

## Chapter 258

# C Charts

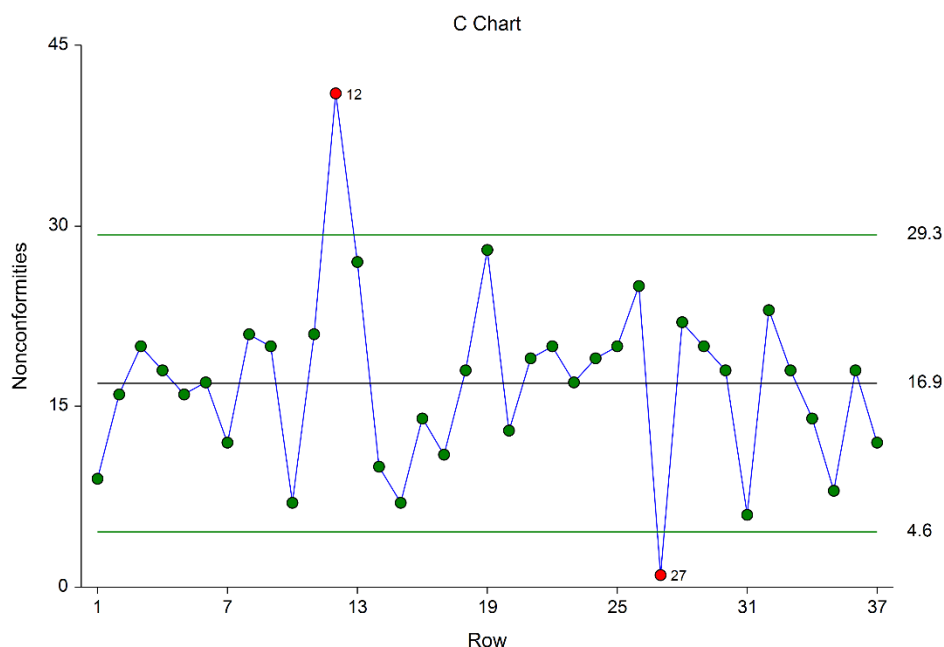
## Introduction

This procedure generates the C control chart for the number of nonconformities (defects) on a unit. The format of the control chart is fully customizable. This procedure permits the defining of stages. For the C chart, the value for C (the average number of nonconformities) can be entered directly or estimated from the data, or a sub-set of the data. A list of out-of-control points can be produced in the output, if desired.

## C Control Charts

C charts are used to monitor the number of nonconformities on a unit of a process based on units taken from the process at given times (hours, shifts, days, weeks, months, etc.). Typically, an initial series of units is used to estimate the average number of nonconformities of a process. The estimated value is then used to produce control limits for the number of nonconformities. During this initial phase, the process should be in control. If points are out-of-control during the initial (estimation) phase, the assignable cause should be determined, and the unit should be removed from estimation.

Once the control limits have been established for the C chart, these limits may be used to monitor the number of nonconformities (defects) of the process going forward. When a point is outside these established control limits it indicates that the number of nonconformities for a unit of the process is out-of-control. An assignable cause is suspected whenever the control chart indicates an out-of-control process.



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## The C Chart versus the U Chart

The C chart is used when a single unit will be examined for nonconformities at each time point. The U chart is used when a number of units will be sampled at each time point, and a per unit average number of nonconformities will be obtained.

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## C Chart Formulas

Suppose we have sampled  $k$  units, one at each time period. Let  $D_i$  represent the number of nonconformities of the  $i^{\text{th}}$  unit.

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### Formulas for the Points on the Chart

Each point on the chart is given by  $D_i$ , the number of nonconformities of the  $i^{\text{th}}$  unit.

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### Estimating the C Chart Center Line

In the C Charts procedure, the center line (average number of nonconformities) may be input directly, or it may be estimated from a series of units. If it is estimated from the units, the formula for the center line is

$$\bar{c} = \frac{\sum_{i=1}^k D_i}{k}$$

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### C Chart Limits

The lower and upper control limits for the C chart are calculated using the formulas

$$LCL = \bar{c} - m\sqrt{\bar{c}}$$

$$UCL = \bar{c} + m\sqrt{\bar{c}}$$

where  $m$  is a multiplier (usually set to 3) chosen to control the likelihood of false alarms (out-of-control signals when the process is in control).

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## Runs Tests

The strength of control charts comes from their ability to detect sudden changes in a process that result from the presence of assignable causes. Unfortunately, the C chart is poor at detecting drifts (gradual trends) or small shifts in the process. For example, there might be a positive trend in the last ten units, but until a number of nonconformities goes above the upper control limit, the chart gives no indication that a change has taken place in the process.

Runs tests can be used to check control charts for unnatural patterns that are most likely caused by assignable causes. Runs tests are sometimes called "pattern tests", "out-of-control" tests, or "zones rules".

## C Charts

While runs tests may be helpful in identifying patterns or smaller shifts in the proportion, they also increase the likelihood of false positive indications. The rate of false positives is typically measured using the average run length (the average length of a run before a false positive is indicated). When several runs tests are used the average run length of the control chart becomes very short.

In order to perform the runs tests, the control chart is divided into six equal zones (three on each side of the center line). Since the control limit is three sigma limits (three standard deviations of the proportion) in width, each zone is one sigma wide and is labeled A, B, or C, with the C zone being the closest to the center line. There is a lower zone A and an upper zone A. The same is true for B and C. The runs tests look at the pattern in which points fall in these zones.

The runs tests used in this procedure are described below.

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### Test 1: Any Single Point Beyond Zone A

This runs test simply indicates a single point is beyond one of the two three-sigma limits.

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### Test 2: Two of Three Successive Points in Zone A or Beyond

This usually indicates a shift in the process average. Note that the two points have to be in the same Zone A, upper or lower. They cannot be on both sides of the center line. The third point can be anywhere.

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### Test 3: Four of Five Successive Points in Zone B or Beyond

This usually indicates a shift in the process average. Note that the odd point can be anywhere.

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### Test 4: Eight Successive Points in Zone C or Beyond

All eight points must be on one side of the centerline. This is another indication of a shift in the process average.

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### Test 5: Fifteen Successive Points Fall in Zone C on Either Side of the Centerline

Although this pattern might make you think that the variation in your process has suddenly decreased, this is usually not the case. It is usually an indication of stratification in the sample. This happens when the samples come from two distinct distributions having different means. Perhaps there are two machines that are set differently. Try to isolate the two processes and check each one separately.

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### Test 6: Eight of Eight Successive Points Outside of Zone C

This usually indicates a mixture of processes. This can happen when two supposedly identical production lines feed a single production or assembly process. You must separate the processes to find and correct the assignable cause.

There are, of course, many other sets of runs tests that have been developed. You should watch your data for trends, zig-zags, and other nonrandom patterns. Any of these conditions could be an indication of an assignable cause and would warrant further investigation.

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## Issues in Using Control Charts

There are several additional considerations surrounding the use of control charts that will not be addressed here. Some important questions are presented below without discussion. For a full treatment of these issues, you should consider a statistical quality control text such as Ryan (2011) or Montgomery (2013).

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### Dealing with Out-of-Control Points

How do you deal with out-of-control points once they have been detected? Should they be included or excluded in the process estimate?

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### Control Limit Multiplier

Three-sigma limits are very common. When should one consider a value other than three?

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### Startup Time

How many units should be used to establish control for my process?

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## Data Structure

In this procedure, the data are entered in a single column. This column contains the number of nonconformities (defects) for each unit.

### Example Dataset

Defects
14
7
16
8
4
22
11
15
19
8
10
13
.
.
.

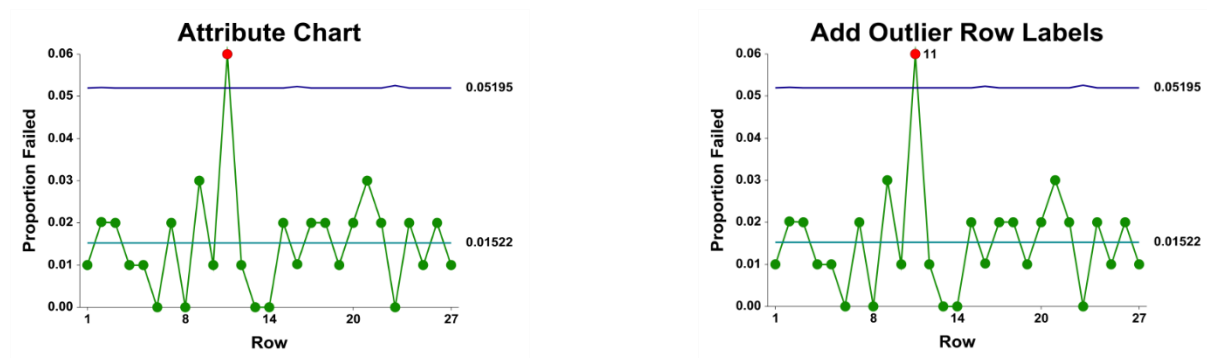
## Attribute Chart Format Window Options

This section describes the specific options available on the Attribute Chart Format window, which is displayed when the Attribute Chart Format button is clicked. Common options, such as axes, labels, legends, and titles are documented in the Graphics Components chapter.

### Attribute Chart Tab

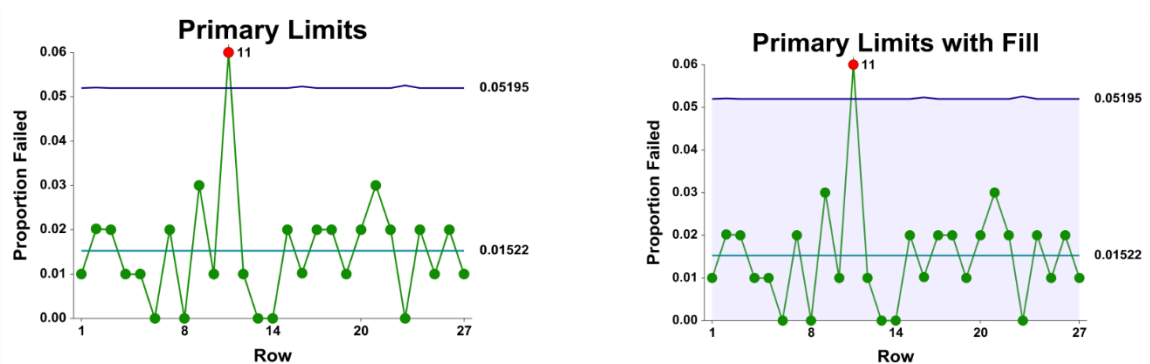
#### Symbols Section

You can modify the attributes of the symbols using the options in this section.



#### Lines Section

You can specify the format of the various lines using the options in this section. Note that when shading is desired, the fill will be to the bottom for single lines (such as the mean line), and between the lines for pairs of lines (such as primary limits).



### Titles, Legend, Numeric Axis, Group Axis, Grid Lines, and Background Tabs

Details on setting the options in these tabs are given in the Graphics Components chapter. The legend does not show by default but can easily be included by going to the Legend tab and clicking the Show Legend checkbox.

## Example 1 – C Chart Analysis (Phase I)

This section presents an example of how to run an initial C Chart analysis to establish control limits. In this example, the welding of large pipeline joints (as the pipeline extends) will be examined for nonconformities. Thirty-seven joints are examined to establish initial control of the process. The data used are in the Weld dataset.

### Setup

To run this example, complete the following steps:

#### 1 Open the Weld example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Weld** and click **OK**.

#### 2 Specify the C Charts procedure options

- Find and open the **C Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Number of Nonconformities Variable ..... **Defects**

#### 3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

### Chart Summary (Estimation and Control Limits)

#### Chart Summary (Estimation and Control Limits) for Rows 1 to 37

Number of Nonconformities Variable: Defects

Number of Units	Number of Nonconformities			
	Total	Average	Control Limits	
			Lower	Upper
37	626	16.91892	4.579135	29.258703

This section displays the calculation values and limits that are to be used in the C chart. The formulas for each calculation are described in the C chart formula section toward the beginning of this chapter.

#### Number of Units

This is the number of units that were examined for nonconformities.

## C Charts

**Total Number of Nonconformities**

This is the sum of all nonconformities across all units.

**Average Number of Nonconformities**

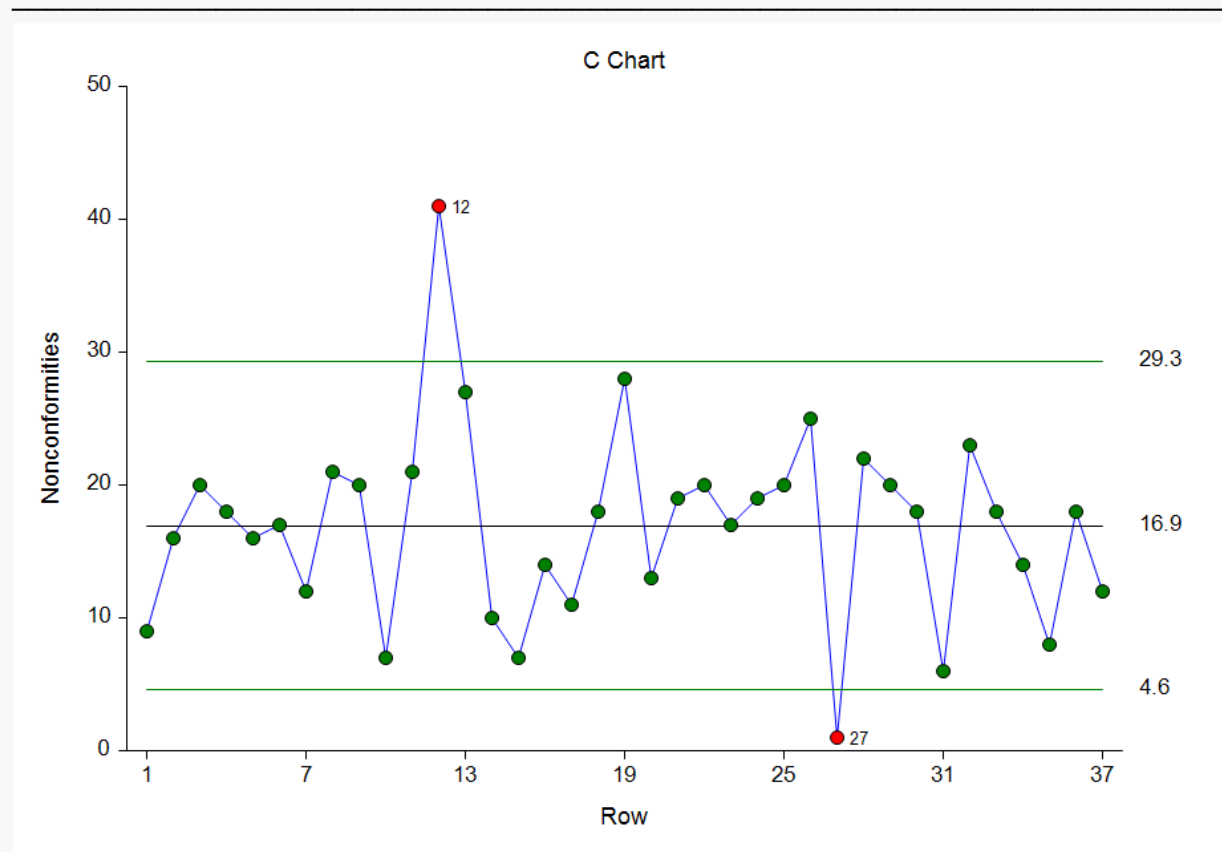
This is the number of nonconformities divided by the number of units.

**Number of Nonconformities Control Limits**

These are the lower and upper control limits for the C chart.

## C Chart

C Chart for Rows 1 to 37



The C chart shows the sequence of nonconformities for the joints. The chart shows that joints 12 and 27 merit further investigation for an assignable cause. After reviewing the records, the investigators found that the 12<sup>th</sup> joint was a different type of joint with more extensive welding than the others. The 27<sup>th</sup> joint had only a partial inspection due to difficult access to the joint. It is determined that these two units should be removed from the initial C chart estimation phase (see Example 2).

## Out-of-Control List

### Out-of-Control List for Rows 1 to 37

Row	Label	Number of Nonconformities	Reason
12	12	41	beyond control limits
13	13	27	2 of 3 in zone A
14	14	10	2 of 3 in zone A
27	27	1	beyond control limits

This report provides a list of the samples that failed one of the runs tests (including points outside the control limits). The report shows that joints 12 and 27 are beyond the control limits. Joints 13 and 14 show runs in the data, but they do seem to signal an important change in the process.



## Example 2 – C Chart Revised (Phase I)

This section presents a continuation of the previous example. In this example the limits are revised to exclude the data from samples 12 and 27, due to assignable cause.

### Setup

To run this example, complete the following steps:

#### 1 Open the Weld example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Weld** and click **OK**.

#### 2 Specify the C Charts procedure options

- Find and open the **C Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

##### Variables Tab

Number of Nonconformities Variable ..... **Defects**  
 Specification Method ..... **Remove Rows Variable**  
 Use all rows except rows where variable ..... **Remove**  
 = ..... **1**

#### 3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

### Output

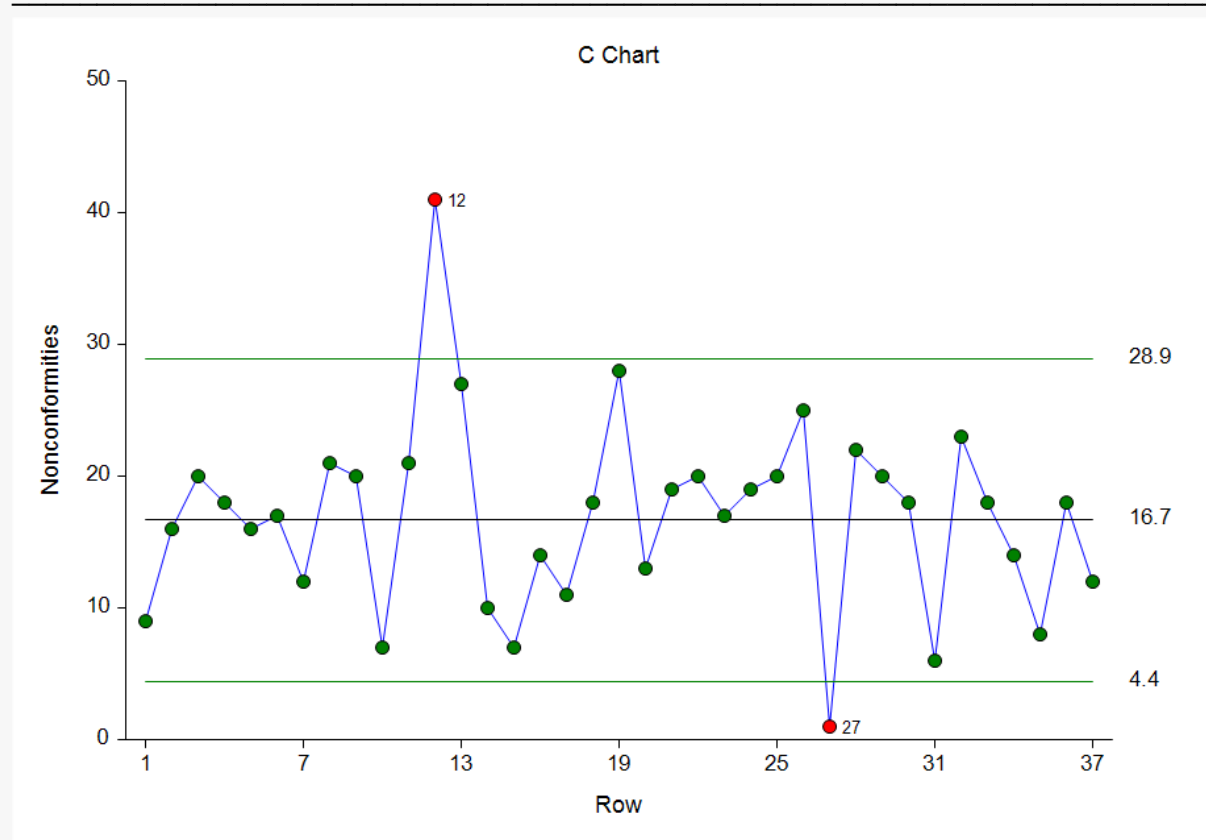
#### Chart Summary (Estimation and Control Limits) for Rows 1 to 37 where Remove ≠ 1

Number of Nonconformities Variable: Defects

Number of Units	Number of Nonconformities			
	Total	Average	Control Limits	
			Lower	Upper
35	584	16.68571	4.431269	28.94016

## C Charts

## C Chart for Rows 1 to 37



## Out-of-Control List for Rows 1 to 37

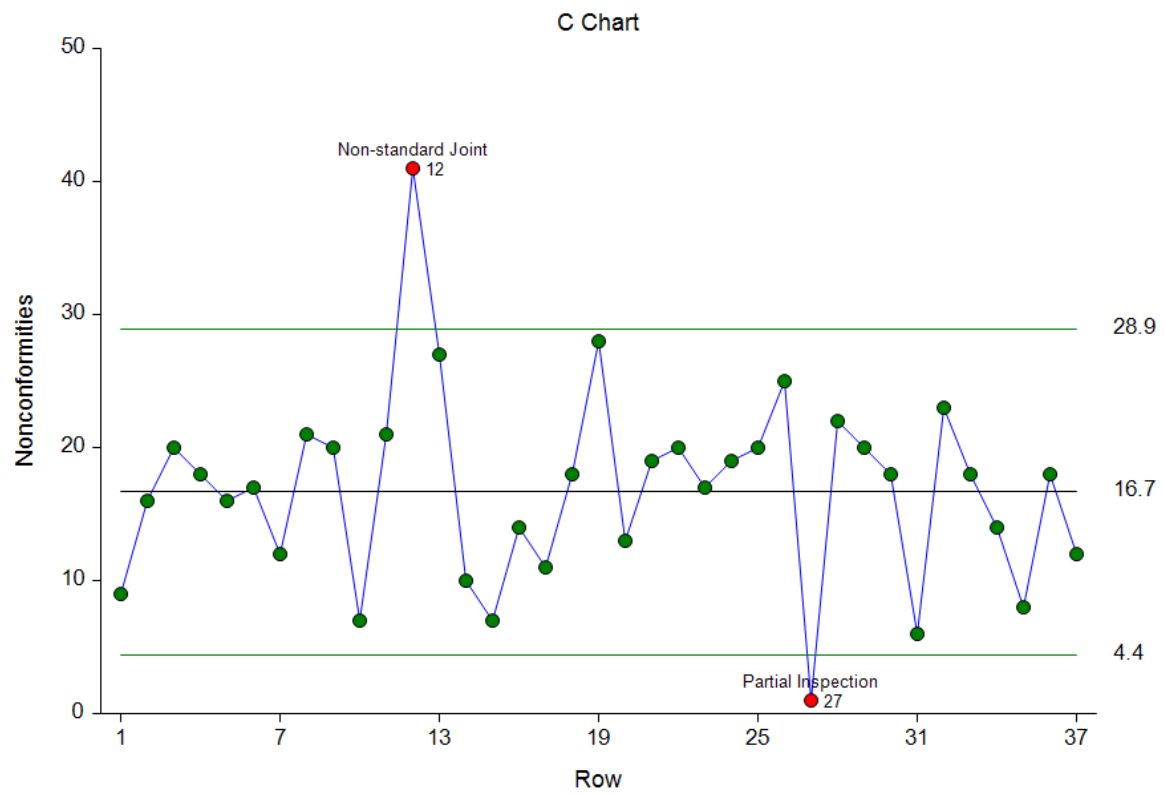
Row	Label	Number of Nonconformities	Reason
12	12	41	beyond control limits
13	13	27	2 of 3 in zone A
14	14	10	2 of 3 in zone A
27	27	1	beyond control limits

The limits in this report and chart are now based on all joints except joints 12 and 27. The number of nonconformities for joints 12 and 27 are included in the chart, but they are not included in the calculations. Because two of the more extreme nonconformity counts were removed, the control limits are narrower than those of Example 1.

The C chart can be additionally enhanced by labeling the points that were not included in the calculations. To do this, set the Point Label Variable to Label.

## C Charts

C Chart for Rows 1 to 37



## Example 3 – C Chart Analysis (Phase II)

Continuing with Examples 1 and 2, the investigators inspect an additional 25 joints. They wish to use the limits based on the first 37 joints, excluding joints 12 and 27.

### Setup

To run this example, complete the following steps:

#### 1 Open the Weld example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Weld** and click **OK**.

#### 2 Specify the C Charts procedure options

- Find and open the **C Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

##### Variables Tab

Number of Nonconformities Variable ..... **DefectsCont**  
 Specification Method ..... **Keep Rows Variable**  
 Use all rows where variable ..... **Keep**  
 = ..... **1**  
 Point Label Variable ..... **Label**

#### 3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

### Output

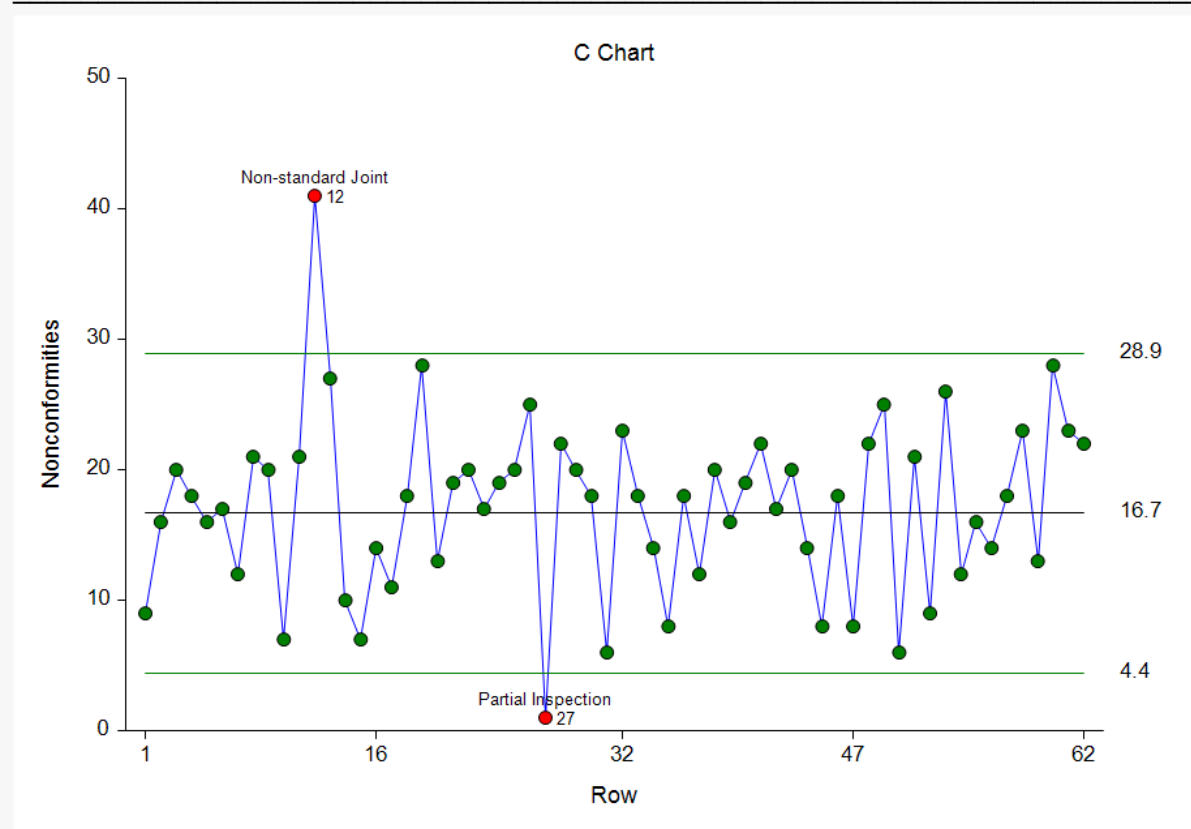
#### Chart Summary (Estimation and Control Limits) for Rows 1 to 62 where Keep = 1

Number of Nonconformities Variable: DefectsCont

Number of Units	Number of Nonconformities			
	Total	Average	Control Limits	
			Lower	Upper
35	584	16.68571	4.431269	28.94016

## C Charts

## C Chart for Rows 1 to 62



## Out-of-Control List for Rows 1 to 62

Row	Label	Number of Nonconformities	Reason
12	12	41	beyond control limits
13	13	27	2 of 3 in zone A
14	14	10	2 of 3 in zone A
27	27	1	beyond control limits
47	47	8	2 of 3 in zone A
54	54	12	8 with none in zone C
62	62	22	4 of 5 in zone B or beyond

The estimation and limits section shows the same results as Example 2, since the calculations are again based only on the first 37 joints, excluding 12 and 27. The plot shows a stable continuation of the process from joints 38 to 62.

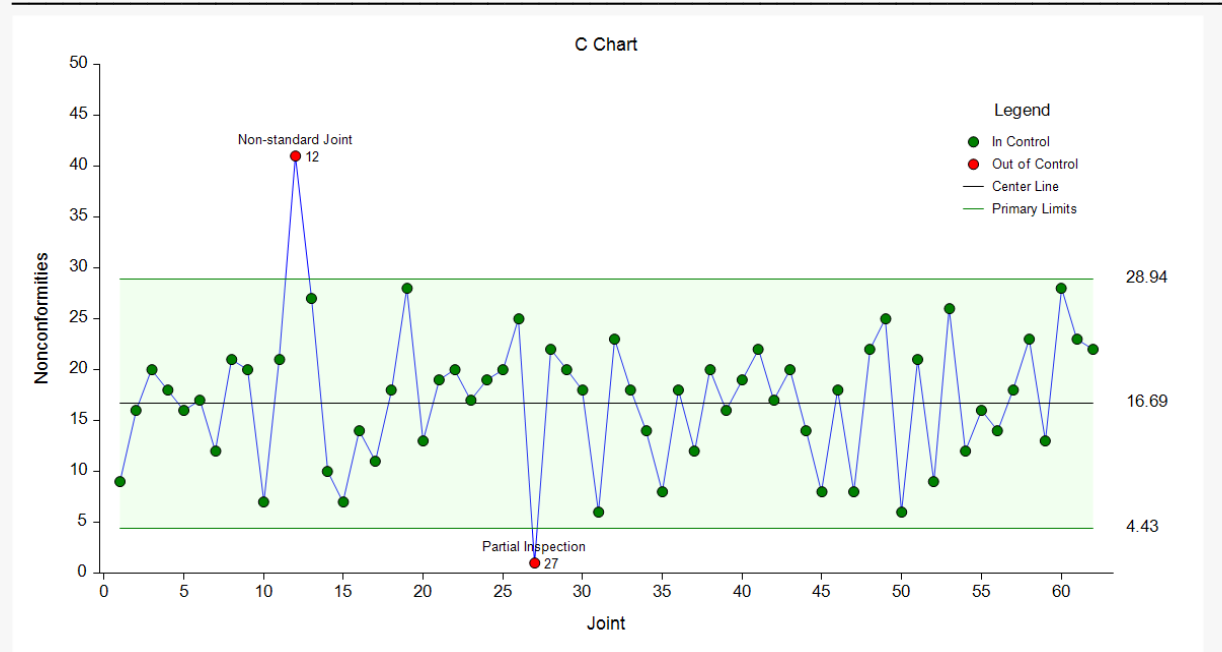
## Example 4 – C Chart with Additional Formatting

This example uses the same setup as Example 3, except that a variety of improvements are made in the plot format. These improvements are made by clicking the C Chart format button on the C Chart tab.

The settings for this example are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

### C Chart

C Chart for Rows 1 to 62



As shown here, a variety of enhancements can be made to the formatting of the control chart to make the chart as easy to read as possible.

## Example 5 – C Chart with Stages

This section presents a continuation of Examples 1, 2, and 3. After the first 62 joints, a new welding process is implemented to reduce the number of nonconformities. The investigators examined 50 additional joints, using the first 25 joints of the additional 50 (63 to 87) to determine the in-control limits of the updated process. The investigators wish to view the entire progress of the process, beginning with the first joint. This example shows the use of stages to monitor a process with a change in center line and limit calculations.

### Setup

To run this example, complete the following steps:

#### 1 Open the Weld example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Weld** and click **OK**.

#### 2 Specify the C Charts procedure options

- Find and open the **C Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 5** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

##### Variables Tab

Number of Nonconformities Variable .....	<b>DefectsStage</b>
Number of Stages.....	<b>Multiple Stages</b>
Stage Variable .....	<b>Stage</b>
Specification Method .....	<b>Keep Rows Variable</b>
Use all rows where variable.....	<b>KeepStage</b>
=.....	<b>1</b>
Point Label Variable.....	<b>Label</b>

#### 3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

## Chart Summary (Estimation and Control Limits)

### Chart Summary (Estimation and Control Limits) for Rows 1 to 62 where KeepStage = 1

Number of Nonconformities Variable: DefectsStage

Number of Units	Number of Nonconformities			
	Total	Average	Control Limits	
			Lower	Upper
35	584	16.68571	4.431269	28.94016

### Chart Summary (Estimation and Control Limits) for Rows 63 to 112 where KeepStage = 1

Number of Nonconformities Variable: DefectsStage

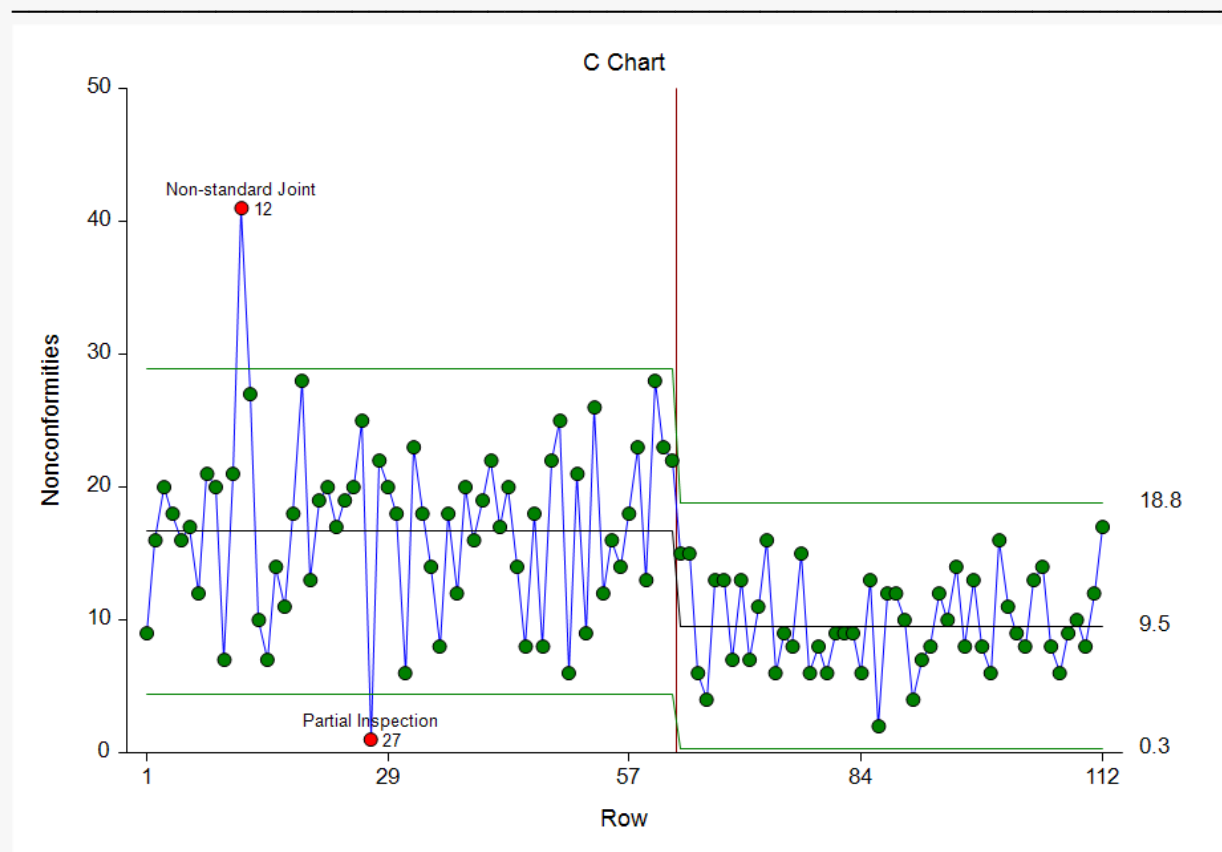
Number of Units	Number of Nonconformities			
	Total	Average	Control Limits	
			Lower	Upper
25	238	9.52	0.263651	18.776349

This section displays the estimation for each stage separately. The estimation results for the first stage are the same as the results in Examples 2 and 3.



## C Chart

C Chart for Rows 1 to 112



The C chart shows the substantial decrease in the number of nonconformities in the second stage. The center line and limits of the second stage are estimated from samples 63 to 87.

## Out-of-Control List

Out-of-Control List for Rows 1 to 62

Row	Label	Number of Nonconformities	Reason
12	12	41	beyond control limits
13	13	27	2 of 3 in zone A
14	14	10	2 of 3 in zone A
27	27	1	beyond control limits
47	47	8	2 of 3 in zone A
54	54	12	8 with none in zone C
62	62	22	4 of 5 in zone B or beyond

With no new out-of-control points, the out-of-control list is the same as that of the previous examples. There is not an Out-of-Control List for joints 63 to 112 since there are no out-of-control points in this group.