

Chapter 123

Data Matching – Optimal and Greedy

Introduction

This procedure is used to create treatment-control matches based on propensity scores and/or observed covariate variables. Both optimal and greedy matching algorithms are available (as two separate procedures), along with several options that allow the user to customize each algorithm for their specific needs. The user is able to choose the number of controls to match with each treatment (e.g., 1:1 matching, 1:k matching, and variable (full) matching), the distance calculation method (e.g., Mahalanobis distance, propensity score difference, sum of rank differences, etc.), and whether or not to use calipers for matching. The user is also able to specify variables whose values must match exactly for both treatment and controls in order to assign a match. **NCSS** outputs a list of matches by match number along with several informative reports and optionally saves the match numbers directly to the database for further analysis.

Matching Overview

Observational Studies

In observational studies, investigators do not control the assignment of treatments to subjects. Consequently, a difference in covariates may exist between treatment and control groups, possibly resulting in undesired biases. Matching is often used to balance the distributions of observed (and possibly confounding) covariates. Furthermore, in many observational studies, there exist a relatively small number of treatment group subjects as compared to control group subjects, and it is often the case that the costs associated with obtaining outcome or response data is high for both groups. Matching is used in this scenario to reduce the number of control subjects included in the study. Common matching methods include Mahalanobis metric matching, propensity score matching, and average rank sum matching. Each of these will be discussed later in this chapter. For a thorough treatment of data matching for observational studies, the reader is referred to chapter 1.2 of D'Agostino, Jr. (2004).

The Propensity Score

Ideally, one would match each treatment subject with a control subject (or subjects) that was an exact match on each of the observed covariates. As the number of covariates increases or the ratio of the number of control subjects to treatment subjects decreases, it becomes less and less likely that an exact match will be found for each treatment subject. *Propensity scores* can be used in this situation to simultaneously control for the presence of several covariate factors. The propensity score was introduced by Rosenbaum and Rubin (1983). The propensity score for subject i ($i = 1, \dots, N$) is defined as the conditional probability of assignment to a treatment ($Z_i = 1$)

Data Matching – Optimal and Greedy

versus the control ($Z_i = 0$), given a set (or vector) of observed covariates, \mathbf{x}_i . Mathematically, the propensity score for subject i can be expressed as

$$e(\mathbf{x}_i) = \text{pr}(Z_i = 1 \mid \mathbf{X}_i = \mathbf{x}_i).$$

It is assumed that the Z_i 's are independent, given the X 's. The observed covariates, \mathbf{x}_i , are not necessarily the same covariates used in the matching algorithm, \mathbf{y}_i , although they could be. Rosenbaum and Rubin (1985a) suggest using the logit of the estimated propensity score for matching because the distribution of transformed scores is often approximately normal. The logit of the propensity score is defined as

$$q(\mathbf{x}) = \log\left(\frac{1 - e(\mathbf{x})}{e(\mathbf{x})}\right),$$

Matching on the observed propensity score (or logit propensity score) can balance the overall distribution of observed covariates between the treatment and control groups. The propensity score is often calculated using logistic regression or discriminant analysis with the treatment variable as the dependent (group) variable and the background covariates as the independent variables. Research suggests that care must be taken when creating the propensity score model (see Austin et al. (2007)). For more information about logistic regression or discriminant analysis, see the corresponding chapters in the NCSS manuals.

Optimal vs. Greedy Matching

Two separate procedures are documented in this chapter, *Optimal Data Matching* and *Greedy Data Matching*. The goal of both algorithms is to produce a matched sample that balances the distribution of observed covariates between the treatment and matched-control groups. Both algorithms allow for the creation of 1:1 or 1:k matched pairings. Gu and Rosenbaum (1993) compared the greedy and optimal algorithms and found that “optimal matching is sometimes noticeably better than greedy matching in the sense of producing closely matched pairs, sometimes only marginally better, but it is no better than greedy matching in the sense of producing balanced matched samples.” The choice of the algorithm depends on the research objectives, the desired analysis, and cost considerations. We recommend using the optimal matching algorithm where possible.

The optimal and greedy algorithms differ in three fundamental ways:

1. Treatment of Previously Matched Subjects
2. Complete vs. Incomplete Matched-Pair Samples
3. Variable (Full) Matching

Treatment of Previously-Matched Subjects

Optimal matching refers to the use of an optimization method based on the Relax-IV algorithm written by Dimitri P. Bertsekas (see Bertsekas (1991)), which minimizes the overall sum of pair-wise distances between treatment subjects and matched control subjects. The Relax-IV algorithm is based on network flow theory, and matching is just one of its many uses. Optimal matching is not a linear matching algorithm in the sense that as the algorithm proceeds, matches are created, broken, and rearranged in order to minimize the overall sum of match distances.

Greedy matching, on the other hand, is a linear matching algorithm: when a match between a treatment and control is created, the control subject is removed from any further consideration for matching. When the number of matches per treatment is greater than one (i.e., 1:k matching), the greedy algorithm finds the best match (if possible) for each treatment before returning and creating the second match, third match, etc. Once a treatment subject has been matched with the user-specified number of control subjects, the treatment subject is also removed from further consideration. A familiar example of a greedy algorithm is forward selection used in multiple regression model creation.

Complete vs. Incomplete Matched-Pair Samples

Optimal matching only allows for *complete matched-pair samples*, while greedy matching also allows for *incomplete matched-pair samples*. A complete matched-pair sample is a sample for which every treatment is matched with at least one control. An incomplete matched-pair sample is a sample for which the number of treatment subjects matched is less than the total number of treatment subjects in the reservoir. Rosenbaum and Rubin (1985b) present strong reasons for avoiding incomplete matched-pair samples.

Variable (Full) Matching

Variable (or “Full”) matching is only available using the optimal matching algorithm. In variable matching, a different number of controls may be matched with each treatment. Each control is used only once, and each treatment receives at least one control. All eligible controls (e.g. all controls for which at least one treatment-control distance is non-infinite) are matched. Results from Gu and Rosenbaum (1993) suggest that in terms of bias reduction, full matching performs much better than 1:k matching. If we require that every treatment have the same number of controls, and the distributions between the two groups of covariates are not the same, then some treatments will be paired with controls that are not good matches. Variable matching, on the other hand, is more flexible in allowing control subjects to pair with the closest treatment subject in every case.

The gains in bias reduction for variable matching over 1:k matching, however, must be weighed against other considerations such as simplicity and aesthetics. The analysis after 1:k matching would arguably be simpler; a more complex analysis method (e.g. stratified analysis) would be employed after variable matching than would be after 1:k matching.

The Distance Calculation Method

Several different distance calculation methods are available in the matching procedures in NCSS. The different methods are really variations of three common distance measures:

1. Mahalanobis Distance
2. Propensity Score Difference
3. Sum of Rank Differences

The variations arise when using calipers for matching or when using forced match variables. A *caliper* is defined in this context a restricted subset of controls whose propensity score is within a specified amount (*c*) of the treatment subject’s propensity score. A *forced match variable* contains values which must match exactly in the treatment and control for the subjects to be considered for matching. If the values for the forced match variables do not agree, then the distance between the two subjects is set equal to ∞ (infinity), and a match between the two is not allowed.

Distance Measures

The complete list of possible distance measures available in NCSS is as follows:

1. Mahalanobis Distance within Propensity Score Calipers (no matches outside calipers)
2. Mahalanobis Distance within Propensity Score Calipers (matches allowed outside calipers)
3. Mahalanobis Distance including the Propensity Score (if specified)
4. Propensity Score Difference within Propensity Score Calipers (no matches outside calipers)
5. Propensity Score Difference
6. Sum of Rank Differences within Propensity Score Calipers (no matches outside calipers)
7. Sum of Rank Differences within Propensity Score Calipers (matches allowed outside calipers)
8. Sum of Rank Differences including the Propensity Score (if specified)

Data Matching – Optimal and Greedy

Distance measures #2 and #7, where matches are allowed outside calipers in caliper matching, are only available with greedy matching. All others can be used with both the greedy and optimal matching algorithms.

For distance measures that involve propensity score calipers, the caliper size is determined by the user-specified radius, c . For any treatment subject, i , the j^{th} , control subject is included in the i^{th} treatment caliper if

$$|q(\mathbf{x}_i) - q(\mathbf{x}_j)| \leq c$$

where $q(\mathbf{x}_i) = e(\mathbf{x}_i)$ is the propensity score based on the covariates \mathbf{x}_i . If the logit transformation is used in the analysis, then $q(\mathbf{x}) = \log((1 - e(\mathbf{x})) / e(\mathbf{x}))$. The width of each caliper is equal to $2c$.

Which Distance Measure to Use?

The best distance measure depends on the number of covariate variables, the variability within the covariate variables, and possibly other factors. Gu and Rosenbaum (1993) compared the imbalance of Mahalanobis distance metrics versus the propensity score difference in optimal 1:1 matching for numbers of covariates (P) between 2 and 20 and control/treatment subject ratios between 2 and 6. Mahalanobis distance within propensity score calipers was always best or second best. When there are many covariates ($P = 20$), the article suggests that matching on the propensity score difference is best. The use of Mahalanobis distance (with or without calipers) is best when there are few covariates on which to match ($P = 2$). In all cases considered by Gu and Rosenbaum (1993), the Mahalanobis distance within propensity score calipers was never the worst method of the three. Rosenbaum and Rubin (1985a) conducted a study of the performance of three different matching methods (Mahalanobis distance, Mahalanobis distance within propensity score calipers, and propensity score difference) in a greedy algorithm with matches allowed outside calipers and concluded that the Mahalanobis distance within propensity score calipers is the best technique among the three. Finally, Rosenbaum (1989) reports parenthetically that he has had “unpleasant experiences using standard deviations to scale covariates in multivariate matching, and [he] is inclined to think that either ranks or some more resistant measure of spread should routinely be used instead.”

Based on these results and suggestions, we recommend using the Mahalanobis Distance within Propensity Score Calipers as the distance calculation method where possible. The caliper radius to use is based on the amount of bias that you want removed.

What Caliper Radius to Use?

The performance of distance metrics involving calipers depends to some extent on the caliper radius used. For instances in the literature where we found reports, comparisons, or studies based on caliper matching, Cochran and Rubin (1973) was nearly always mentioned as the literature used in determining the caliper radius (or “caliper width” as they call it) for the study. The following table (Table 2.3.1 from Cochran and Rubin (1973)) can be used to determine the appropriate coefficient and/or caliper radius to use:

Table 2.3.1 from Cochran and Rubin (1973). Percent Reduction in bias of x for caliper matching to within $\pm a\sqrt{(\sigma_1^2 + \sigma_2^2)/2}$

| a | $\sigma_1^2/\sigma_2^2 = 1/2$ | $\sigma_1^2/\sigma_2^2 = 1$ | $\sigma_1^2/\sigma_2^2 = 2$ |
|------------|-------------------------------|-----------------------------|-----------------------------|
| 0.2 | 0.99 | 0.99 | 0.98 |
| 0.4 | 0.96 | 0.95 | 0.93 |
| 0.6 | 0.91 | 0.89 | 0.86 |
| 0.8 | 0.86 | 0.82 | 0.77 |
| 1.0 | 0.79 | 0.74 | 0.69 |

Data Matching – Optimal and Greedy

The caliper radius to use depends on the desired bias reduction (table body), the coefficient a , and the ratio of the treatment group sample variance of $q(\mathbf{x})$, σ_1^2 , to the control group sample variance of $q(\mathbf{x})$, σ_2^2 . “Loose Matching” corresponds to $a \geq 1.0$, while “Tight Matching” corresponds to $a \leq 0.2$. The caliper radius is calculated as

$$c = a\sqrt{(\sigma_1^2 + \sigma_2^2)/2} = a \times \text{SIGMA}$$

NCSS allows you to choose the caliper radius using the syntax “ a *SIGMA”, where you specify the value for a (e.g. “0.2*SIGMA”) or by entering the actual value directly for c (e.g. “0.5”). In the case of the former, the program calculates the variances of the treatment and control group propensity scores for you and determines the pooled standard deviation, sigma. You may want to run descriptive statistics on the treatment and control group propensity scores to determine the variance ratio of your data in order to find the appropriate value of a (from the table above) for your research objectives.

Data Structure

The propensity scores and covariate variables must each be entered in individual columns in the database. Only numeric values are allowed in propensity score and covariate variables. Blank cells or non-numeric (text) entries are treated as missing values. If the logit transformation is used, values in the propensity score variable that are not between zero and one are also treated as missing. A grouping variable containing two (and only two) unique groups must be present. A data label variable is optional. The following is a subset of the Propensity dataset, which illustrates the data format required for the greedy and optimal data matching procedures.

Propensity dataset (subset)

| ID | Exposure | X1 | ... | Age | Race | Gender | Propensity |
|----|-------------|----|-----|-----|-----------|--------|-----------------|
| A | Exposed | 50 | ... | 45 | Hispanic | Male | 0.7418116515 |
| B | Not Exposed | 4 | ... | 71 | Hispanic | Male | 0.01078557025 |
| C | Not Exposed | 81 | ... | 70 | Caucasian | Male | 0.0008716385678 |
| D | Exposed | 31 | ... | 33 | Hispanic | Female | 0.5861360724 |
| E | Not Exposed | 65 | ... | 38 | Black | Male | 0.1174339761 |
| F | Exposed | 22 | ... | 29 | Black | Female | 0.07538899371 |
| G | Not Exposed | 36 | ... | 57 | Black | Female | 0.008287371892 |
| H | Not Exposed | 31 | ... | 52 | Caucasian | Male | 0.4250166047 |
| I | Not Exposed | 46 | ... | 39 | Hispanic | Female | 0.2630767334 |
| J | Exposed | 3 | ... | 58 | Hispanic | Male | 0.4858799526 |
| K | Not Exposed | 84 | ... | 24 | Black | Female | 0.1251753736 |

Procedure Options

This section describes the options available in both the optimal and greedy matching procedures.

Variables Tab

Specify the variables to be used in matching and storage, along with the matching options.

Data Variables

Grouping Variable

Specify the variable that contains the group assignment for each subject. The values in this variable may be text or numeric, but only two unique values are allowed. One value should designate the treatment group and the other should designate the control group. The value assigned to the treatment group should be entered under “Treatment Group” to the right.

Treatment Group

Specify the value in the Grouping Variable that is used to designate the treatment group. This value can be text or numeric.

Propensity Score Variable

Specify the variable containing the propensity scores to be used for matching. This variable is optional if the Distance Calculation Method is not specifically based on the propensity score. If no covariate variables are specified, then you must specify a propensity score variable. If caliper matching is used, this variable must be specified. Only numeric values are allowed. Text values are treated as missing values in the reports. If the logit transformation is used, all values in this variable must be between zero and one, otherwise they are treated as missing. Propensity scores are often obtained using logistic regression or discriminant analysis.

Use Logit

This option specifies whether or not to use the logit transformation on the propensity score. If selected, all calculations and reports will be based on the logit propensity score (where applicable).

Forced Match Variable(s)

Specify variables for which the treatment and control values must match exactly in order to create a match. More than one variable may be specified. This variable is optional. Variables such as gender and race are commonly used as forced match variables.

The use of forced match variables may greatly restrict the number of possible matches for each treatment. If you are using greedy matching, the use of this variable may result in unmatched treatment subjects. If you are using optimal matching, the use of forced match variable(s) may result in an infeasible (unsolvable) problem for the matching algorithm. If the optimal matching algorithm is unable to find a solution, try eliminating one or more forced match variable(s).

Covariate Variable(s)

Specify variables to be used in distance calculations between treatment and control subjects. Only numeric values are allowed. Text values are treated as missing. Covariate variables are optional, however, if no propensity score variable is specified you must specify at least one covariate variable. If the distance calculation method involves only the propensity score (e.g. propensity score difference) and one or more covariate variables are specified, then the covariate variables are only used in group comparison reports (they are not used in matching nor to determine whether or not a row contains missing values during matching).

Data Matching – Optimal and Greedy

Data Label Variable

The values in this variable contain text (or numbers) and are used to identify each row. This variable is optional.

Storage Variable

Store Match Numbers In

Specify a variable to store the match number assignments for each row. This variable is optional. If no storage variable is specified, the match numbers will not be stored in the database, but matching reports can still be generated.

Optimization Algorithm Options

Maximum Iterations

Specify the number of optimization iterations to perform before exiting. You may choose a value from the list, or enter your own. This option is available in order to avoid an infinite loop. We have found that as the number of Matches per Treatment increases, it takes more and more iterations in order to arrive at a solution.

Matching Options

Distance Calculation Method

Specify the method to be used in calculating distances between treatment and control subjects. If the distance method involves propensity score calipers, then a Propensity Score Variable must also be specified. Eight different distance measures are available in NCSS. For the formulas that follow, we will adopt the following notation:

1. The subscript i refers to the i^{th} treatment subject.
2. The subscript j refers to the j^{th} control subject.
3. $d(i, j)$ is the estimated distance between subjects i and j .
4. \mathbf{x} is the vector of observed covariates used to estimate the propensity score.
5. $q(\mathbf{x}) = e(\mathbf{x})$ is the propensity score based on the covariates \mathbf{x} . If the logit transformation is used in the analysis, then $q(\mathbf{x}) = \log((1 - e(\mathbf{x})) / e(\mathbf{x}))$.
6. \mathbf{y} is the vector of observed covariates used in the distance calculation. \mathbf{y} is not necessary equivalent to \mathbf{x} , although it could be.
7. $\mathbf{u} = (\mathbf{y}, q(\mathbf{x}))$ is the vector of observed covariates and the propensity score (or logit propensity score).
8. C is the sample covariance matrix of the matching variables (including the propensity score) from the full set of control subjects.
9. c is the caliper radius. The width of each caliper is $2c$.
10. $FM_{i,l}$ and $FM_{j,l}$ are the values of the l^{th} forced match variable for subjects i and j , respectively. If no forced match variables are specified, then $FM_{i,l} = FM_{j,l}$ for all l .
11. $R_{i,p}$ and $R_{j,p}$ are the ranks of the p^{th} covariate values or propensity score for subjects i and j , respectively. Average ranks are used in the case of ties.

Data Matching – Optimal and Greedy

The options are:

- **Mahalanobis Distance within Propensity Score Calipers (no matches outside calipers)**

$$d(i, j) = \begin{cases} (\mathbf{u}_i - \mathbf{u}_j)^T C^{-1} (\mathbf{u}_i - \mathbf{u}_j) & \text{if } |q(\mathbf{x}_i) - q(\mathbf{x}_j)| \leq c \text{ and } FM_{i,l} = FM_{j,l} \text{ for all } l \\ \infty & \text{otherwise} \end{cases}$$

- **Mahalanobis Distance within Propensity Score Calipers (matches allowed outside calipers)**

$$d(i, j) = \begin{cases} (\mathbf{u}_i - \mathbf{u}_j)^T C^{-1} (\mathbf{u}_i - \mathbf{u}_j) & \text{if } |q(\mathbf{x}_i) - q(\mathbf{x}_j)| \leq c \text{ and } FM_{i,l} = FM_{j,l} \text{ for all } l \\ |q(\mathbf{x}_i) - q(\mathbf{x}_j)| & \text{if } |q(\mathbf{x}_i) - q(\mathbf{x}_j)| > c \text{ for all unmatched } j \\ & \text{and } FM_{i,l} = FM_{j,l} \text{ for all } l \\ \infty & \text{otherwise} \end{cases}$$

The absolute difference, $|q(\mathbf{x}_i) - q(\mathbf{x}_j)|$, is only used in assigning matches if there are no available controls for which $|q(\mathbf{x}_i) - q(\mathbf{x}_j)| \leq c$.

- **Mahalanobis Distance including the Propensity Score (if specified)**

$$d(i, j) = \begin{cases} (\mathbf{u}_i - \mathbf{u}_j)^T C^{-1} (\mathbf{u}_i - \mathbf{u}_j) & \text{if } FM_{i,l} = FM_{j,l} \text{ for all } l \\ \infty & \text{otherwise} \end{cases}$$

- **Propensity Score Difference within Propensity Score Calipers (no matches outside calipers)**

$$d(i, j) = \begin{cases} |q(\mathbf{x}_i) - q(\mathbf{x}_j)| & \text{if } |q(\mathbf{x}_i) - q(\mathbf{x}_j)| \leq c \text{ and } FM_{i,l} = FM_{j,l} \text{ for all } l \\ \infty & \text{otherwise} \end{cases}$$

- **Propensity Score Difference**

$$d(i, j) = \begin{cases} |q(\mathbf{x}_i) - q(\mathbf{x}_j)| & \text{if } FM_{i,l} = FM_{j,l} \text{ for all } l \\ \infty & \text{otherwise} \end{cases}$$

- **Sum of Rank Differences within Propensity Score Calipers (no matches outside calipers)**

$$d(i, j) = \begin{cases} \sum_p |R_{i,p} - R_{j,p}| & \text{if } |q(\mathbf{x}_i) - q(\mathbf{x}_j)| \leq c \text{ and } FM_{i,l} = FM_{j,l} \text{ for all } l \\ \infty & \text{otherwise} \end{cases}$$

- **Sum of Rank Differences within Propensity Score Calipers (matches allowed outside calipers)**

$$d(i, j) = \begin{cases} \sum_p |R_{i,p} - R_{j,p}| & \text{if } |q(\mathbf{x}_i) - q(\mathbf{x}_j)| \leq c \text{ and } FM_{i,l} = FM_{j,l} \text{ for all } l \\ |q(\mathbf{x}_i) - q(\mathbf{x}_j)| & \text{if } |q(\mathbf{x}_i) - q(\mathbf{x}_j)| > c \text{ for all unmatched } j \\ & \text{and } FM_{i,l} = FM_{j,l} \text{ for all } l \\ \infty & \text{otherwise} \end{cases}$$

The absolute difference, $|q(\mathbf{x}_i) - q(\mathbf{x}_j)|$, is only used in assigning matches if there are no available controls for which $|q(\mathbf{x}_i) - q(\mathbf{x}_j)| \leq c$.

- **Sum of Rank Differences including the Propensity Score (if specified)**

$$d(i, j) = \begin{cases} \sum_p |R_{i,p} - R_{j,p}| & \text{if } FM_{i,l} = FM_{j,l} \text{ for all } l \\ \infty & \text{otherwise} \end{cases}$$

Data Matching – Optimal and Greedy

In the Greedy Data Matching procedure, two distance calculation methods are available that are not in the Optimal Data Matching procedure (option #2 and option #7). Both involve caliper matching with matches allowed outside calipers. When matches are allowed outside calipers, the algorithm always tries to find matches inside the calipers first, and only assigns matches outside calipers if a match was not found inside. Matches outside calipers are created based solely on the propensity score, i.e., if matches outside calipers are allowed and no available control subject exists that is within c propensity score units of a treatment subject, then the control subject with the nearest propensity score is matched with the treatment. This type of matching algorithm is described in Rosenbaum and Rubin (1985a).

Matches per Treatment

Choose the number of controls to match with each treatment. You may choose one of the values for the list or enter an integer value of your own. For greedy matching, the value you enter can be no larger than controls/treatments rounded up to the next highest integer. When the number of matches per treatment is greater than one, the greedy algorithm finds the best match (if possible) for each treatment before returning and creating the second match, third match, etc. For optimal matching, the value can be no larger than controls/treatments rounded down to the next lowest integer. The options are:

- **Variable (Full Matching) (Optimal Data Matching Only)**

This option causes the optimal matching algorithm to match a variable number of controls to each treatment. Each control is used only once, and each treatment is matched with at least one control. All eligible controls (e.g. all controls where at least one treatment-control distance is non-infinite) are matched.

- **Maximum Possible**

This option causes the program to assign the maximum number (k) of matches that can be made between treatments and controls. If greedy matching is used and controls/treatments is not an integer, then using this option will result in incomplete pair-matching.

- **Integer Values**

If an integer value is entered or selected, then the program attempts to create the specified number of control matches for each treatment.

Order for Matching

This option specifies the order in which subjects are entered into the matching algorithm. In the case of tied distance values, the matches created depend on the order in which the treatment and control subjects are considered. The options are:

- **Random**

Both treatment and control subjects are randomly ordered before entering into the matching algorithm. When the number of matches per treatment is greater than one, the greedy algorithm finds the best match (if possible) for each treatment before returning and creating the second match, third match, etc. It is likely that match assignments will change from run-to-run when using random ordering.

- **Sorted by Distance (Greedy Data Matching Only)**

This option causes the program to sort the matrix of all pair-wise treatment-control distances, and assign matches starting with the smallest distance and working toward the largest until all treatments have been matched with the specified number of controls.

- **Sorted by Row Number**

Both treatment and control subjects are entered into the matching algorithms according to their location in the database. When the number of matches per treatment is greater than one, the greedy algorithm finds the best match (if possible) for each treatment before returning and creating the second match, third match, etc.

Data Matching – Optimal and Greedy

Caliper Radius

This option specifies the caliper radius, c , to be used in caliper matching. The caliper radius is calculated as

$$c = a\sqrt{(\sigma_1^2 + \sigma_2^2)/2} = a \times SIGMA$$

where a is a user-specified coefficient, σ_1^2 is the sample variance of $q(\mathbf{x})$ for the treatment group, and σ_2^2 is the sample variance of $q(\mathbf{x})$ for the control group. NCSS allows you to enter the caliper radius using the syntax “ $a \times SIGMA$ ”, where you specify the value for a (e.g. “0.2* $SIGMA$ ”) or by entering the actual value directly for c (e.g. “0.5”). In the case of the former, the program calculates the variances of the treatment and control group propensity scores for you. You may want to run descriptive statistics on the treatment and control group propensity scores to determine the variance ratio of your data in order to find the appropriate value of a (from the table above) for your research objectives.

Reports Tab

The following options control the format of the reports that are displayed.

Select Reports

Data Summary Report ... Matching Detail Report

Indicate whether to display the indicated reports.

Incomplete Matching Report (Greedy Data Matching Only)

Indicate whether to display the incomplete matching report that lists the treatments that were not paired with the specified number of controls.

Report Options

Variable Names

This option lets you select whether to display variable names, variable labels, or both.

Report Options – Decimals

Propensity Scores/Covariates ... Standardized Differences

Specify the number of digits after the decimal point to be displayed for output values of the type indicated.

Example 1 – Optimal (1:1) Matching using the Mahalanobis Distance within Propensity Score Calipers

This tutorial describes how to create 1:1 treatment-control matches using the Mahalanobis Distance within Propensity Score Calipers distance metric. The data used in this example are contained in the PROPENSITY database. The propensity scores were created using logistic regression with Exposure as the dependent variable, X1 – Age as numeric independent variables, and Race and Gender as categorical independent variables. The propensity score represents the probability of being exposed given the observed covariate values. The optimal matching algorithm will always produce a complete matched-pair sample.

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Data Matching – Optimal window.

1 Open the Propensity dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Propensity.NCSS**.
- Click **Open**.

2 Open the Data Matching - Optimal window.

- Using the Data or Tools menu or the Procedure Navigator, find and select the **Data Matching - Optimal** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the Data Matching - Optimal window, select the **Variables tab**.
- Enter **Exposure** in the **Grouping Variable** box.
- Enter “**Exposed**” (no quotes) in the **Treatment Group** box.
- Enter **Propensity** in the **Propensity Score Variable** box.
- Make sure **Use Logit** is checked.
- Enter **X1-Age** in the **Covariate Variable(s)** box.
- Enter **ID** in the **Data Label Variable** box.
- Enter **C11** in the **Store Match Numbers In** box.
- Enter **1.5*Sigma** in the **Caliper Radius** box.
- Leave all other options at their default values.

4 Specify the reports.

- On the Data Matching - Optimal window, select the **Reports tab**.
- Put a check mark next to **Matching Detail Report**. Leave all other options at their default values.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

The following reports will be generated for both optimal and greedy matching with slight variations depending on the algorithm selected.

Data Summary Report

| Data Summary Report | |
|---|-------------------|
| Rows Read | 30 |
| Rows with Missing Data | 0 |
| Treatment Rows | 8 |
| Control Rows | 22 |
| ----- Data Variables ----- | |
| Grouping Variable | Exposure |
| - Treatment Group | "Exposed" |
| - Control Group | "Not Exposed" |
| Data Label Variable | ID |
| ----- Variables Used in Distance Calculations ----- | |
| Propensity Score Variable | Logit(Propensity) |
| Covariate Variable 1 | X1 |
| Covariate Variable 2 | X2 |
| Covariate Variable 3 | X3 |
| Covariate Variable 4 | X4 |
| Covariate Variable 5 | X5 |
| Covariate Variable 6 | X6 |
| Covariate Variable 7 | Age |
| ----- Storage Variable ----- | |
| Match Number Storage Variable | C11 |

This report gives a summary of the data and variables used for matching.

Matching Summary Report

| Matching Summary Report | | | | | |
|------------------------------------|--|----------------|------------------------|------------------|--------------------------|
| Distance Calculation Method | Mahalanobis Distance within Propensity Score Calipers (no matches outside calipers) | | | | |
| Caliper Radius | 2.63288 | | | | |
| Order for Matching | Random | | | | |
| Controls Matched per Treatment | 1 | | | | |
| Sum of Match Mahalanobis Distances | 53.94887 | | | | |
| Average Match Mahalanobis Distance | 6.74361 | | | | |
| Group | N | Matched | Percent Matched | Unmatched | Percent Unmatched |
| Exposed | 8 | 8 | 100.00% | 0 | 0.00% |
| Not Exposed | 22 | 8 | 36.36% | 14 | 63.64% |

This report gives a summary of the matches created, as well as a summary of the matching parameters used by the matching algorithm.

Distance Calculation Method

This is the method used to calculate distances between treatment and control subjects.

Caliper Radius

This is the caliper radius entered or calculated by the program. This line is only displayed if caliper matching based on propensity scores was used.

Order for Matching

This is the order used in matching as selected on the procedure window.

Data Matching – Optimal and Greedy

Controls Matched per Treatment

This is the target number of controls to match with each treatment. This value is specified on the procedure window.

Sum of Match Mahalanobis Distances (Sum of Match Propensity Score Differences or Sum of Match Rank Differences)

This is the sum of Mahalanobis distances, propensity score differences, or rank differences (depending on the distance calculation method selected) for all matched pairs.

Average Match Mahalanobis Distance (Average Match Propensity Score Difference or Average Match Rank Differences)

This is the average Mahalanobis distances, propensity score difference, or rank difference (depending on the distance calculation method selected) for all matched pairs. This is calculated as the [Sum of Match Distances (or Differences)]/[Number of Matches Formed].

Group (e.g. Exposure)

This specifies either the treatment or the control group. The title of this column is the Grouping Variable name (or label).

N

This is the number of candidates for matching in each group, i.e. the number of subjects with non-missing values for all matching variables in each group.

Matched (Unmatched)

This is the number of subjects that were matched (unmatched) from each group.

Percent Matched (Percent Unmatched)

This is the percent of subjects that were matched (unmatched) from each group.

Group Comparison Reports

| Group Comparison Report for Variable = Logit(Propensity) | | | | | | |
|--|-------------|----|----------|-------|-----------------|-----------------------------|
| Group Type | Exposure | N | Mean | SD | Mean Difference | Standardized Difference (%) |
| Before Matching | Exposed | 8 | -0.18344 | 1.39 | -2.81410 | -160.32% |
| | Not Exposed | 22 | 2.63066 | 2.06 | | |
| After Matching | Exposed | 8 | -0.18344 | 1.39 | -1.00503 | -73.88% |
| | Not Exposed | 8 | 0.82159 | 1.33 | | |
| Group Comparison Report for Variable = X1 | | | | | | |
| Group Type | Exposure | N | Mean | SD | Mean Difference | Standardized Difference (%) |
| Before Matching | Exposed | 8 | 39.50000 | 20.96 | -6.40909 | -27.07% |
| | Not Exposed | 22 | 45.90909 | 26.11 | | |
| After Matching | Exposed | 8 | 39.50000 | 20.96 | 13.00000 | 73.58% |
| | Not Exposed | 8 | 26.50000 | 13.60 | | |
| . | | | | | | |
| . | | | | | | |
| . | | | | | | |

(output reports continue for each covariate variable specified)

This report provides summary statistics by group for the data in the propensity score variable and each covariate variable both before and after matching. Notice that the matching seemed to improve the balance of the propensity

Data Matching – Optimal and Greedy

scores (Standardized Difference dropped from -160% to -73%) between the treatment and control groups, but worsened the balance for the covariate X1 (Standardized Difference increased from -27% to 73.58%).

Group Type

This specifies whether the summary statistics refer to groups before or after matching.

Group (e.g. Exposure)

This specifies either the treatment or the control group. The title of this column is the grouping variable name (or label).

N

This is the number of non-missing values in each variable by group. If there are missing values in covariates that were not used for matching, then these numbers may be different from the total number of subjects in each group.

Mean

This is the average value for each variable by group.

SD

This is the standard deviation for each variable by group.

Mean Difference

This is the difference between the mean of the treatment group and the mean of the control group.

Standardized Difference (%)

The standardized difference can be used to measure the balance between the treatment and control groups before and after matching. If a variable is balanced, then the standardized difference should be close to zero. The standardized difference is the mean difference as a percentage of the average standard deviation

$$\text{Standardized Difference (\%)} = \frac{100(\bar{x}_{t,p} - \bar{x}_{c,p})}{\sqrt{(s_{t,p}^2 + s_{c,p}^2)/2}}$$

where $\bar{x}_{t,p}$ and $\bar{x}_{c,p}$ are the treatment and control group means for the p^{th} covariate variable, respectively, and $s_{t,p}^2$ and $s_{c,p}^2$ are the treatment and control group sample variances for the p^{th} covariate variable, respectively.

Matching Detail Report

| Matching Detail Report | | | | | | | |
|--|----------------------|-----------------------|------------|----|-----------------------------|------------|----|
| Treatment = "Exposed", Control = "Not Exposed" | | | | | | | |
| Match Number | Mahalanobis Distance | ----- Treatment ----- | | | ----- Matched Control ----- | | |
| | | Row | Propensity | ID | Row | Propensity | ID |
| 1 | 4.32807 | 1 | -1.05541 | A | 8 | 0.30221 | H |
| 2 | 5.05385 | 4 | -0.34801 | D | 22 | -1.28232 | V |
| 3 | 9.07686 | 6 | 2.50671 | F | 16 | 3.28652 | P |
| 4 | 3.99318 | 10 | 0.05650 | J | 24 | 1.73357 | X |
| 5 | 13.85904 | 14 | -1.11718 | N | 28 | -0.07642 | BB |
| 6 | 9.25961 | 19 | -1.31100 | S | 27 | 0.85319 | AA |
| 7 | 5.06011 | 26 | 1.16584 | Z | 29 | 0.72590 | CC |
| 8 | 3.31815 | 30 | -1.36499 | DD | 9 | 1.03004 | I |

This report provides a list of all matches created and important information about each match.

Data Matching – Optimal and Greedy

Match

This is the match number assigned by the program to each match and stored to the database (if a storage variable was specified).

Mahalanobis Distance (Propensity Score |Difference| or Sum of Rank |Differences|)

This is the estimated distance between the treatment and matched control. The column title depends on the distance calculation method selected.

Row

This is the row of the treatment or control subject in the database.

Propensity Score (or first covariate variable)

This is the value of the propensity score (or logit propensity score if ‘Use Logit’ was selected). If no propensity score variable was used in distance calculations, then this is the value of first covariate variable specified. The title of this column is based on the propensity score variable name (or label) or the first covariate variable name (or label).

Data Label (e.g. ID)

This is the identification label of the row in the database. The title of this column is the data label variable name (or label).

Example 2 – Greedy (1:2) Matching using the Propensity Score Difference with Forced Match Variables

Continuing with Example 1, we will now use the greedy matching algorithm to create matches while using race and gender as forced match variables. This will force the algorithm to find control matches for treatments where the gender and race match exactly, i.e., a male can only be matched with a male, and a female can only be matched with a female, etc. Please note that the optimal matching algorithm can also be used with forced match variables, but we use the greedy matching algorithm here to display the incomplete matched-pair sample that results.

You may follow along here by making the appropriate entries or load the completed template **Example 2** by clicking on Open Example Template from the File menu of the Data Matching – Greedy window.

1 Open the Propensity dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Propensity.NCSS**.
- Click **Open**.

2 Open the Data Matching - Greedy window.

- Using the Data or Tools menu or the Procedure Navigator, find and select the **Data Matching - Greedy** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the Data Matching - Greedy window, select the **Variables tab**.
- Enter **Exposure** in the **Grouping Variable** box.
- Enter “**Exposed**” (no quotes) in the **Treatment Group** box.
- Enter **Propensity** in the **Propensity Score Variable** box.
- Make sure **Use Logit** is checked.
- Enter **Race-Gender** in the **Forced Match Variable(s)** box.
- Enter **X1-Age** in the **Covariate Variable(s)** box.
- Enter **ID** in the **Data Label Variable** box.
- Enter **C11** in the **Store Match Numbers In** box.
- Choose **Propensity Score Difference** in the **Distance Calculation Method** box.
- Enter **2** in the **Matches per Treatment** box.
- Leave all other options at their default values.

4 Specify the reports.

- On the Data Matching - Greedy window, select the **Reports tab**.
- Put a check mark next to **Matching Detail Report** and **Incomplete Matching Report**. Leave all other options at their default values.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Greedy Data Matching Output

Data Summary Report

```

Rows Read                30
Rows with Missing Data   0
Treatment Rows           8
Control Rows             22

---- Data Variables ----
Grouping Variable        Exposure
- Treatment Group       "Exposed"
- Control Group         "Not Exposed"
Data Label Variable      ID

---- Variables Used in Distance Calculations ----
Propensity Score Variable Logit(Propensity)
Forced Match Variable 1   Race
Forced Match Variable 2   Gender

---- Storage Variable ----
Match Number Storage Variable C11
    
```

Matching Summary Report

```

Distance Calculation Method Propensity Score Difference
Order for Matching          Sorted by Distance
Controls Matched per Treatment 2
Sum of Match Propensity Score Differences 14.63954
Average Match Propensity Score Difference 1.46395
    
```

| Exposure | N | Matched | Percent Matched | Unmatched | Percent Unmatched |
|-------------|----|---------|-----------------|-----------|-------------------|
| Exposed | 8 | 6 | 75.00% | 2 | 25.00% |
| Not Exposed | 22 | 10 | 45.45% | 12 | 54.55% |

Group Comparison Report for Variable = Logit(Propensity)

| Group Type | Exposure | N | Mean | SD | Mean Difference | Standardized Difference (%) |
|-----------------|-------------|----|----------|------|-----------------|-----------------------------|
| Before Matching | Exposed | 8 | -0.18344 | 1.39 | -2.81410 | -160.32% |
| | Not Exposed | 22 | 2.63066 | 2.06 | | |
| After Matching | Exposed | 6 | 0.11751 | 1.50 | -1.36296 | -89.21% |
| | Not Exposed | 10 | 1.48046 | 1.55 | | |

Group Comparison Report for Variable = X1

| Group Type | Exposure | N | Mean | SD | Mean Difference | Standardized Difference (%) |
|-----------------|-------------|----|----------|-------|-----------------|-----------------------------|
| Before Matching | Exposed | 8 | 39.50000 | 20.96 | -6.40909 | -27.07% |
| | Not Exposed | 22 | 45.90909 | 26.11 | | |
| After Matching | Exposed | 6 | 33.66667 | 20.79 | -11.23333 | -42.97% |
| | Not Exposed | 10 | 44.90000 | 30.57 | | |

.
.

.

(output reports continue for each covariate variable specified)

Data Matching – Optimal and Greedy

Matching Detail Report
Treatment = "Exposed", Control = "Not Exposed"

| Match Number | Logit | ----- Treatment ----- | | | ----- Matched Control ----- | | |
|--------------|-------------------------|-----------------------|------------------|----|-----------------------------|------------------|----|
| | Propensity [Difference] | Row | Logit Propensity | ID | Row | Logit Propensity | ID |
| 1 | 1.20120 | 4 | -0.34801 | D | 27 | 0.85319 | AA |
| 1 | 1.37805 | 4 | -0.34801 | D | 9 | 1.03004 | I |
| 2 | 0.15966 | 6 | 2.50671 | F | 15 | 2.34705 | O |
| 2 | 0.56240 | 6 | 2.50671 | F | 11 | 1.94431 | K |
| 3 | 0.66941 | 10 | 0.05650 | J | 29 | 0.72590 | CC |
| 3 | 4.46221 | 10 | 0.05650 | J | 2 | 4.51870 | B |
| 4 | 1.61321 | 19 | -1.31100 | S | 8 | 0.30221 | H |
| 4 | 3.92267 | 19 | -1.31100 | S | 12 | 2.61167 | L |
| 5 | 0.58806 | 26 | 1.16584 | Z | 23 | 1.75390 | W |
| 6 | 0.08266 | 30 | -1.36499 | DD | 22 | -1.28232 | V |

Notice that only the propensity score variable was used in distance calculations, but group comparison reports were generated for each covariate variable specified. In the Matching Detail Report, you can see that not all treatments were matched (incomplete matching). Finally, notice that race and gender were both used as Forced Match variables.

If you go back to the spreadsheet and sort the data on C11 (click on **Data > Sort** from the **NCSS Home** window), you will notice that matches were only created where the race and gender were identical for both the treatment and control.

| | Exposure | ID | X1 | X2 | X3 | X4 | X5 | X6 | Age | Race | Gender | Propensity | C11 |
|----|-------------|----|----|-----|-----|----|----|----|-----|-----------|--------|------------|-----|
| 11 | Not Exposed | U | 64 | 48 | 48 | 59 | 56 | 50 | 60 | Hispanic | Female | 0.05299638 | |
| 12 | Not Exposed | X | 13 | 25 | 35 | 8 | 6 | 9 | 52 | Hispanic | Female | 0.15013131 | |
| 13 | Not Exposed | Y | 30 | 100 | 105 | 53 | 60 | 94 | 72 | Black | Female | 0.00267398 | |
| 14 | Not Exposed | BB | 20 | 36 | 44 | 17 | 16 | 22 | 33 | Black | Female | 0.51909545 | |
| 15 | Exposed | D | 31 | 81 | 86 | 46 | 50 | 74 | 33 | Hispanic | Female | 0.58613607 | 1 |
| 16 | Not Exposed | I | 46 | 36 | 40 | 39 | 36 | 32 | 39 | Hispanic | Female | 0.26307673 | 1 |
| 17 | Not Exposed | AA | 31 | 22 | 28 | 21 | 17 | 11 | 60 | Hispanic | Female | 0.2987649 | 1 |
| 18 | Exposed | F | 22 | 30 | 39 | 17 | 15 | 17 | 29 | Black | Female | 0.07538899 | 2 |
| 19 | Not Exposed | K | 84 | 86 | 82 | 90 | 91 | 97 | 24 | Black | Female | 0.12517537 | 2 |
| 20 | Not Exposed | O | 82 | 79 | 75 | 86 | 86 | 89 | 36 | Black | Female | 0.08730072 | 2 |
| 21 | Not Exposed | B | 4 | 2 | 5 | 11 | 11 | 5 | 71 | Hispanic | Male | 0.01078557 | 3 |
| 22 | Exposed | J | 3 | 38 | 49 | 5 | 6 | 19 | 58 | Hispanic | Male | 0.48587995 | 3 |
| 23 | Not Exposed | CC | 15 | 35 | 44 | 14 | 13 | 20 | 65 | Hispanic | Male | 0.32609454 | 3 |
| 24 | Not Exposed | H | 31 | 91 | 96 | 50 | 56 | 85 | 52 | Caucasian | Male | 0.4250166 | 4 |
| 25 | Not Exposed | L | 73 | 88 | 86 | 83 | 85 | 96 | 63 | Caucasian | Male | 0.06839106 | 4 |
| 26 | Exposed | S | 64 | 36 | 36 | 53 | 49 | 37 | 47 | Caucasian | Male | 0.78768042 | 4 |
| 27 | Not Exposed | W | 71 | 5 | 5 | 45 | 37 | 7 | 53 | Caucasian | Female | 0.14755618 | 5 |
| 28 | Exposed | Z | 36 | 67 | 71 | 44 | 46 | 61 | 62 | Caucasian | Female | 0.23760766 | 5 |
| 29 | Not Exposed | V | 12 | 63 | 72 | 23 | 26 | 48 | 29 | Caucasian | Female | 0.78284523 | 6 |
| 30 | Exposed | DD | 46 | 60 | 63 | 49 | 49 | 57 | 28 | Caucasian | Female | 0.79656871 | 6 |
| 31 | | | | | | | | | | | | | |

Incomplete Matching Report

Incomplete Matching Report
Exposure = "Exposed"

| Treatment Row | Matches (Target = 2) | Logit Propensity | ID |
|---------------|----------------------|------------------|----|
| 1 | 0 | -1.05541 | A |
| 14 | 0 | -1.11718 | N |
| 26 | 1 | 1.16584 | Z |
| 30 | 1 | -1.36499 | DD |

This report lists the treatments that were not paired with the target number of controls (2 in this case). Rows 1 and 14 were not paired with any controls. Rows 26 and 30 were only paired with 1 control. All other treatment rows

Data Matching – Optimal and Greedy

were paired with 2 treatments. Incomplete matching is usually due to the use of forced match variables, using caliper matching, or setting Matches per Treatment to ‘Maximum Possible’.

Treatment Row

This is the row in the database containing the treatment subject that was not fully matched.

Matches (Target = k)

This is the number of matches that were found for each treatment. The target represents the number of Matches per Treatment specified on the input window.

Propensity Score (or first covariate variable)

This is the value of the propensity score (or logit propensity score if ‘Use Logit’ was selected) for the incompletely matched treatment. If no propensity score variable was used in distance calculations, then this is the value of first covariate variable specified. The title of this column is based on the propensity score variable name (or label) or the first covariate variable name (or label).

Data Label (e.g. ID)

This is the identification label of the incompletely matched row in the database. The title of this column is the data label variable name (or label).

Example 3 – Matching on Forced Match Variables Only

Continuing with Example 2, suppose we wanted to form matches based solely on forced match variables, i.e., we want the matches to have exactly the same values for each covariate. We could enter all of the covariates in as forced match variables, but with a database as small as we are using, we are unlikely to find any matches. We will use the greedy data matching procedure to illustrate how you can assign matches based on the gender and race forced match variables only. Random ordering is used to ensure that the treatments are randomly paired with controls (where the forced match variable values match).

In order to complete this task, you must first create a new column in the database filled with 1's. You can do this by clicking on the first cell in an empty column and selecting **Edit > Fill** from the **NCSS Home** window (for **Fill Value(s)** enter **1**, for **Increment** enter **0**, and click **OK**). A column of ones has already been created for you in the Propensity dataset. This column of ones is necessary because the matching procedure requires either a propensity score variable or a covariate variable to run.

You may follow along here by making the appropriate entries or load the completed template **Example 3** by clicking on Open Example Template from the File menu of the Data Matching – Greedy window.

1 Open the Propensity dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Propensity.NCSS**.
- Click **Open**.

2 Open the Data Matching - Greedy window.

- Using the Data or Tools menu or the Procedure Navigator, find and select the **Data Matching - Greedy** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the Data Matching - Greedy window, select the **Variables tab**.
- Enter **Exposure** in the **Grouping Variable** box.
- Enter **“Exposed”** (no quotes) in the **Treatment Group** box.
- Enter **Ones** (or the name of your variable containing all 1's) in the **Propensity Score Variable** box.
- Make sure **Use Logit** is **unchecked**.
- Enter **Race-Gender** in the **Forced Match Variable(s)** box.
- Enter **ID** in the **Data Label Variable** box.
- Enter **C11** in the **Store Match Numbers In** box.
- Choose **Propensity Score Difference** in the **Distance Calculation Method** box.
- Enter **2** in the **Matches per Treatment** box.
- Choose **Random** in the **Order for Matching** box.
- Leave all other options at their default values.

4 Specify the reports.

- On the Data Matching - Greedy window, select the **Reports tab**.
- Put a check mark next to **Matching Detail Report** and **Incomplete Matching Report**. Leave all other options at their default values.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Matching Reports

Matching Detail Report

Treatment = "Exposed", Control = "Not Exposed"

| Match Number | Logit | ----- Treatment ----- | | | ----- Matched Control ----- | | |
|--------------|-------------------------|-----------------------|------------|----|-----------------------------|------------|----|
| | Propensity [Difference] | Row | Propensity | ID | Row | Propensity | ID |
| 1 | 0.00000 | 1 | 1.00000 | A | 29 | 1.00000 | CC |
| 2 | 0.00000 | 4 | 1.00000 | D | 13 | 1.00000 | M |
| 2 | 0.00000 | 4 | 1.00000 | D | 20 | 1.00000 | T |
| 3 | 0.00000 | 6 | 1.00000 | F | 15 | 1.00000 | O |
| 3 | 0.00000 | 6 | 1.00000 | F | 11 | 1.00000 | K |
| 4 | 0.00000 | 10 | 1.00000 | J | 2 | 1.00000 | B |
| 5 | 0.00000 | 19 | 1.00000 | S | 12 | 1.00000 | L |
| 5 | 0.00000 | 19 | 1.00000 | S | 8 | 1.00000 | H |
| 6 | 0.00000 | 26 | 1.00000 | Z | 23 | 1.00000 | W |
| 7 | 0.00000 | 30 | 1.00000 | DD | 22 | 1.00000 | V |

Incomplete Matching Report

Exposure = "Exposed"

| Treatment Row | Matches (Target = 2) | Logit Propensity | ID |
|---------------|----------------------|------------------|----|
| 1 | 1 | 1.00000 | A |
| 10 | 1 | 1.00000 | J |
| 14 | 0 | 1.00000 | N |
| 26 | 1 | 1.00000 | Z |
| 30 | 1 | 1.00000 | DD |

The matching detail report is not very informative because all of the propensity scores are equal to 1. If you run the procedure several times, you will notice that the controls are randomly pairing with the treatments when the race and gender are the same. Your report may be slightly different from this report because random ordering was used. If you sort on C11, you will see that all matched pairs have the same value for race and gender.

| | | | | | | | | | | | | | |
|----|-------------|----|----|-----|-----|----|----|-----|----|-----------|--------|------------|---|
| 13 | Exposed | Z | 36 | 67 | 71 | 44 | 46 | 61 | 62 | Caucasian | Female | 0.23760766 | |
| 14 | Exposed | A | 50 | 102 | 103 | 70 | 75 | 102 | 45 | Hispanic | Male | 0.74181165 | 1 |
| 15 | Not Exposed | CC | 15 | 35 | 44 | 14 | 13 | 20 | 65 | Hispanic | Male | 0.32609454 | 1 |
| 16 | Exposed | N | 64 | 1 | 2 | 38 | 29 | 0 | 39 | Caucasian | Female | 0.75346448 | 2 |
| 17 | Not Exposed | W | 71 | 5 | 5 | 45 | 37 | 7 | 53 | Caucasian | Female | 0.14755618 | 2 |
| 18 | Exposed | D | 31 | 81 | 86 | 46 | 50 | 74 | 33 | Hispanic | Female | 0.58613607 | 3 |
| 19 | Not Exposed | I | 46 | 36 | 40 | 39 | 36 | 32 | 39 | Hispanic | Female | 0.26307673 | 3 |
| 20 | Not Exposed | AA | 31 | 22 | 28 | 21 | 17 | 11 | 60 | Hispanic | Female | 0.2987649 | 3 |

Example 4 – Validation of the Optimal Data Matching Algorithm using Rosenbaum (1989)

Rosenbaum (1989) provides an example of both optimal and greedy matching using a well-known dataset from Cox and Snell (1981), which involves 26 U.S. light water nuclear power plants (six “partial turnkey” plants are excluded in the analysis). Seven of the plants were constructed on sites where a light water reactor had existed previously; these are the treatments. The 19 remaining plants serve as the controls. The sum of rank differences was used to calculate distances between treatment and control plants. Two covariate variables were used in the analysis: the date the construction permit was issued (Date), and the capacity of the plant (Capacity). Site was used as the grouping variable with “Existing” as the treatment group. Rosenbaum (1989) reports the following optimal pairings by plant number (treatment, control):

(3,2), (3,21), (5,4), (5,7), (9,7), (9,10), (18,8), (18,13), (20,14), (20,15), (22,17), (22,26), (24,23), (24,25)

The data used in this example are contained in the CoxSnell dataset.

You may follow along here by making the appropriate entries or load the completed template **Example 4** by clicking on Open Example Template from the File menu of the Data Matching – Optimal window.

1 Open the CoxSnell dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **CoxSnell.NCSS**.
- Click **Open**.

2 Open the Data Matching - Optimal window.

- Using the Data or Tools menu or the Procedure Navigator, find and select the **Data Matching - Optimal** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the Data Matching - Optimal window, select the **Variables tab**.
- Enter **Site** in the **Grouping Variable** box.
- Enter “**Existing**” (no quotes) in the **Treatment Group** box.
- Make sure nothing is entered in the **Propensity Score Variable** box.
- Enter **Date-Capacity** in the **Covariate Variable(s)** box.
- Enter **Plant** in the **Data Label Variable** box.
- Choose **Sum of Rank Differences including the Propensity Score (if specified)** in the **Distance Calculation Method** box.
- Enter **2** in the **Matches per Treatment** box.
- Leave all other options at their default values.

4 Specify the reports.

- On the Data Matching - Optimal window, select the **Reports tab**.
- Put a check mark next to **Matching Detail Report**. Leave all other options at their default values.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Matching Reports

Matching Summary Report

| | |
|--------------------------------|--|
| Distance Calculation Method | Sum of Rank Differences including the Propensity Score |
| Order for Matching | Random |
| Controls Matched per Treatment | 2 |
| Sum of Match Rank Differences | 74.00000 |
| Average Match Rank Difference | 5.28571 |

| Site | N | Matched | Percent Matched | Unmatched | Percent Unmatched |
|----------|----|---------|-----------------|-----------|-------------------|
| Existing | 7 | 7 | 100.00% | 0 | 0.00% |
| New | 19 | 14 | 73.68% | 5 | 26.32% |

Matching Detail Report

Treatment = "Existing", Control = "New"

| Match Number | Sum of Rank Differences | ----- Treatment ----- | | | ----- Matched Control ----- | | |
|--------------|--------------------------|-----------------------|---------|-------|-----------------------------|---------|-------|
| | | Row | Date | Plant | Row | Date | Plant |
| 1 | 18.50000 | 1 | 2.33000 | 3 | 23 | 3.75000 | 21 |
| 1 | 0.00000 | 1 | 2.33000 | 3 | 9 | 2.33000 | 2 |
| 2 | 0.00000 | 2 | 3.00000 | 5 | 10 | 3.00000 | 4 |
| 2 | 10.50000 | 2 | 3.00000 | 5 | 12 | 3.17000 | 7 |
| 3 | 5.50000 | 3 | 3.42000 | 9 | 20 | 3.42000 | 16 |
| 3 | 5.50000 | 3 | 3.42000 | 9 | 14 | 3.33000 | 10 |
| 4 | 0.00000 | 4 | 3.42000 | 18 | 17 | 3.42000 | 13 |
| 4 | 2.50000 | 4 | 3.42000 | 18 | 13 | 3.42000 | 8 |
| 5 | 0.00000 | 5 | 3.92000 | 20 | 18 | 3.92000 | 14 |
| 5 | 2.50000 | 5 | 3.92000 | 20 | 19 | 3.92000 | 15 |
| 6 | 12.00000 | 6 | 5.92000 | 22 | 26 | 6.08000 | 26 |
| 6 | 5.00000 | 6 | 5.92000 | 22 | 21 | 4.50000 | 17 |
| 7 | 4.00000 | 7 | 5.08000 | 24 | 24 | 4.67000 | 23 |
| 7 | 8.00000 | 7 | 5.08000 | 24 | 25 | 5.42000 | 25 |

The optimal match-pairings found by **NCSS** match those in Rosenbaum (1989) exactly. Notice, however, that the distances (Sum of Rank |Differences|) are slightly different in some instances from those given in Table 1 of the article. This is due to the fact that Rosenbaum (1989) rounds all non-integer distances in their reports. This rounding also affects the overall sum of match rank differences; **NCSS** calculates the overall sum as 74, while Rosenbaum (1989) calculates the overall sum as 71, with the difference due to rounding.

Example 5 – Validation of the Greedy Data Matching Algorithm using Rosenbaum (1989)

Continuing with Example 4, Rosenbaum (1989) also reports the results from the greedy matching algorithm, where the order for matching is sorted by distance. The article reports the following greedy pairings by plant number (treatment, control):

(3,2), (3,19), (5,4), (5,21), (9,10), (9,7), (18,8), (18,13), (20,14), (20,15), (22,17), (22,26), (24,23), (24,25)

You may follow along here by making the appropriate entries or load the completed template **Example 5** by clicking on Open Example Template from the File menu of the Data Matching – Greedy window.

1 Open the CoxSnell dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **CoxSnell.NCSS**.
- Click **Open**.

2 Open the Data Matching - Greedy window.

- Using the Data or Tools menu or the Procedure Navigator, find and select the **Data Matching - Greedy** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the Data Matching - Greedy window, select the **Variables tab**.
- Enter **Site** in the **Grouping Variable** box.
- Enter “**Existing**” (no quotes) in the **Treatment Group** box.
- Make sure nothing is entered in the **Propensity Score Variable** box.
- Enter **Date-Capacity** in the **Covariate Variable(s)** box.
- Enter **Plant** in the **Data Label Variable** box.
- Choose **Sum of Rank Differences including the Propensity Score (if specified)** in the **Distance Calculation Method** box.
- Enter **2** in the **Matches per Treatment** box.
- Leave all other options at their default values.

4 Specify the reports.

- On the Data Matching - Greedy window, select the **Reports tab**.
- Put a check mark next to **Matching Detail Report**. Leave all other options at their default values.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Data Matching – Optimal and Greedy

Output

Matching Summary Report

Distance Calculation Method Sum of Rank Differences including the Propensity Score
 Order for Matching Sorted by Distance
 Controls Matched per Treatment 2
 Sum of Match Rank Differences 80.00000
 Average Match Rank Difference 5.71429

| Site | N | Matched | Percent Matched | Unmatched | Percent Unmatched |
|----------|----|---------|-----------------|-----------|-------------------|
| Existing | 7 | 7 | 100.00% | 0 | 0.00% |
| New | 19 | 14 | 73.68% | 5 | 26.32% |

Matching Detail Report

Treatment = "Existing", Control = "New"

| Match Number | Sum of Rank Differences | ----- Treatment ----- | | | ----- Matched Control ----- | | |
|--------------|--------------------------|-----------------------|---------|-------|-----------------------------|---------|-------|
| | | Row | Date | Plant | Row | Date | Plant |
| 1 | 0.00000 | 1 | 2.33000 | 3 | 9 | 2.33000 | 2 |
| 1 | 21.00000 | 1 | 2.33000 | 3 | 22 | 4.17000 | 19 |
| 2 | 0.00000 | 2 | 3.00000 | 5 | 10 | 3.00000 | 4 |
| 2 | 15.50000 | 2 | 3.00000 | 5 | 23 | 3.75000 | 21 |
| 3 | 4.00000 | 3 | 3.42000 | 9 | 12 | 3.17000 | 7 |
| 3 | 5.50000 | 3 | 3.42000 | 9 | 14 | 3.33000 | 10 |
| 4 | 0.00000 | 4 | 3.42000 | 18 | 17 | 3.42000 | 13 |
| 4 | 2.50000 | 4 | 3.42000 | 18 | 13 | 3.42000 | 8 |
| 5 | 0.00000 | 5 | 3.92000 | 20 | 18 | 3.92000 | 14 |
| 5 | 2.50000 | 5 | 3.92000 | 20 | 19 | 3.92000 | 15 |
| 6 | 5.00000 | 6 | 5.92000 | 22 | 21 | 4.50000 | 17 |
| 6 | 12.00000 | 6 | 5.92000 | 22 | 26 | 6.08000 | 26 |
| 7 | 4.00000 | 7 | 5.08000 | 24 | 24 | 4.67000 | 23 |
| 7 | 8.00000 | 7 | 5.08000 | 24 | 25 | 5.42000 | 25 |

The greedy match-pairings found by NCSS match those in Rosenbaum (1989) exactly. Again, some of the distances are different from those in Table 1 of the article because of rounding. NCSS calculates the overall sum of rank differences as 80, while Rosenbaum (1989) calculates the overall sum as 79 with the difference due to rounding.