables and then click **Ok**. “Subject” will appear in this box.
- Double-click in the **Method 1 Variable** box. This will bring up the variable selection window.
- Select **RV** from the list of variables and then click **Ok**. “RV” will appear in this box.
- Double-click in the **Method 2 Variable** box. This will bring up the variable selection window.
- Select **IC** from the list of variables and then click **Ok**. “IC” will appear this box.

4 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

The following reports and charts will be displayed in the Output window.

Bland-Altman Plot



This is an example of the Bland-Altman plot for the case when there are multiple replicates per subject and pairing does matter. Each point represents a row of the dataset.

Look for outliers and changes in the vertical spread as you move your eye horizontally across the plot.

Descriptive Statistics

Variable	Subject Count n	Observation Count N	Mean	Mean-Square Between-Subjects MSB	Mean-Square Within-Subjects MSE = s²[w]	Variance Between-Subjects s²[I]	Variance Between and Within s²
RV	12	60	5.3895	9.066264	0.1072278	1.782619	1.889847
IC	12	60	4.680264	8.359395	0.1378741	0.9259933	1.063867
Difference	12	60	0.7092361	4.209086	0.170714	0.8768886	1.047603

Correlation Between Subject Means = 0.734134

This report provides the means, mean squares, and variances for each set of variables. The mean-square between and within subjects were computed from a one-way analysis of variance on each variable.

Mean Difference Between Methods

Parameter	Subject Count	Value	Standard Deviation	95.0% LCL of Mean	95.0% UCL of Mean
Mean of Subject Differences	12	0.7092361	0.2757854	0.1687066	1.249766

This report provides the mean of the subject differences, their standard deviations, and confidence limits for the mean.

Limits of Agreement with Confidence Intervals using MOVER

Limit of Agreement	Subject Count	Value	Standard Deviation	95.0% LCL of Value	95.0% UCL of Value
Lower Limit of Agreement	12	-1.296872	0.4643287	-2.662969	-0.5610639
Upper Limit of Agreement	12	2.715344	0.4643287	1.979536	4.081441

This report provides the limits of agreement (LoA) along with their confidence limits.

Variance and Standard Deviation Estimates for Difference

Variance Estimates for Difference between RV and IC
Limits of Agreement = Diff ± 1.96 x (Std Dev of Difference)

Bias Estimation Method	Variance of Mean Difference $s^2[B]$	Variance Between-Subjects $s^2[dI]$	Variance Within-Subject $s^2[dw]$	Variance Between + Within $s^2[d]$	Variance of of Limits of Agreement $s^2[LoA]$
Subject Difference	0.0760576	0.8768886	0.170714	1.047603	0.2156011

Standard Deviation Estimates for Difference between RV and IC
Limits of Agreement = Diff ± 1.96 x (Std Dev of Difference)

Bias Estimation Method	Std Dev of Mean Difference s[B]	Std Dev Between-Subjects s[dI]	Std Dev Within-Subject s[dw]	Std Dev Between + Within s[d]	Std Dev of of Limits of Agreement s[LoA]
Subject Difference	0.2757854	0.9364233	0.4131755	1.023525	0.4643287

This report provides estimates for the variances (and standard deviations) of the quantities used in the calculations of the limits of agreement and their confidence intervals.