

Chapter 241

Individuals and Moving Range Charts

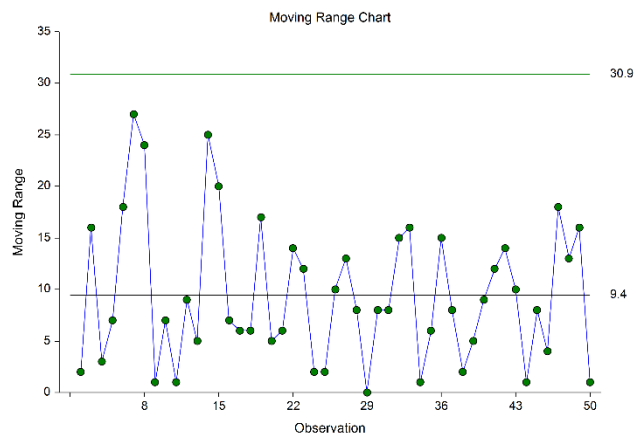
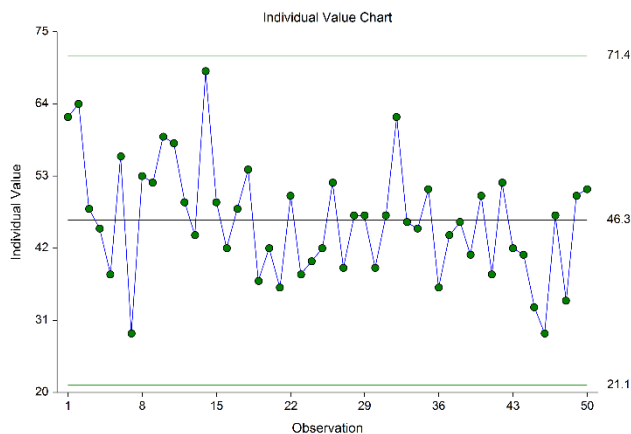
Introduction

This procedure generates individual value and moving range control charts. The format of the control charts is fully customizable. This procedure permits the defining of stages. For the individuals chart, the center line can be entered directly or estimated from the data, or a subset of the data. Similarly, sigma may be estimated from the data or a standard sigma value may be entered. A list of out-of-control points can be produced in the output, if desired, and ranges may be stored to the spreadsheet.

Individuals and Moving Range Control Charts

Individuals and moving range charts are used to monitor individual values and the variation of a process based on samples taken from a process over time (hours, shifts, days, weeks, months, etc.). Typically, an initial series of observations is used to estimate the mean and standard deviation of a process. The mean and standard deviation are then used to produce control limits for the individual values and ranges. During this initial phase, the process should be in control. If points are out-of-control during the initial (estimation) phase, the assignable cause should be determined, and the observation should be removed from estimation.

Once the control limits have been established of the individuals and moving range charts, these limits may be used to monitor the values and variation of the process going forward. When a point is outside these established control limits it indicates that the mean (or variation) of the process is out-of-control. An assignable cause is suspected whenever the control chart indicates an out-of-control process.



Other Control Charts for the Individuals

Two additional control charts available for monitoring the process mean for individual observations are the cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts. The CUSUM and EWMA charts differ from the individuals charts in that they take into account the information of previous observations rather than just the current value. The CUSUM and EWMA charts are more sensitive to smaller shifts in the mean since they use the cumulative information of the sequence of values. However, CUSUM and EWMA methods also assume a reliable estimate or known value for the true standard deviation is available.

Control Chart Formulas

Suppose we have k observations from a process. Let x_i represent the measurement of the i^{th} individual.

Formulas for the Points on the Chart

The value for each point of the individuals chart is simply the measurement value x_i .

Each moving range is calculated as

$$R_i = |x_i - x_{i-1}|,$$

which is the absolute value of each value minus the previous value.

Estimating the Individuals Chart Center Line

In the Individuals Charts procedure, the average value may be input directly, or it may be estimated from a series of values. If it is estimated from the values, the formula for the average is

$$\bar{x} = \frac{\sum_{i=1}^k x_i}{k}.$$

Estimating Sigma

The true standard deviation (sigma) may be input directly, or it may be estimated from the moving ranges by

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

where

$$\bar{R} = \frac{\sum_{i=2}^k R_i}{k-1}$$

$$d_2 = \frac{E(R)}{\sigma} = \frac{\mu_R}{\sigma}$$

Individuals and Moving Range Charts

The calculation of $E(R)$ requires the knowledge of the underlying distribution of the x_i 's. Making the assumption that the x_i 's follow the normal distribution with constant mean and variance, the values for d_2 are derived through the use of numerical integration. It is important to note that the normality assumption is used and that the accuracy of this estimate requires that this assumption be valid.

Individuals Chart Limits

The lower and upper control limits for the individuals chart are calculated using the formulas

$$LCL = \bar{x} - m\hat{\sigma}$$

$$UCL = \bar{x} + m\hat{\sigma}$$

where m is a multiplier (usually set to 3) chosen to control the likelihood of false alarms (out-of-control signals when the process is in control).

Estimating the Moving Range Chart Center Line

If a standard sigma (standard deviation) value is entered by the user, the Moving Range Chart center line is computed using

$$\bar{R} = d_2\sigma$$

If the standard deviation is estimated from a series of values, the R chart center line is given by

$$\bar{R} = \frac{\sum_{i=2}^k R_i}{k-1}$$

Moving Range Chart Limits

The lower and upper control limits for the Moving Range chart are calculated using the formula

$$LCL = \bar{R} - md_3\hat{\sigma}$$

$$UCL = \bar{R} + md_3\hat{\sigma}$$

where m is a multiplier (usually set to 3) chosen to control the likelihood of false alarms, and d_3 is a constant that is calculated by numerical integration and is based on the assumption of normality. The relation for d_3 is

$$d_3 = \frac{\sigma_R}{\sigma}$$

Runs Tests

The strength of control charts comes from their ability to detect sudden changes in a process that result from the presence of assignable causes. Unfortunately, the individuals chart is poor at detecting drifts (gradual trends) or small shifts in the process. For example, there might be a positive trend in the last ten subgroups, but until a mean goes above the upper control limit, the chart gives no indication that a change has taken place in the process.

Runs tests can be used to check control charts for unnatural patterns that are most likely caused by assignable causes. Runs tests are sometimes called “pattern tests”, “out-of-control” tests, or “zones rules”.

While runs tests may be helpful in identifying patterns or smaller shifts in the mean, they also increase the likelihood of false positive indications. The rate of false positives is typically measured using the average run length (the average length of a run before a false positive is indicated). When several runs tests are used the average run length of the control chart becomes very short. Two alternatives to consider before using runs tests are the CUSUM and EWMA control charts. Runs tests are generally advised against when there is only one observation per subgroup. In this case, the rate of false positives is quite high (average run length is short).

In order to perform the runs tests, the control chart is divided into six equal zones (three on each side of the centerline). Since the control limit is three sigma limits (three standard deviations of the mean) in width, each zone is one sigma wide and is labeled A, B, or C, with the C zone being the closest to the centerline. There is a lower zone A and an upper zone A. The same is true for B and C. The runs tests look at the pattern in which points fall in these zones.

The runs tests used in this procedure are described below.

Test 1: Any Single Point Beyond Zone A

This runs test simply indicates a single point is beyond one of the two three-sigma limits.

Test 2: Two of Three Successive Points in Zone A or Beyond

This usually indicates a shift in the process average. Note that the two points have to be in the same Zone A, upper or lower. They cannot be on both sides of the centerline. The third point can be anywhere.

Test 3: Four of Five Successive Points in Zone B or Beyond

This usually indicates a shift in the process average. Note that the odd point can be anywhere.

Test 4: Eight Successive Points in Zone C or Beyond

All eight points must be on one side of the centerline. This is another indication of a shift in the process average.

Test 5: Fifteen Successive Points Fall in Zone C on Either Side of the Centerline

Although this pattern might make you think that the variation in your process has suddenly decreased, this is usually not the case. It is usually an indication of stratification in the sample. This happens when the samples come from two distinct distributions having different means. Perhaps there are two machines that are set differently. Try to isolate the two processes and check each one separately.

Test 6: Eight of Eight Successive Points Outside of Zone C

This usually indicates a mixture of processes. This can happen when two supposedly identical production lines feed a single production or assembly process. You must separate the processes to find and correct the assignable cause.

There are, of course, many other sets of runs tests that have been developed. You should watch your data for trends, zig-zags, and other nonrandom patterns. Any of these conditions could be an indication of an assignable cause and would warrant further investigation.

Data Structure

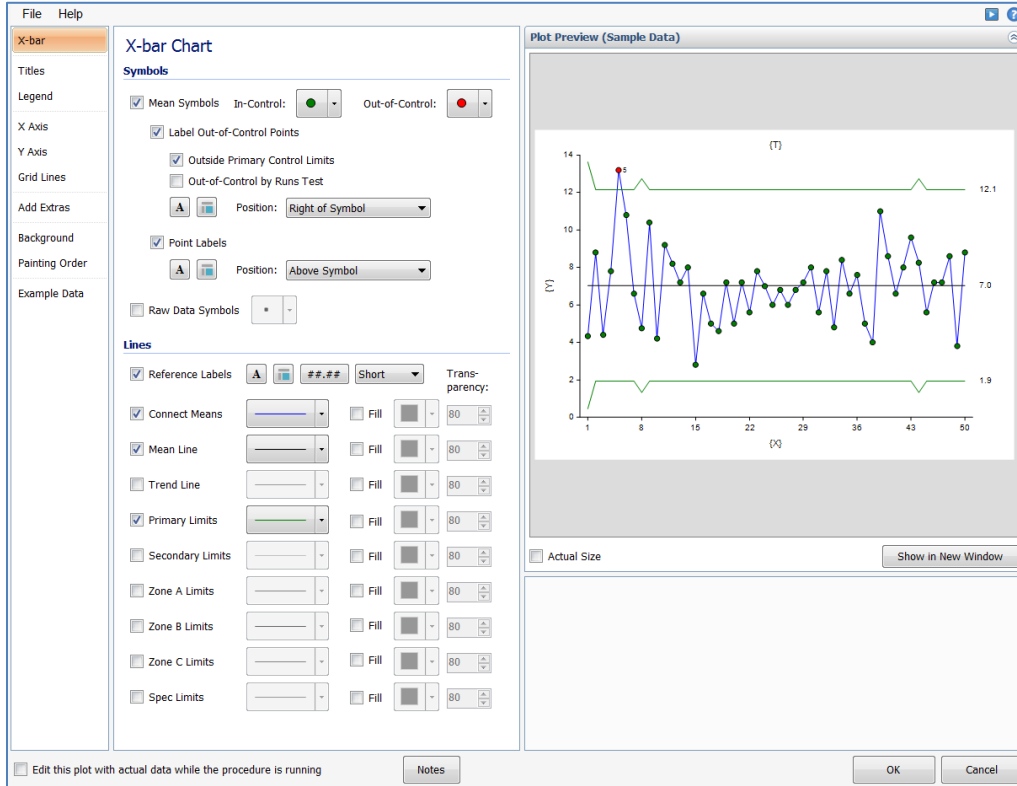
In this procedure, the data should be entered as a single column. If stages are used, a column defining the stage may be used.

Example Dataset

Response	Stage
2	1
6	1
8	2
8	2
9	2
.	.
.	.
.	.

Quality Control Chart Format Window Options

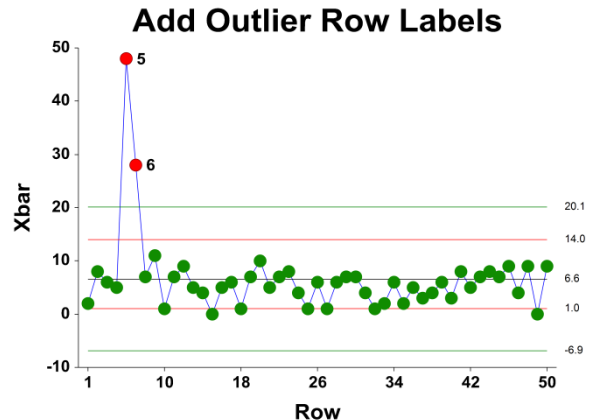
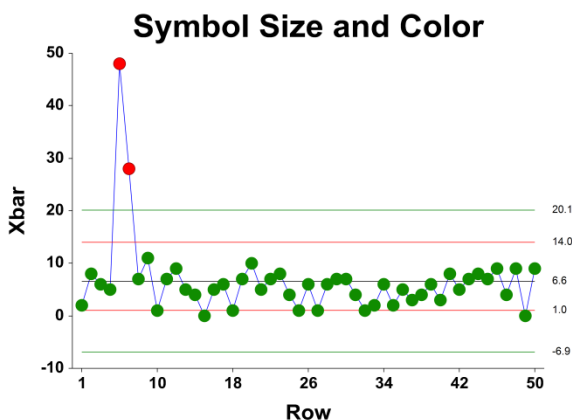
This section describes a few of the specific options available on the first tab of the control chart format window, which is displayed when a quality control chart format button is pressed. Common options, such as axes, labels, legends, and titles are documented in the Graphics Components chapter.



[Xbar] / [Range] Chart Tab

Symbols Section

You can modify the attributes of the symbols using the options in this section.



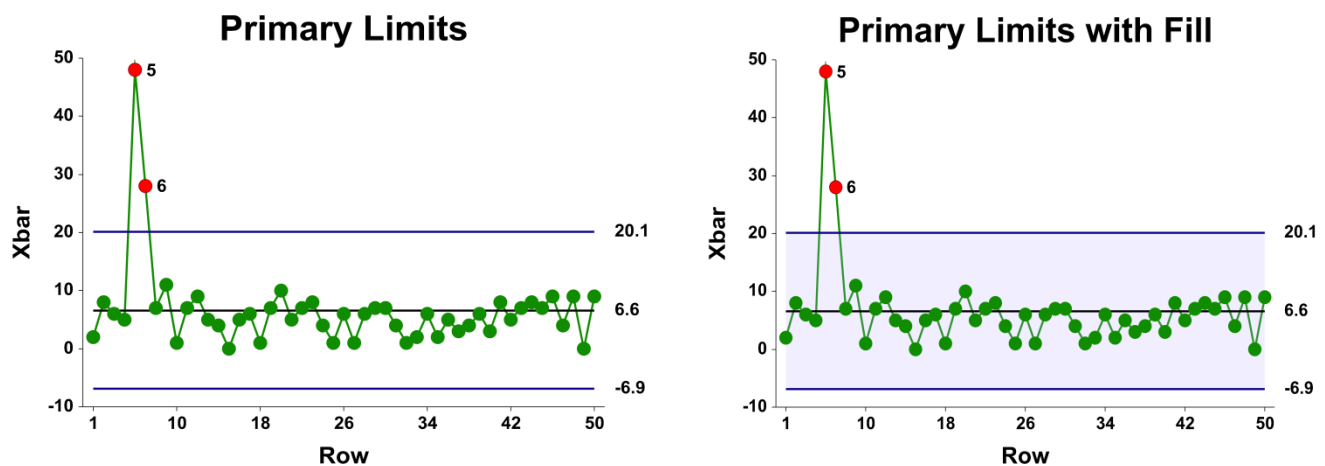
Individuals and Moving Range Charts

A wide variety of sizes, shapes, and colors are available for the symbols. The symbols for in-control and out-of-control points are specified independently. There are additional options to label out-of-control points. The label for points outside the primary control limits is the subgroup number. The label for points that are out-of-control based on the runs test is the number of the first runs test that is signaled by this point.

The user may also specify a column of point labels on the procedure variables tab, to be used to label all or some of the points of the chart. The raw data may also be shown, based on customizable raw data symbols.

Lines Section

You can specify the format of the various lines using the options in this section. Note that when shading is desired, the fill will be to the bottom for single lines (such as the mean line), and between the lines for pairs of lines (such as primary limits).



Lines for the zones, secondary limits, and specification limits are also specified here.

Titles, Legend, Numeric Axis, Group Axis, Grid Lines, and Background Tabs

Details on setting the options in these tabs are given in the Graphics Components chapter. The legend does not show by default but can easily be included by going to the Legend tab and clicking the Show Legend checkbox.

Example 1 – Individuals and Moving Range Charts

This section presents an example of how to produce individuals and moving range charts in NCSS. The data represent 50 ordered width observations. The data used are in the Width dataset. We will analyze the variable Width of this dataset.

Setup

To run this example, complete the following steps:

1 Open the Width example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Width** and click **OK**.

2 Specify the Individuals and Moving Range Charts procedure options

- Find and open the **Individuals and Moving Range Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

```
Variables Tab
-----
Response Variable .....Width
```

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Center Line Section

Center Line Section for Observations 1 to 50	
Number of Observations: 50	
Center Line	Estimate
Estimated Grand Average	46.26
R-bar	9.448979

This section displays the center line values that are to be used in the individuals and moving range charts.

Estimated Grand Average

This value is the average of all the observations.

R-bar

This is the average of the ranges.

Primary Control Limit Section

Primary Control Limit Section for Observations 1 to 50

Chart Type	Primary Control Limits	
	Lower	Upper
Individuals	21.12974	71.39027
Moving Range	0	30.8851

This report gives the lower and upper limits for each of the charts.

Individuals Lower and Upper Primary Control Limits

These limits are the primary control limits of the Individuals chart, as defined in the sub-section Individuals Chart Limits of the Control Chart Formulas section toward the beginning of this chapter.

Moving Range Lower and Upper Primary Control Limits

These limits are the primary control limits of the moving range chart, as defined in the sub-section Moving Range Chart Limits of the Control Chart Formulas section toward the beginning of this chapter. Since the lower limit for the R chart is less than 0, it has been reset to 0.

Sigma Estimation Section

Sigma Estimation Section for Observations 1 to 50

Estimation Type	Estimated Value	Estimated Sigma
Moving Ranges (R-bar)*	9.448979	8.376755

* Indicates the estimation type used in this report.

This report gives the estimation of the population standard deviation (sigma) based on the moving range average.

Estimation Type

The formula for estimating sigma based on the moving ranges is shown earlier in this chapter in the Control Chart Formulas section.

Estimated Value

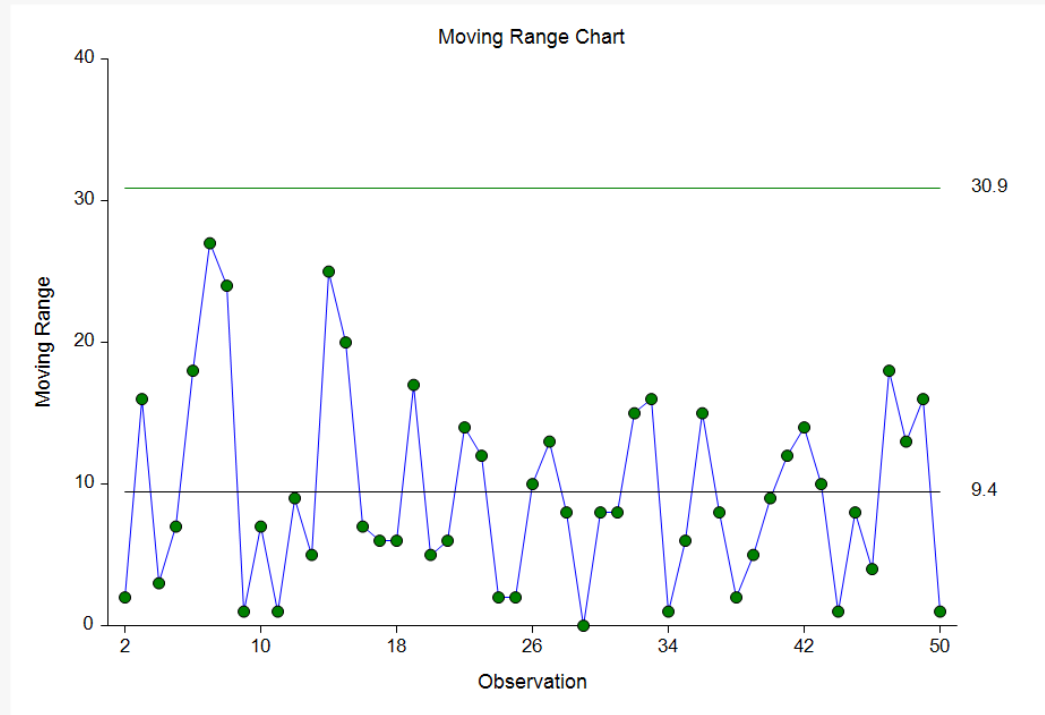
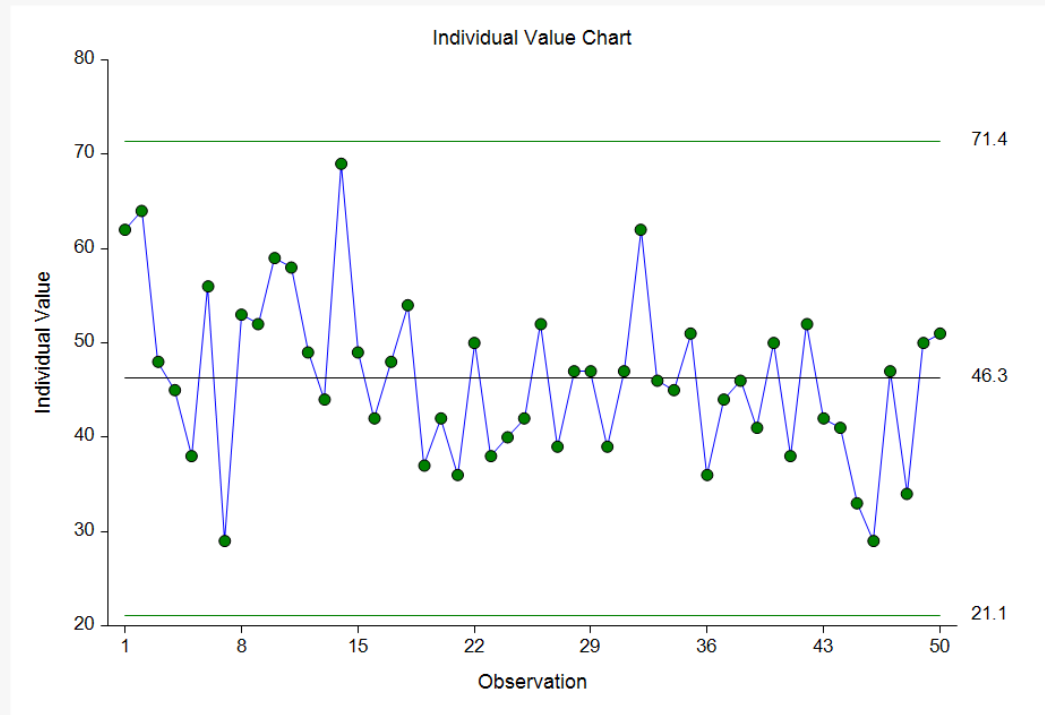
This column gives the R-bar estimate based on the corresponding formulas.

Estimated Sigma

This column gives the estimate of the population standard deviation (sigma) based on moving range estimation.

X-bar and R Charts

Chart Section for Observations 1 to 50



The first plot shows the observation values, as well as the center line and control limits for the process mean, based on the 50 observations. This process appears to be in control. The second plot shows the moving range for each observation (starting with the second observation), as well as the corresponding center line and limits. The Moving Range chart seems to indicate the variation is also in control.

Out-of-Control List

Out-of-Control List for Observations 1 to 50

Obs	Value	Range	Obs Label	Reason
8	53	24	8	Range: 2 of 3 in zone A
9	52	1	9	Range: 2 of 3 in zone A

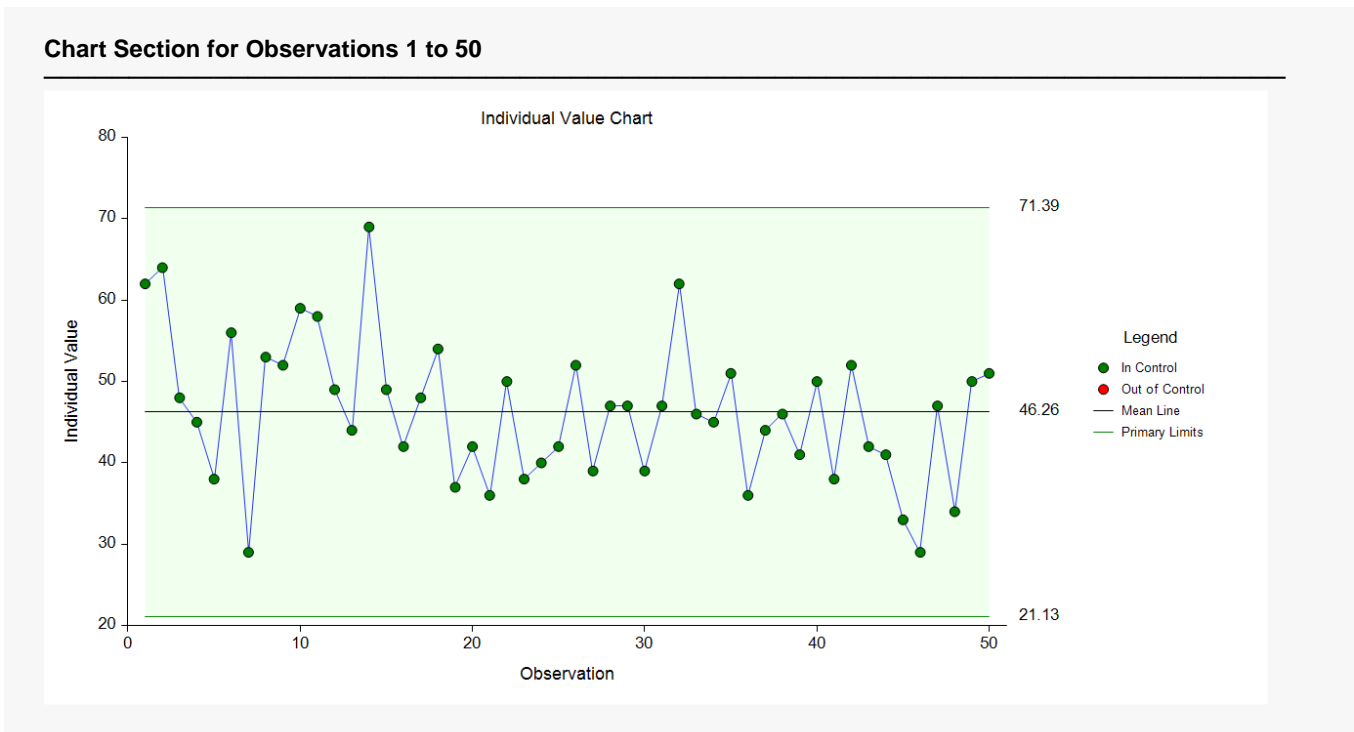
This report provides a list of the observations that failed one of the runs tests (including points outside the control limits). The report shows that observations 8 and 9 are the final points of 2 out of 3 points in Zone A. This run does not appear to indicate a clear shift in the process variation.

Example 2 – Individuals Chart with Additional Formatting

This example uses the same setup as Example 1, except that a variety of improvements are made in the plot format. These improvements are made by clicking the Individuals Chart format button on the I-MR Charts tab.

The settings for this example are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Individuals Chart



As shown here, a variety of enhancements can be made to the formatting of the control charts to make the chart as easy to read as possible.

Example 3 – Individuals and Moving Range Charts with Stages

This section presents Individuals and Moving Range charts for the scenario of multiple stages in a process. In the Width Stage dataset, a significant change had been made in the process after observation 70. This change merits a new evaluation of the process mean and standard deviation, beginning at observation 71. In this example, the process mean and standard deviation for each stage are to be determined based on the first 30 subgroups.

Setup

To run this example, complete the following steps:

1 Open the Width Stage example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Width Stage** and click **OK**.

2 Specify the Individuals and Moving Range Charts procedure options

- Find and open the **Individuals and Moving Range Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

```

Response Variable .....Width
Number of Stages.....Multiple Stages
Stage Variable .....Stage
Specification Method .....Keep Rows Variable
Use all rows where variable.....CalcRows
equals .....1
    
```

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Center Line, Control Limits, and Estimation Sections

Center Line Section for Observations 1 to 70 where CalcRows equals 1

Number of Observations: 30

Center Line	Estimate
Estimated Grand Average	70.63333
R-bar	7.827586

Individuals and Moving Range Charts

Primary Control Limit Section for Observations 1 to 70 where CalcRows equals 1

Chart Type	Primary Control Limits	
	Lower	Upper
Individuals	49.81528	91.45139
Moving Range	0	25.58538

Sigma Estimation Section for Observations 1 to 70 where CalcRows equals 1

Estimation Type	Estimated Value	Estimated Sigma
Moving Ranges (R-bar)*	7.827586	6.93935

* Indicates the estimation type used in this report.

Center Line Section for Observations 71 to 150 where CalcRows equals 1

Number of Observations: 30

Center Line	Estimate
Estimated Grand Average	88.3
R-bar	12.72414

Primary Control Limit Section for Observations 71 to 150 where CalcRows equals 1

Chart Type	Primary Control Limits	
	Lower	Upper
Individuals	54.45921	122.1408
Moving Range	0	41.59033

Sigma Estimation Section for Observations 71 to 150 where CalcRows equals 1

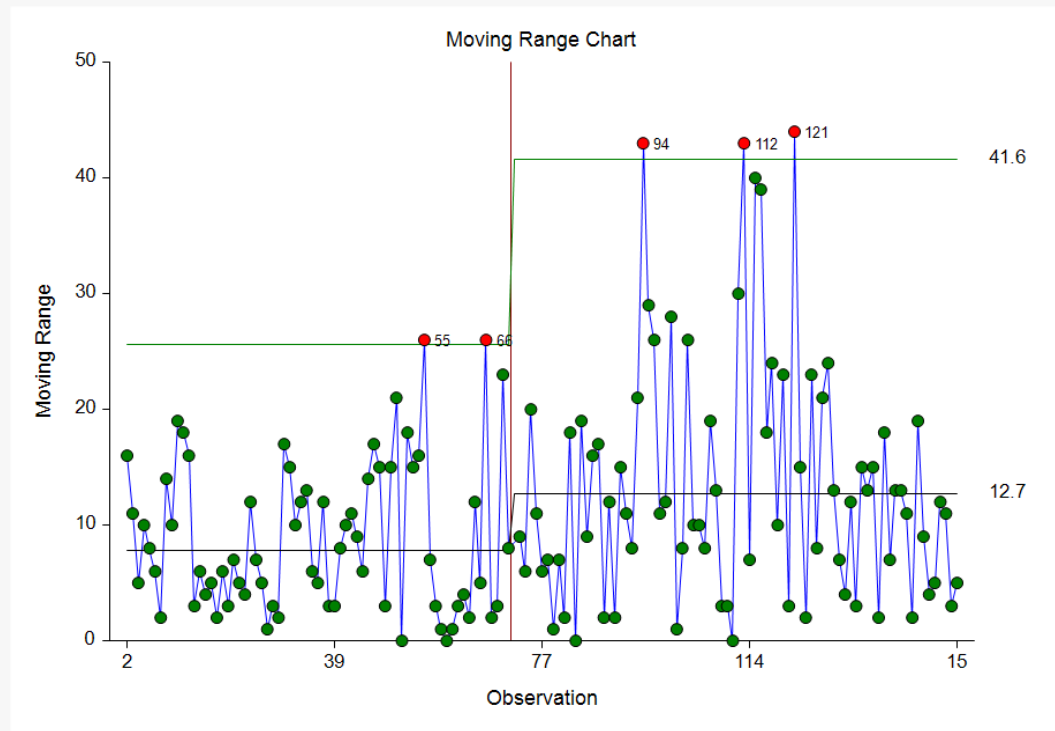
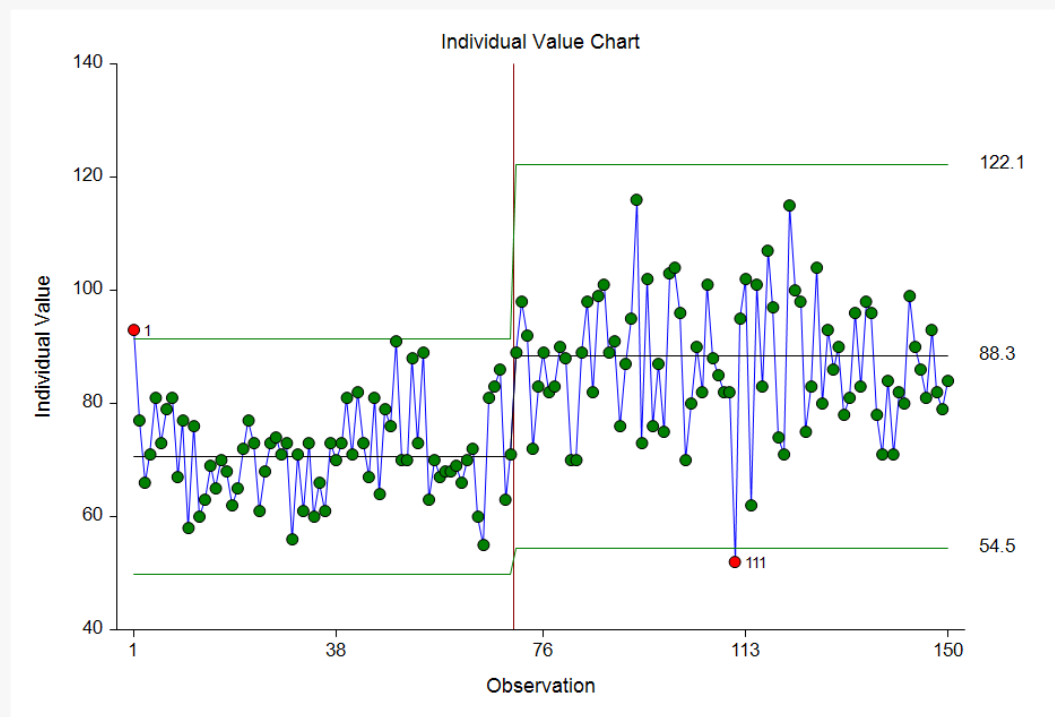
Estimation Type	Estimated Value	Estimated Sigma
Moving Ranges (R-bar)*	12.72414	11.28026

* Indicates the estimation type used in this report.

This report shows the center line, limit, and sigma estimates for each of the two stages. The center line estimate changes from about 71 to about 88. There also appears to be a change in sigma of about 7 to about 11.

Individuals and Moving Range Charts

Chart Section for Observations 1 to 150



The change in process mean and variation from the first stage to the second is reflected in the individuals and moving range charts. The mean and variation appear to be reasonably stable within the stages.