# Chapter 258

C Charts

Introduction

This procedure generates the C control chart for the number of nonconformities (defects) on a unit. The format of the control chart is fully customizable. This procedure permits the defining of stages. For the C chart, the value for C (the average number of nonconformities) can be entered directly or estimated from the data, or a sub-set of the data. A list of out-of-control points can be produced in the output, if desired.

C Control Charts

C charts are used to monitor the number of nonconformities on a unit of a process based on units taken from the process at given times (hours, shifts, days, weeks, months, etc.). Typically, an initial series of units is used to estimate the average number of nonconformities of a process. The estimated value is then used to produce control limits for the number of nonconformities. During this initial phase, the process should be in control. If points are out-of-control during the initial (estimation) phase, the assignable cause should be determined and the unit should be removed from estimation.

Once the control limits have been established for the C chart, these limits may be used to monitor the number of nonconformities (defects) of the process going forward. When a point is outside these established control limits it indicates that the number of nonconformities for a unit of the process is out-of-control. An assignable cause is suspected whenever the control chart indicates an out-of-control process.
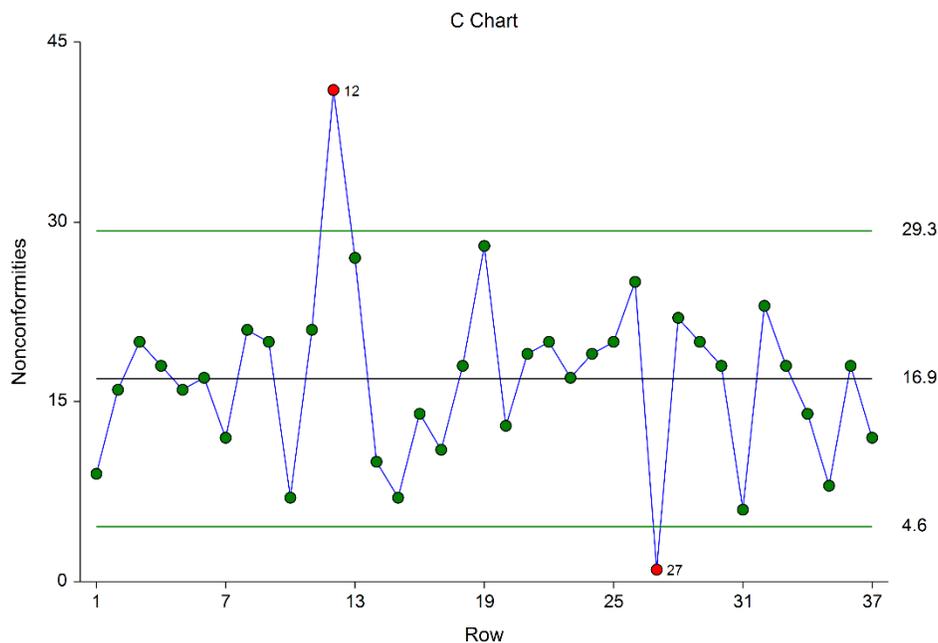
The C Chart versus the U Chart

The C chart is used when a single unit will be examined for nonconformities at each time point. The U chart is used when a number of units will be sampled at each time point, and a per unit average number of nonconformities will be obtained.

C Chart Formulas

Suppose we have sampled k units, one at each time period. Let D_i represent the number of nonconformities of the ith unit.

Formulas for the Points on the Chart

Each point on the chart is given by D_i, the number of nonconformities of the ith unit.

Estimating the C Chart Center Line

In the C Charts procedure, the center line (average number of nonconformities) may be input directly, or it may be estimated from a series of units. If it is estimated from the units, the formula for the center line is

$$\bar{c} = \frac{\sum_{i=1}^{k} D_i}{k}$$

C Chart Limits

The lower and upper control limits for the C chart are calculated using the formulas

$$LCL = \bar{c} - m\sqrt{\bar{c}}$$

$$UCL = \bar{c} + m\sqrt{\bar{c}}$$

where m is a multiplier (usually set to 3) chosen to control the likelihood of false alarms (out-of-control signals) when the process is in control).

Runs Tests

The strength of control charts comes from their ability to detect sudden changes in a process that result from the presence of assignable causes. Unfortunately, the C chart is poor at detecting drifts (gradual trends) or small shifts in the process. For example, there might be a positive trend in the last ten units, but until a number of nonconformities goes above the upper control limit, the chart gives no indication that a change has taken place in the process.

Runs tests can be used to check control charts for unnatural patterns that are most likely caused by assignable causes. Runs tests are sometimes called "pattern tests", "out-of-control" tests, or "zones rules".

While runs tests may be helpful in identifying patterns or smaller shifts in the proportion, they also increase the likelihood of false positive indications. The rate of false positives is typically measured using the average run length (the average length of a run before a false positive is indicated). When several runs tests are used the average run length of the control chart becomes very short.

In order to perform the runs tests, the control chart is divided into six equal zones (three on each side of the center line). Since the control limit is three sigma limits (three standard deviations of the proportion) in width, each zone

is one sigma wide and is labeled A, B, or C, with the C zone being the closest to the center line. There is a lower zone A and an upper zone A. The same is true for B and C. The runs tests look at the pattern in which points fall in these zones.

The runs tests used in this procedure are described below.

Test 1: Any Single Point Beyond Zone A

This runs test simply indicates a single point is beyond one of the two three-sigma limits.

Test 2: Two of Three Successive Points in Zone A or Beyond

This usually indicates a shift in the process average. Note that the two points have to be in the same Zone A, upper or lower. They cannot be on both sides of the center line. The third point can be anywhere.

Test 3: Four of Five Successive Points in Zone B or Beyond

This usually indicates a shift in the process average. Note that the odd point can be anywhere.

Test 4: Eight Successive Points in Zone C or Beyond

All eight points must be on one side of the centerline. This is another indication of a shift in the process average.

Test 5: Fifteen Successive Points Fall in Zone C on Either Side of the Centerline

Although this pattern might make you think that the variation in your process has suddenly decreased, this is usually not the case. It is usually an indication of stratification in the sample. This happens when the samples come from two distinct distributions having different means. Perhaps there are two machines that are set differently. Try to isolate the two processes and check each one separately.

Test 6: Eight of Eight Successive Points Outside of Zone C

This usually indicates a mixture of processes. This can happen when two supposedly identical production lines feed a single production or assembly process. You must separate the processes to find and correct the assignable cause.

There are, of course, many other sets of runs tests that have been developed. You should watch your data for trends, zig-zags, and other nonrandom patterns. Any of these conditions could be an indication of an assignable cause and would warrant further investigation.

Issues in Using Control Charts

There are several additional considerations surrounding the use of control charts that will not be addressed here. Some important questions are presented below without discussion. For a full treatment of these issues you should consider a statistical quality control text such as Ryan (2011) or Montgomery (2013).

Dealing with Out-of-Control Points

How do you deal with out-of-control points once they have been detected? Should they be included or excluded in the process estimate?

Control Limit Multiplier

Three-sigma limits are very common. When should one consider a value other than three?

Startup Time

How many units should be used to establish control for my process?

Data Structure

In this procedure, the data are entered in a single column. This column contains the number of nonconformities (defects) for each unit.

Example dataset

Defects
14
7
16
8
4
22
11
15
19
8
10
13
.
.
.

Procedure Options

This section describes the options available in this procedure. To find out more about using a procedure, go to the Procedures chapter.

Variables Tab

This panel specifies the variables that will be used in the analysis.

Variables

Number of Nonconformities Variable

Specify the column containing the number of nonconformities or defects of each unit.

Stages

Number of Stages

Specify whether the analyses and charts are to be produced based on a single set of units, or a series of sets of units. Typically a single stage is used unless you wish to have separate estimation for various segments of the units. When using multiple stages, the software requires that there be at least one unit in every stage specified.

Stage Specification

Specify whether the various stages will be defined using a variable (column) with a unique value for each stage, or by entering a range of rows for each stage.

- **Stage Variable**

NC	Stage
6	1
7	1
5	1
5	1
7	1
8	2
6	2
6	2
5	2
.	.
.	.
.	.

- **Enter a range for each stage**

  1-50, 51-100, 101-150

  This would produce three stages. The first stage would be made up of rows 1 to 50, the second stage would be rows 51 to 100, and the third stage would be rows 101 to 150.

Stage Variable

Specify the variable (column) that contains the identifiers for each stage.

NC	Stage
6	1
7	1
5	1
5	1
7	1
8	2
6	2
6	2
5	2
.	.
.	.
.	.

A new stage is created for each change in the Stage Variable, going down.

Stage Ranges

Enter a range for each stage using a dash. Separate each stage with a comma.

Example: 1-50, 51-100, 101-150

This would produce three stages. The first stage would be made up of rows 1 to 50, the second stage would be rows 51 to 100, and the third stage would be rows 101 to 150.

Specify Rows in Calculations

Specification Method

Select which method will be used to specify the rows of the data to be used.

- **All Rows**

  All rows in the response column(s) will be used.

- **Enter First Row and Last Row**

  Specify the first row and the last row of the data for use in calculations.

- **First N Rows (Enter N)**

  The data beginning at Row 1 and ending at Row N will be used in calculations.

- **Last N Rows (Enter N)**

  The last N rows of the dataset will be used.

- **Keep Rows Variable**

  Specify a variable and a value in that variable column that will be used to determine which rows are used.

- **Remove Rows Variable**

  Specify a variable and a value in that variable column that will be used to determine which rows will not be used.

First Row

Specify the beginning row to be used.

Last Row

Specify the last row to be used.

N

Enter the number of rows to be used.

Keep Rows Variable

This variable (column) is used to specify which rows of the data will be used for the calculations.

Keep Rows Value

This value determines which rows of the Keep Rows Variable will be used in the calculation portion of the analysis.

Remove Rows Variable

This variable (column) is used to specify which rows of the data will not be used for the calculations.

Remove Rows Value

This value determines which rows of the Remove Rows Variable will not be used in the calculation portion of the analysis.

Specify Rows in Charts

Specification Method

Select which method will be used to specify the rows of the data to be used for the charts.

- **All Rows**

  All rows in the response column(s) will be used.

- **Enter First Row and Last Row**

  Specify the first row and the last row of the data for use in the plots.

- **First N Rows (Enter N)**

  The data beginning at Row 1 and ending at Row N will be used in the plots.

- **Last N Rows (Enter N)**

  The last N rows of the dataset will be used.

- **Keep Rows Variable**

  Specify a variable and a value in that variable column that will be used to determine which rows are used.

- **Remove Rows Variable**

  Specify a variable and a value in that variable column that will be used to determine which rows will not be used.

First Row

Specify the beginning row to be used.

Last Row

Specify the last row to be used.

N

Enter the number of rows to be used.

Keep Rows Variable

This variable (column) is used to specify which rows of the data will be used for the plots.

Keep Rows Value

This value determines which rows of the Keep Rows Variable will be used in the plots.

Remove Rows Variable

This variable (column) is used to specify which rows of the data will not be used for the plots.

Remove Rows Value

This value determines which rows of the Remove Rows Variable will not be used in the plots.

Labels (Optional)

Unit Label Variable

Specify a variable (column) that contains the desired x axis (sample) labels for plots. If left blank, the plot will use the row number. The format of the labels is controlled on the x axis tab of the plot format window.

Point Label Variable

Specify a variable (column) that contains the desired individual point labels for plots. If left blank, no point labels are shown. The format of the labels is controlled on the main chart tab of the plot format window.

Estimation & Limits Tab

The options on the Estimation & Limits tab are used to specify the estimation of P, the specification limits, and the limit multipliers.

P Chart Options

Estimation of C

Specify whether the C chart center line will be estimated from the data, or whether it will be specified directly.

- **From Data**

  Estimate C from the units specified for calculations.

- **Enter Value(s) of C**

  Specify directly the value of C. If multiple stages are used, separate the center line values for each stage by spaces.

- **Use a Variable with Value(s) of C**

  Specify a column containing the value for C in row 1. If multiple stages are used, a value for C should be entered in a separate cell for each stage, beginning with row 1 for the first stage.

Value(s) of C

Enter the value to be used for the center line of the C chart. If multiple stages are used, separate the center line values for each stage by spaces.

Variable with C

Specify a column containing the value of C in row 1. If multiple stages are used, a value for C should be entered in a separate cell for each stage, beginning with row 1 for the first stage.

Specification Limits

Lower Limit

Enter an optional lower specification limit for display on the C chart. These limits are not the control limits.

Upper Limit

Enter an optional upper specification limit for display on the C chart. These limits are not the control limits.

Limit Multipliers

Primary Limit Multiplier

This option specifies the multiplier of the standard deviation for the primary control limits. For the well-known 3-sigma limits, the multiplier is set to 3.

Secondary Limit Multiplier

This option specifies the multiplier of the standard deviation for the optional, secondary control limits. The secondary limits may be shown on the plot by checking the 'Secondary Limit' option on the plot format window.

Reports Tab

The following options control the format of the reports.

Specify Reports

Chart Summary (Control Limits and Estimation) Section

This report gives the numeric values of the center lines and limits.

Out-of-Control List Section

This report shows the out-of-control points and the corresponding reason (runs test or outside control limit).

Report Options

Precision

Specify the precision of numbers in the report. A single-precision number will show seven-place accuracy, while a double-precision number will show thirteen-place accuracy. Note that the reports are formatted for single precision. If you select double precision, some numbers may run into others. Also note that all calculations are performed in double precision regardless of which option you select here. This is for reporting purposes only.

Variable Names

This option lets you select whether to display variable names, variable labels, or both.

Page Title

This option specifies a title to appear at the top of each page.

Plot Subtitle

This option specifies a subtitle to appear at the top of each plot.

C Chart Tab

This panel sets the options used to define the appearance of the P chart.

Select Plots

C Chart

Each chart is controlled by three form objects:

1. A checkbox to indicate whether the chart is displayed.

2. A format button used to call up the plot format window (see Attribute Chart Format Options below for more formatting details).

3.   A second checkbox used to indicate whether the chart can be edited during the run.

Attribute Chart Format Window Options

This section describes the specific options available on the Attribute Chart Format window, which is displayed when the Attribute Chart Format button is clicked. Common options, such as axes, labels, legends, and titles are documented in the Graphics Components chapter.

Attribute Chart Tab

Symbols Section

You can modify the attributes of the symbols using the options in this section.
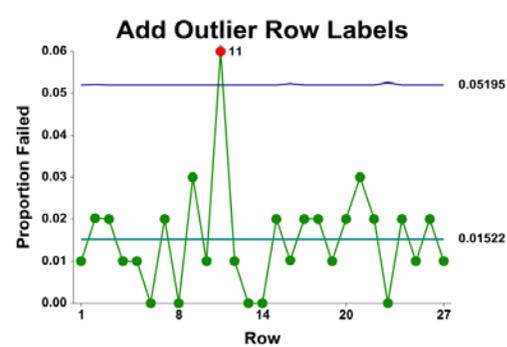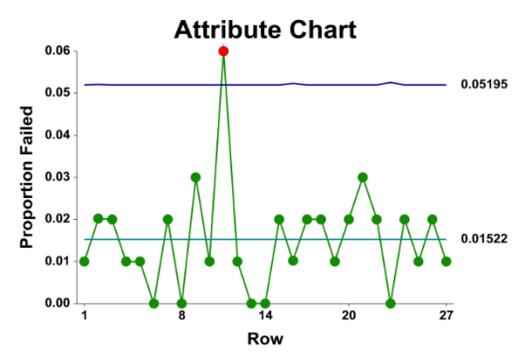
Lines Section

You can specify the format of the various lines using the options in this section. Note that when shading is desired, the fill will be to the bottom for single lines (such as the mean line), and between the lines for pairs of lines (such as primary limits).
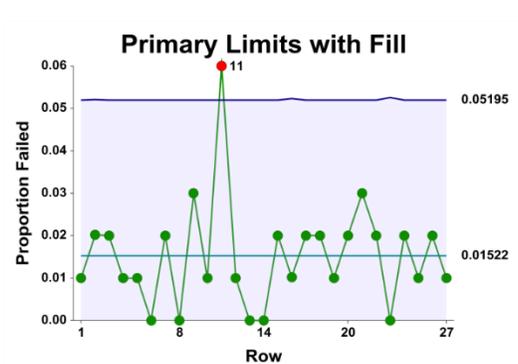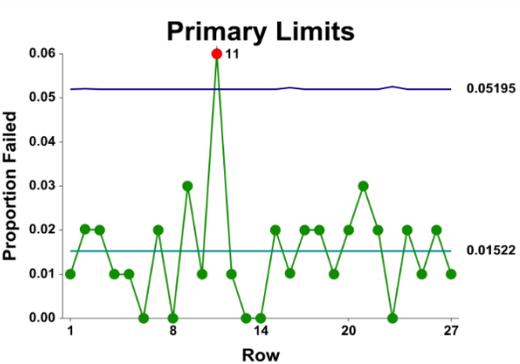
Titles, Legend, Numeric Axis, Group Axis, Grid Lines, and Background Tabs

Details on setting the options in these tabs are given in the Graphics Components chapter. The legend does not show by default, but can easily be included by going to the Legend tab and clicking the Show Legend checkbox.

Example 1 – C Chart Analysis (Phase I)

This section presents an example of how to run an initial C Chart analysis to establish control limits. In this example, the welding of large pipeline joints (as the pipeline extends) will be examined for nonconformities. Thirty-seven joints are examined to establish initial control of the process. The data used are in the Weld dataset.

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the C Charts window.

1   Open the Weld dataset.
- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Weld.NCSS**.
- Click **Open**.

2   Open the C Charts window.
- Using the Analysis or Graphics menu or the Procedure Navigator, find and select the **C Charts** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3   Specify the variables.
- On the C Charts window, select the **Variables tab**.
- Double-click in the **Number of Nonconformities Variable** text box. This will bring up the variable selection window.
- Select **Defects** from the list of variables and then click **Ok**. "Defects" will appear in the Number of Nonconformities Variable box.

4   Run the procedure.
- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Estimation and Limit Section

Estimation and Limit Section for Rows 1 to 37
Number of Nonconformities Variable   Defects

Number of Units	Number of Nonconformities	Average # of Nonconformities	-- Nonconformities Limits --	
			Lower Limit	Upper Limit
37	626	16.91892	4.579135	29.258703

This section displays the calculation values and limits that are to be used in the C chart. The formulas for each calculation are described in the C chart formula section toward the beginning of this chapter.

Number of Units

This is the number of units that were examined for nonconformities.

Number of Nonconformities

This is the sum of all nonconformities across all units.
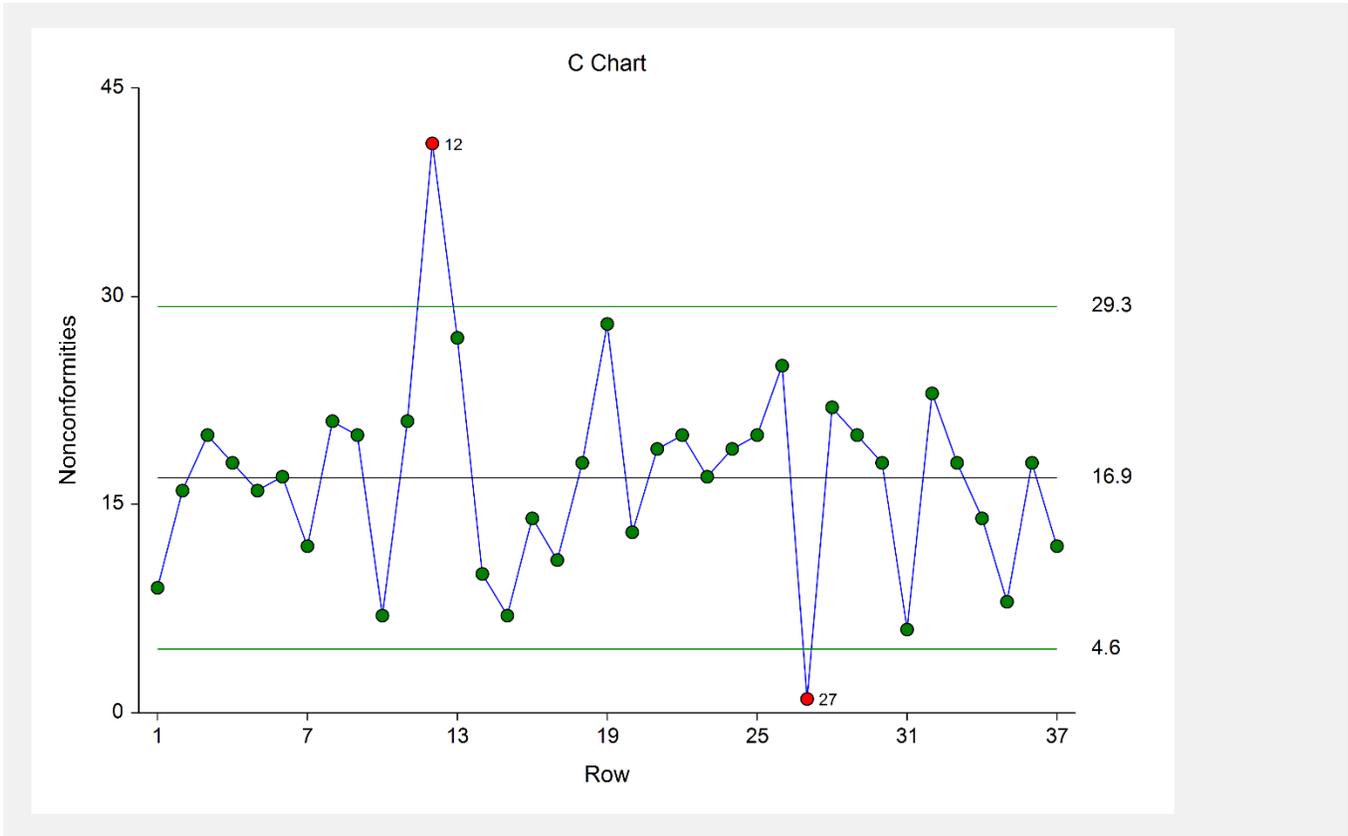
Average Number of Nonconformities

This is number of nonconformities divided by the number of units.

Nonconformities Limits

These are the lower and upper control limits for the C chart.

C Chart

The C chart shows the sequence of nonconformities for the joints. The chart show that joints 12 and 27 merit further investigation for an assignable cause. After reviewing the records, the investigators found that the 12th joint was a different type of joint with more extensive welding than the others. The 27th joint had only a partial inspection due to difficult access to the joint. It is determined that these two units should be removed from the initial C chart estimation phase (see Example 2).

Out-of-Control List

Out-of-Control List for Rows 1 to 37

Row	Number of Nonconformities	Row Label	Reason
12	41.0	12	beyond control limits
13	27.0	13	2 of 3 in zone A
14	10.0	14	2 of 3 in zone A
27	1.0	27	beyond control limits

This report provides a list of the samples that failed one of the runs tests (including points outside the control limits). The report shows that joints 12 and 27 are beyond the control limits. Joints 13 and 14 show runs in the data, but they do seem to signal an important change in the process.

Example 2 – C Chart Revised (Phase I)

This section presents a continuation of the previous example. In this example the limits are revised to exclude the data from samples 12 and 27, due to assignable cause.

You may follow along here by making the appropriate entries or load the completed template **Example 2** by clicking on Open Example Template from the File menu of the C Charts window.

1   After all the steps of Example 1, additionally specify the revised Rows in Calculations.

- On the C Charts window, select the **Variables tab**.
- Under **Specify Rows in Calculations**, change the **Specification Method** from **All Rows** to **Remove Rows Variable**.
- Double-click in the variable box after **Use all rows except rows where variable**. Select **Remove** from the list of variables and then click **Ok**. "Remove" will appear in the variables box. After **equals** enter **1**. The value of the Remove column is 1 for rows 12 and 27.

2   Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.
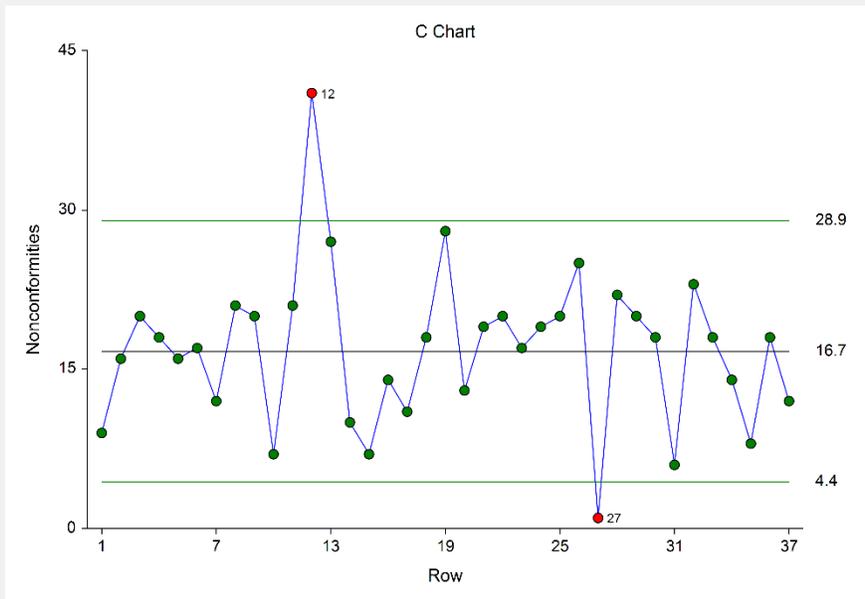
Estimation and Limit Section

Estimation and Limit Section for Rows 1 to 37
Number of Nonconformities Variable   Defects

Number of Units	Number of Nonconformities	Average # of Nonconformities	-- Nonconformities Limits -- Lower Limit	Upper Limit
35	584	16.68571	4.431269	28.940160

Chart Section for Rows 1 to 37

Out-of-Control List for Rows 1 to 37

Row	Number of Nonconformities	Row Label	Reason
12	41.0	12	beyond control limits
13	27.0	13	2 of 3 in zone A
14	10.0	14	2 of 3 in zone A
27	1.0	27	beyond control limits

The limits in this report and chart are now based on all joints except joints 12 and 27. The number of nonconformities for joints 12 and 27 are included in the chart, but they are not included in the calculations. Because two of the more extreme nonconformity counts were removed, the control limits are narrower than those of Example 1.

The C chart can be additionally enhanced by labeling the points that were not included in the calculations. To do this, set the Point Label Variable to Label.

Chart Section for Rows 1 to 37
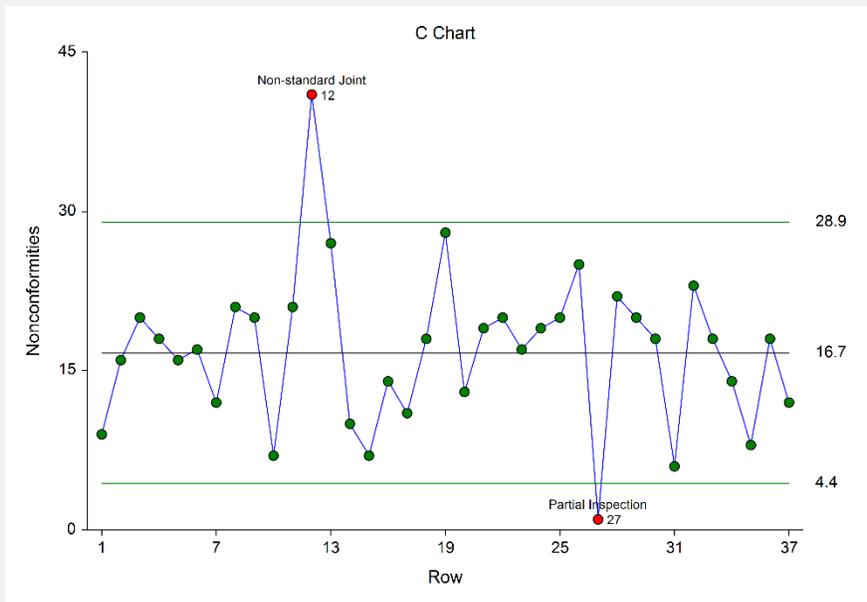
Example 3 – C Chart Analysis (Phase II)

Continuing with Examples 1 and 2, the investigators inspect an additional 25 joints. They wish to use the limits based on the first 37 joints, excluding joints 12 and 27.

You may follow along here by making the appropriate entries or load the completed template **Example 3** by clicking on Open Example Template from the File menu of the C Charts window.

1   Open the Weld dataset.
- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Weld.NCSS**.
- Click **Open**.

2   Open the C Charts window.
- Using the Analysis or Graphics menu or the Procedure Navigator, find and select the **C Charts** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3   Specify the variables.
- On the C Charts window, select the **Variables tab**.
- Double-click in the **Number of Nonconformities Variable** text box. This will bring up the variable selection window.
- Select **Defects** from the list of variables and then click **Ok**. "Defects" will appear in the Number of Nonconformities Variable box.

4   Specify the Rows in Calculations.
- On the C Charts window, select the **Variables tab**.
- Under **Specify Rows in Calculations**, change the **Specification Method** from **All Rows** to **Keep Rows Variable**.
- Double-click in the variable box after **Use all rows where variable**. Select **Keep** from the list of variables and then click **Ok**. "Keep" will appear in the variables box. After **equals** enter **1**.

5   Specify the labels.
- On the C Charts window, select the **Variables tab**.
- Double-click in the **Point Label Variable** text box. This will bring up the variable selection window.
- Select **Label** from the list of variables and then click **Ok**. "Label" will appear in the Point Label Variable box.

6   Run the procedure.
- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.
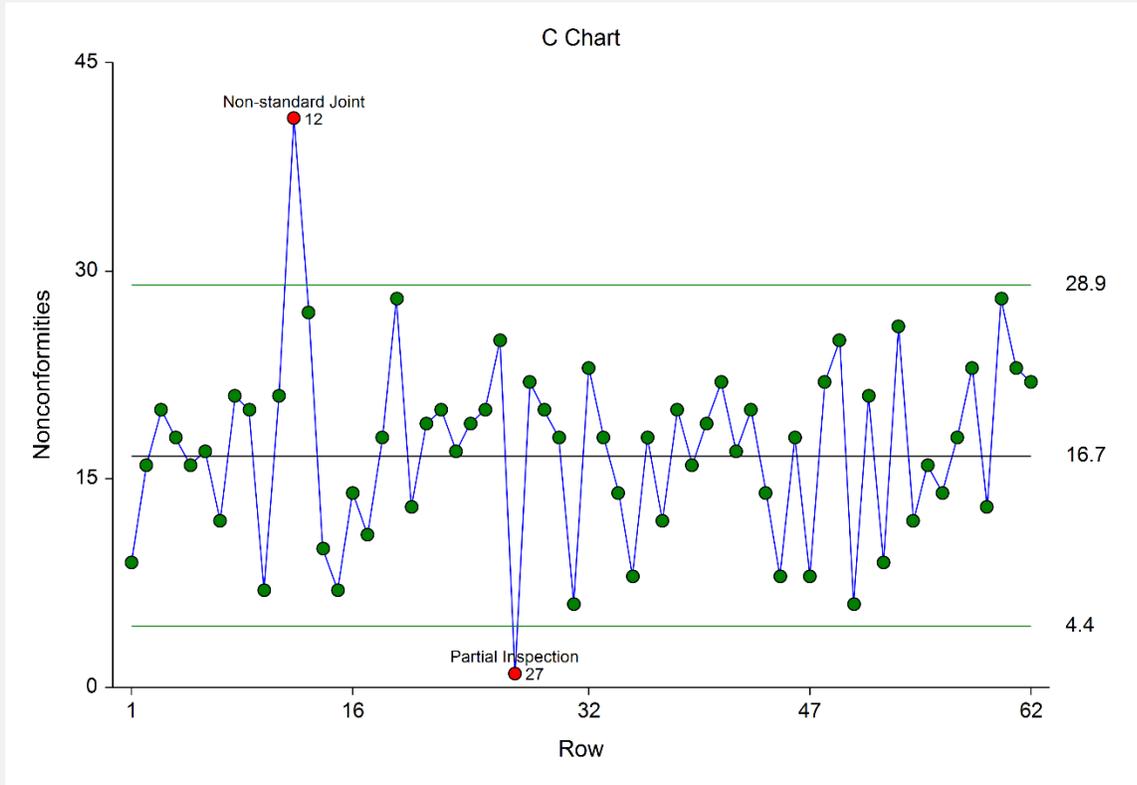
C Chart Analysis Output

Estimation and Limit Section for Rows 1 to 62 where Keep equals 1
Number of Nonconformities Variable   DefectsCont

Number of Units	Number of Nonconformities	Average # of Nonconformities	-- Nonconformities Limits --	
			Lower Limit	Upper Limit
35	584	16.68571	4.431269	28.940160

Chart Section for Rows 1 to 62

Out-of-Control List for Rows 1 to 62

| Row	Number of Nonconformities	Row Label	Reason
12	41.0	12	beyond control limits
13	27.0	13	2 of 3 in zone A
14	10.0	14	2 of 3 in zone A
27	1.0	27	beyond control limits
47	8.0	47	2 of 3 in zone A
54	12.0	54	8 with none in zone C
62	22.0	62	4 of 5 in zone B or beyond

The estimation and limits section shows the same results as Example 2, since the calculations are again based only on the first 37 joints, excluding 12 and 27. The plot shows a stable continuation of the process from joints 38 to 62.

Example 4 – C Chart with Additional Formatting

This example uses the same setup as Example 3, except that a variety of improvements are made in the plot format. These improvements are made by clicking the C Chart format button on the C Chart tab.

You can load the completed template **Example 4** by clicking on Open Example Template from the File menu of the C Charts window.

C Chart
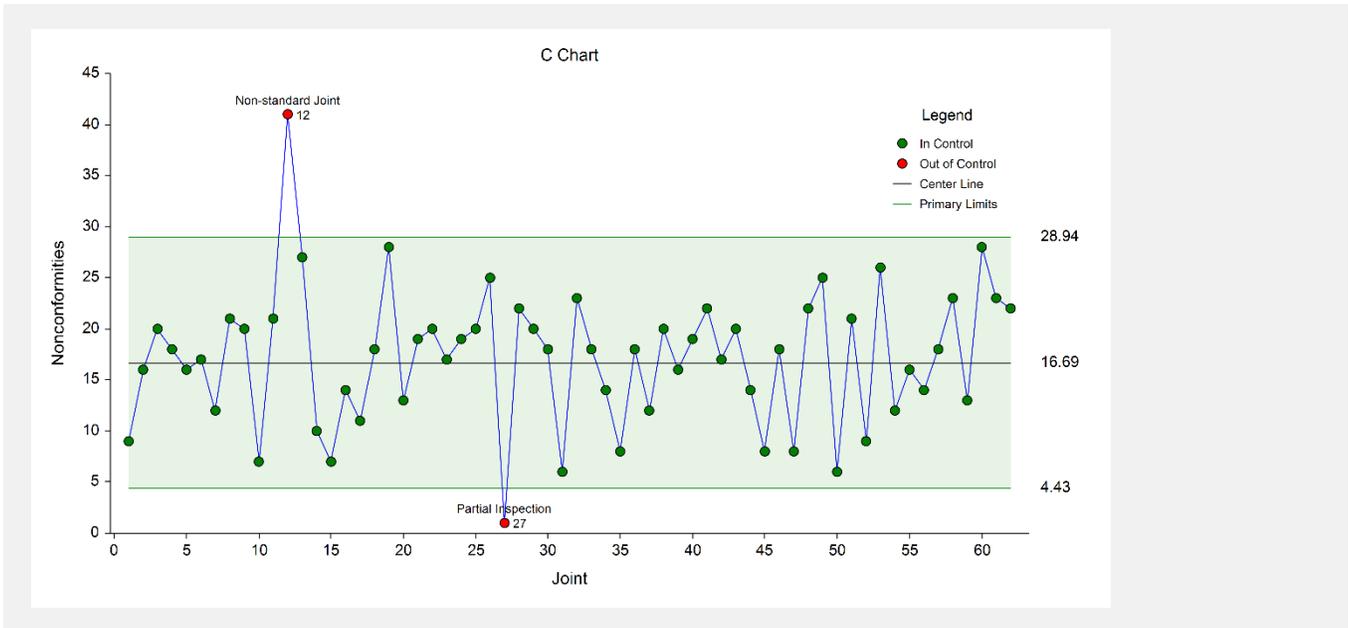
As shown here, a variety of enhancements can be made to the formatting of the control chart to make the chart as easy to read as possible.

Example 5 – C Chart with Stages

This section presents a continuation of Examples 1, 2, and 3. After the first 62 joints, a new welding process is implemented to reduce the number of nonconformities. The investigators examined 50 additional joints, using the first 25 joints of the additional 50 (63 to 87) to determine the in-control limits of the updated process. The investigators wish to view the entire progress of the process, beginning with the first joint. This example shows the use of stages to monitor a process with a change in center line and limit calculations.

You may follow along here by making the appropriate entries or load the completed template **Example 5** by clicking on Open Example Template from the File menu of the C Charts window.

1   Open the Weld dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Weld.NCSS**.
- Click **Open**.

2   Open the C Charts window.

- Using the Analysis or Graphics menu or the Procedure Navigator, find and select the **C Charts** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3   Specify the variables.

- On the C Charts window, select the **Variables tab**.
- Double-click in the **Number of Nonconformities Variable** text box. This will bring up the variable selection window.
- Select **DefectsStage** from the list of variables and then click **Ok**. "DefectsStage" will appear in the Number of Nonconformities Variable box.

4   Specify the stages.

- On the C Charts window, select the **Variables tab**.
- For **Number of Stages** select **Multiple Stages**.
- For **Stage Variable** select **Stage**.

5   Specify the Rows in Calculations.

- On the C Charts window, select the **Variables tab**.
- Under **Specify Rows in Calculations**, change the **Specification Method** from **All Rows** to **Keep Rows Variable**.
- Double-click in the variable box after **Use all rows where variable**. Select **KeepStage** from the list of variables and then click **Ok**. "KeepStage" will appear in the variables box. After **equals** enter **1**.

6   Specify the labels.

- On the C Charts window, select the **Variables tab**.
- Double-click in the **Point Label Variable** text box. This will bring up the variable selection window.
- Select **Label** from the list of variables and then click **Ok**. "Label" will appear in the Point Label Variable box.

7   Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Estimation and Limit Section

Estimation and Limit Section for Rows 1 to 62 where KeepStage equals 1
Number of Nonconformities Variable   DefectsStage

Number of Units	Number of Nonconformities	Average # of Nonconformities	-- Nonconformities Limits --	
			Lower Limit	Upper Limit
35	584	16.68571	4.431269	28.940160

Estimation and Limit Section for Rows 63 to 112 where KeepStage equals 1
Number of Nonconformities Variable   DefectsStage

Number of Units	Number of Nonconformities	Average # of Nonconformities	-- Nonconformities Limits --	
			Lower Limit	Upper Limit
25	238	9.52	0.263651	18.776349

This section displays the estimation for each stage separately. The estimation results for the first stage are the same as the results in Examples 2 and 3.

C Chart
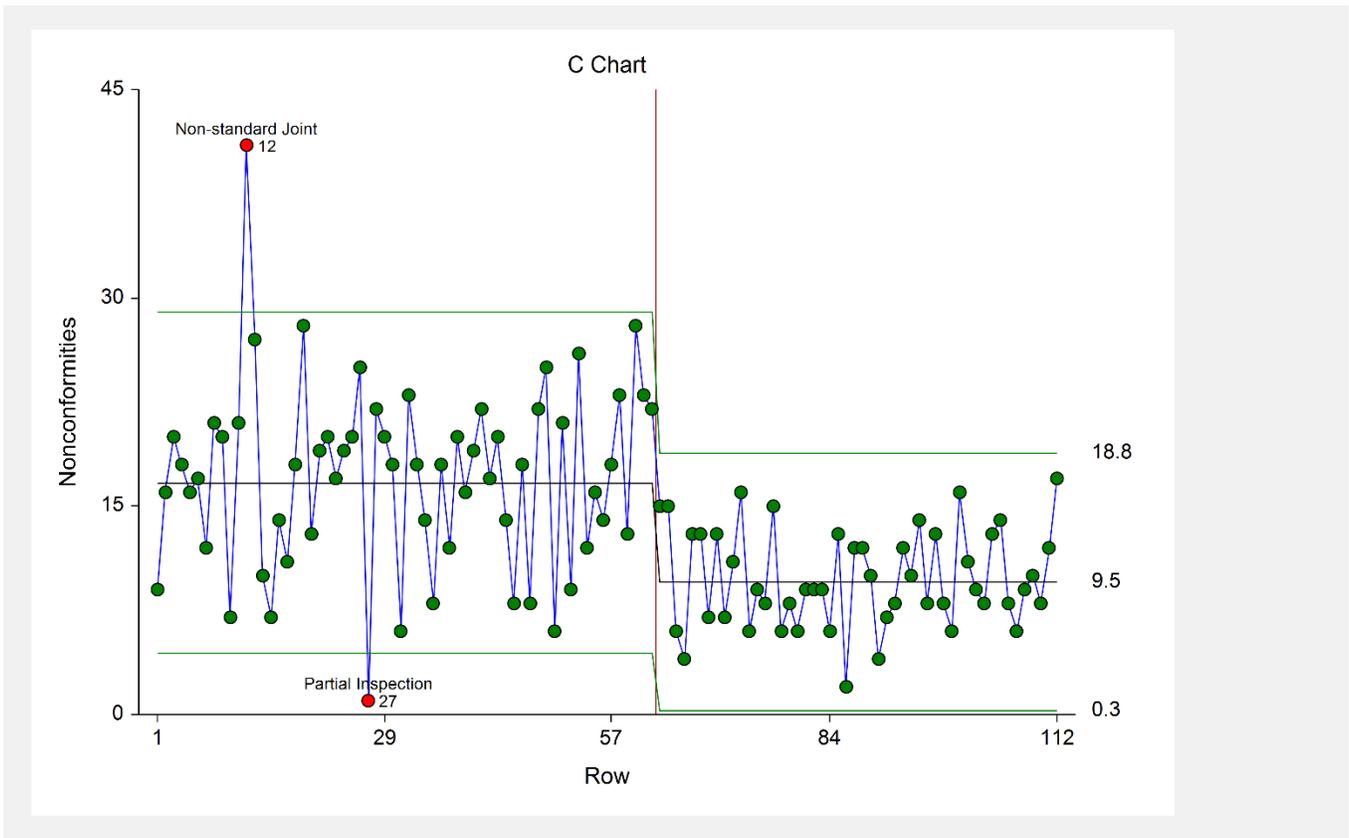
The C chart shows the substantial decrease in the number of nonconformities in the second stage. The center line and limits of the second stage are estimated from samples 63 to 87.

Out-of-Control List

Out-of-Control List for Rows 1 to 62

Row	Number of Nonconformities	Row Label	Reason
12	41.0	12	beyond control limits
13	27.0	13	2 of 3 in zone A
14	10.0	14	2 of 3 in zone A
27	1.0	27	beyond control limits
47	8.0	47	2 of 3 in zone A
54	12.0	54	8 with none in zone C
62	22.0	62	4 of 5 in zone B or beyond

With no new out-of-control points, the out-of-control list is the same as that of the previous examples. There is not an Out-of-Control List for joints 63 to 112 since there are no out-of-control points in this group.