Chapter 570

Life-Table Analysis

Introduction

A life table presents the proportion surviving, the cumulative hazard function, and the hazard rates of a large group of subjects followed over time. The analysis accounts for subjects who die (fail) as well as subjects who are censored (withdrawn). The life-table method competes with the Kaplan-Meier product-limit method as a technique for survival analysis. The life-table method was developed first, but the Kaplan-Meier method has been shown to be superior and with the advent of computers is now the method of choice. However, for large samples, the life-table method is still popular in that it provides a simple summary of a large set of data.

Construction of a Life Table

We will give a brief introduction to the subject in this section. For a complete account of life-table analysis, we suggest the books by Lee (1992) and Elandt-Johnson and Johnson (1980). We will use the same terminology as in the Kaplan-Meier Survival Curves chapter. We suggest that you read the introduction to survival analysis given in that chapter if you are not familiar with common survival analysis terms such as *cumulative survival distribution*, *cumulative hazard function*, and *hazard rates*.

A life table is constructed from a set of grouped or ungrouped failure data. The columns of the table are created using a set of formulas. The rows of the table represent various time intervals. We will now define each of the columns in the life table. Note, however, that because of the large number of columns required to display all of the items, there will be several output reports produced.

Time Interval

Each time interval is represented by $T_t \leq T < T_{t+1}$ or $[T_t, T_{t+1})$, where $t = 1, \cdots, s$. The interval is from T_t up to but not including T_{t+1}. The intervals are assumed to be fixed. The intervals do not have to be of equal length, but it is often convenient to make them so.

The midpoint of the interval, T_{mt}, is defined as halfway through the interval.

The width of the interval is b_t where $b_t = T_{t+1} - T_t$. The width of the last interval, b_s, is theoretically infinite, so items requiring this value will be left blank.

Number Lost to Follow-Up

The number lost to follow-up, l_t, is the number of individuals who were loss to observation during this interval, so their survival status is unknown.

Number Withdrawn Alive

The number withdrawn alive, w_t, is the number of individuals who had not died (failed) by the end of the study.

Number Dying

The number dying, d_t, is the number of individuals who die (fail) during the interval.

Number Entering the tth Interval

The number entering the *t*th interval, n'_t, is computed as follows. In the first interval, it is the total sample size. In the remaining intervals, it is computed using the formula

$$n'_t = n'_{t-1} - l_{t-1} - w_{t-1} - d_{t-1}$$

Number Exposed to Risk

The number exposed to risk, n_t, is computed using the formula

$$n_t = n'_{t-1} - \tfrac{1}{2}\left(l_{t-1} + w_{t-1}\right)$$

This formula assumes that times to loss or withdrawal are distributed uniformly across the interval.

Conditional Proportion Dying

The conditional proportion dying, q_t, is an estimate of the conditional probability of death in the interval given exposure to the risk of death in the interval. It is computed using the formula

$$q_t = \frac{d_t}{n_t}$$

Conditional Proportion Surviving

The conditional proportion surviving, p_t, is an estimate of the conditional probability of surviving through the interval. It is computed using the formula

$$p_t = 1 - q_t$$

Cumulative Proportion Surviving

The cumulative proportion surviving, $S(T_t)$, is an estimate of cumulative survival rate at time T_t. It is computed using the formula

$$S(T_t) = S(T_{t-1})p_{t-1}$$

where

$$S(T_1) = 1$$

The variance of this estimate is itself estimated using the formula

$$V[S(T_t)] = S(T_t)^2 \sum_{j=1}^{t-1} \frac{q_j}{n_j p_j}$$

Using these estimates, pointwise confidence intervals are given using the Kaplan-Meier product-limit formulas given in the Kaplan-Meier chapter.

Estimated Death Density Function

The estimated death density function, $f(T_{mt})$, is an estimate of the probability of dying in this interval per unit width. At the interval midpoint, it is computed using the formula

$$f(T_{mt}) = \frac{S(T_t) - S(T_{t+1})}{b_t} = \frac{S(T_t)q_t}{b_t}$$

The variance of this estimate is itself estimated using the formula

$$V[f(T_{mt})] = \frac{S(T_t)^2 q_t^2}{b_t} \sum_{j=1}^{t-1} \left(\frac{q_j}{n_j p_j} + \frac{p_j}{n_j q_j} \right)$$

Hazard Rate Function

The estimated hazard rate function, $h(T_{mt})$, is an estimate of the number of deaths per unit time divided by the average number of survivors at the interval midpoint. It is computed using the formula

$$h(T_{mt}) = \frac{f(T_{mt})}{S(T_{mt})} = \frac{d_t}{b_t\left(n_t - \frac{1}{2}d_t\right)} = \frac{2q_t}{b_t(1 + p_t)}$$

The variance of this estimate is itself estimated using the formula

$$V[h(T_{mt})] = \frac{h(T_{mt})^2}{n_t q_t} \left(1 - \left[\frac{h(T_{mt})b_t}{2_t} \right]^2 \right)$$

Using these estimates, pointwise confidence intervals are given using the cumulative hazard confidence interval formulas given in the Kaplan-Meier chapter.

Cumulative Hazard Function

The cumulative hazard function, $H(T_t)$, is estimated using the Nelson-Aalen method. It is computed using the formula

$$\tilde{H}(T_t) = \sum_{j=1}^{t} \frac{d_j}{n_j}$$

The variance of this estimate is itself estimated using the formula

$$V\left[\tilde{H}(T_t)\right] = \sum_{j=1}^{t} \frac{d_j}{n_j^2}$$

Using these estimates, pointwise confidence intervals are given using the Nelson-Aalen formulas given in the Kaplan-Meier chapter.

Median Remaining Lifetime

The median remaining lifetime, MRT_t, is the time value at which exactly one-half of those who survived until T_t are still alive.

To compute this value, find the value *j* such that $S(T_j) \geq \frac{1}{2} S(T_t)$ and $S(T_{j+1}) < \frac{1}{2} S(T_t)$. Next, computed the median remaining lifetime using the formula

$$MRT_t = (T_j - T_t) + \frac{b_j \left(S(T_j) - \frac{1}{2} S(T_t) \right)}{S(T_j) - S(T_{j+1})}$$

The variance of this estimate is itself estimated using the formula

$$V(MRT_t) = \frac{S(T_t)^2}{4n_i \left[f(T_{mj}) \right]^2}$$

Using these estimates, pointwise confidence intervals are given using the Nelson-Aalen formulas given in the Kaplan-Meier chapter. Note that in this case, the confidence intervals are very crude since the MRT_t are not necessarily distributed normally, even in large samples.

Data Structure

Survival datasets require the ending survival time and an indicator of whether an observation was censored or failed. Additionally, you may also include a frequency variable the gives the count for each row.

The table below shows a dataset from which Lee (1992) constructs a life table. The survival experience of 2418 males with angina pectoris is recorded in years. The life table will use 16 intervals of one year each. These data are contained in the Lee91 dataset. Note that two rows are required for each data value, one for the failed individuals and another for the censored individuals.

Lee91 dataset

Time	Censor	Count
0.5	1	456
1.5	1	226
2.5	1	152
3.5	1	171
4.5	1	135
5.5	1	125
6.5	1	83
7.5	1	74
8.5	1	51
9.5	1	42
10.5	1	43
11.5	1	34
12.5	1	18
13.5	1	9
14.5	1	6
15.5	1	0
0.5	0	0
1.5	0	39
2.5	0	22
3.5	0	23
4.5	0	24
5.5	0	107

Lee91 dataset (cont'd)

Time	Censor	Count
6.5	0	133
7.5	0	102
8.5	0	68
9.5	0	64
10.5	0	45
11.5	0	53
12.5	0	33
13.5	0	27
14.5	0	23
15.5	0	30

Procedure Options

This section describes the options available in this procedure.

Variables Tab

This panel specifies the variables used in the analysis.

Time Variables

Time Variable

This variable contains the length of time that an individual was observed. This may represent a failure time or a censor time. Whether the subject actually died is specified by the Censor Variable. Since the values are elapsed times, they must be positive. Zeroes and negative values are treated as missing values. If you have date values, you much subtract them so that you have a column of elapsed times.

Time Interval Boundaries

Specify a list of times to be used as boundary points along the time scale. These become the left boundaries of the time intervals. Care should be taken to specify a left-most boundary that is less than the smallest time value. This number is often zero.

It is often convenient to make all intervals of the same width, but it is not necessary to do so.

Each interval is closed on the left and open on the right. That is, the interval is T(i) <= T < T(i+1).

Numbers representing the times are separated by blanks or commas. Specify sequences with a colon, putting the increment inside parentheses. For example: 5:25(5) means 5 10 15 20 25. Avoid negative numbers.

Use '(10)' alone to specify ten, equal-spaced values between zero and the maximum.

Frequency Variable

Frequency Variable

This variable gives the count, or frequency, of the time displayed on that row. This is the number of subjects represented by each row. When omitted, each row receives a frequency of one. Frequency values should be positive integers.

Censor Variable

Censor Variable

The values in this variable indicate whether the value of the Time Variable represents a censored time or a failure time. These values may be text or numeric. The interpretation of these codes is specified by the Failed and Censored options to the right of this option.

Only two values are used, the Failure code and the Censor code. The Unknown Type option specifies what is to be done with values that do not match either the Failure code or the Censor code.

Rows with missing values (blanks) in this variable are omitted.

Failed

This value identifies those values of the Censor Variable that indicate that the Time Variable gives a failure time. The value may be a number or a letter.

We suggest the letter 'F' or the number '1' when you are in doubt as to what to use.

Censored

This value identifies those values of the Censor Variable that indicate that the individual recorded on this row was censored. That is, the actual failure time occurs sometime after the value of the Time Variable.

We suggest the letter 'C' or the number '0' when you are in doubt as to what to use.

A censored observation is one in which the time until the event of interest is not known because the individual withdrew from the study, the study ended before the individual failed, or for some similar reason.

Unknown Censor

This option specifies what the program is to assume about rows whose censor value is not equal to either the Failed code or the Censored code. Note that observations with missing censor values are always treated as missing.

- **Censored**
  Observations with unknown censor values are assumed to have been censored.

- **Failed**
  Observations with unknown censor values are assumed to have failed.

- **Missing**
  Observations with unknown censor values are assumed to be missing and they are removed from the analysis.

Group Variable

Group Variable

An optional categorical (grouping) variable may be specified. If it is used, a separate analysis is conducted for each unique value of this variable.

Options

Confidence Limits

This option specifies the method used to estimate the confidence limits of the survival and hazard values that are displayed. The options are:

- **Linear**
  This is the classical method which uses Greenwood's estimate of the variance.

- **Log Transform**
  This method uses the logarithmic transformation of Greenwood's variance estimate. It produces better limits than the Linear method and has better small sample properties.

- **ArcSine**
  This method uses the arcsine square-root transformation of Greenwood's variance estimate to produce better limits.

Variance

The option specifies which estimator of the variance of the Nelson-Aalen cumulative hazard estimate is to be used. Three estimators have been proposed. When there are no event-time ties, all three give about the same results.

We recommend that you use the Simple estimator unless ties occur naturally in the theoretical event times.

- **Simple**
  This estimator should be used when event-time ties are caused by rounding and lack of measurement precision. This estimate gives the largest value and hence the widest, most conservation, confidence intervals.

- **Plug In**
  This estimator should be used when event-time ties are caused by rounding and lack of measurement precision.

- **Binomial**
  This estimator should be used when ties occur in the theoretical distribution of event times.

Reports Tab

The following options control which reports are displayed and the format of those reports.

Select Reports

Data Summary - Median Remaining Lifetime

These options indicate whether to display the corresponding report.

Alpha Level

This is the value of alpha used in the calculation of confidence limits. For example, if you specify 0.04 here, then 96% confidence limits will be calculated.

Report Options

Precision

Specify the precision of numbers in the report. A single-precision number will show seven-place accuracy, while a double-precision number will show thirteen-place accuracy. Note that the reports are formatted for single precision. If you select double precision, some numbers may run into others. Also note that all calculations are performed in double precision regardless of which option you select here. Single precision is for reporting purposes only.

Variable Names

This option lets you select whether to display only variable names, variable labels, or both.

Value Labels

This option lets you select whether to display only values, only value labels, or both for values of the group variable. Use this option if you want to automatically attach labels to the values of the group variable (like 1=Male, 2=Female, etc.). See the section on specifying *Value Labels* elsewhere in this manual.

Report Options – Decimal Places

Time

This option specifies the number of decimal places shown on reported time values.

Probability

This option specifies the number of decimal places shown on reported probability and hazard values.

N

This option specifies the number of decimal places shown on the number exposed.

Plots Tab

The following options control the plots that are displayed.

Select Plots

Survival/Reliability Plot - Hazard Rate Plot

These options specify which plots type of plots are displayed. Check the plots that you want to see.

Select Plots – Plots Displayed

Individual-Group Plots

When checked, this option specifies that a separate chart of each designated type is displayed.

Combined Plots

When checked, this option specifies that a chart combining all groups is to be displayed.

Plot Options – Plot Arrangement

These options control the size and arrangement of the plots.

Two Plots Per Line

When a man charts are specified, checking this option will cause the size of the charts to be reduced so that they can be displayed two per line. This will reduce the overall size of the output.

Storage Tab

These options let you specify if, and where on the dataset, various statistics are stored.

Warning: Any data already in these columns are replaced by the new data. Be careful not to specify columns that contain important data.

Data Storage Options

Storage Option

This option controls whether the values indicated below are stored on the dataset when the procedure is run.

- **Do not store data**
  No data are stored even if they are checked.

- **Store in empty columns only**
  The values are stored in empty columns only. Columns containing data are not used for data storage, so no data can be lost.

- **Store in designated columns**
  Beginning at the *Store First Item In* column, the values are stored in this column and those to the right. If a column contains data, the data are replaced by the storage values. Care must be used with this option because it cannot be undone.

Store First Item In

The first item is stored in this column. Each additional item that is checked is stored in the columns immediately to the right of this column.

Leave this value blank if you want the data storage to begin in the first blank column on the right-hand side of the data.

Warning: any existing data in these columns is automatically replaced, so be careful.

Data Storage Options – Select Items to Store on the Spreadsheet

Group - Median R.L. UCL

Indicate whether to store these values, beginning at the column indicated by the *Store First Item In* option.

Example 1 – Creating a Life Table

This section presents an example of how to create a life table. This example will use the survival data contained in the Lee91 dataset .

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Life-Table Analysis window.

1 Open the Lee91 dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Lee91.NCSS**.
- Click **Open**.

2 Open the Life-Table Analysis window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Life-Table Analysis** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the Life-Table Analysis window, select the **Variables tab**.
- Set the Time Variable to **Time**.
- Set the Time Interval Boundaries to **0:15(1)**.
- Set the Frequency Variable to **Count**.
- Set the Censor Variable to **Censor**.

4 Specify the plots.

- On the Life-Table Analysis window, select the **Plots tab**.
- Check the **Hazard Function Plot** box.
- Check the **Hazard Rate Plot** box.
- Click on the **Kaplan-Meier Survival/Reliability Plot Format** button.
- Click on the **At-Risk Table** tab on the left and check **Show At-Risk Table**.
- Click **OK** to save the plot settings.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Data Summary

Type	Rows	Count	Percent (%)	Minimum	Maximum
Failed	15	1625	67.2%	0.5	14.5
Censored	15	793	32.8%	1.5	15.5
Total	30	2418	100.0%	0.5	15.5

This report displays a summary of the amount of data that were analyzed. Scan this report to determine if there were any obvious data errors by double checking the counts and the minimum and maximum times.

Life-Table Analysis Detail

Time	Number Starting Interval	Number Lost	Number Died	Number Exposed to Risk	Conditonal Proportion Surviving	Cumulative Proportion Surviving	Hazard Rate
0.0	2418	0	456	2418.0	0.81141	1.00000	0.20822
1.0	1962	39	226	1942.5	0.88366	0.81141	0.12353
2.0	1697	22	152	1686.0	0.90985	0.71701	0.09441
3.0	1523	23	171	1511.5	0.88687	0.65237	0.11992
4.0	1329	24	135	1317.0	0.89749	0.57856	0.10804
5.0	1170	107	125	1116.5	0.88804	0.51926	0.11860
6.0	938	133	83	871.5	0.90476	0.46112	0.10000
7.0	722	102	74	671.0	0.88972	0.41721	0.11672
8.0	546	68	51	512.0	0.90039	0.37120	0.10483
9.0	427	64	42	395.0	0.89367	0.33422	0.11230
10.0	321	45	43	298.5	0.85595	0.29868	0.15523
11.0	233	53	34	206.5	0.83535	0.25566	0.17942
12.0	146	33	18	129.5	0.86100	0.21356	0.14938
13.0	95	27	9	81.5	0.88957	0.18388	0.11688
14.0	59	23	6	47.5	0.87368	0.16357	0.13483
15.0	30	30	0	15.0	1.00000	0.14291	

This report displays the standard life table. The formulas used were presented earlier.

Time

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \leq T < T_{t+1}$.

Number Starting Interval

This is the number entering the *t*th interval. In the first interval, it is the total sample size.

Number Lost

This is the number lost to follow-up and the number withdrawn from the study alive.

Number Died

This is the number of individuals who died (failed) during the interval.

Number Exposed to Risk

This is the average number exposed to risk in the interval. It is calculated under the assumption that losses and withdrawals are distributed uniformly across the interval.

Conditional Proportion Surviving

This is the conditional proportion surviving through the interval.

Cumulative Proportion Surviving

This is the estimate of the survivorship function, $S(T_t)$. It is also called the cumulative survival rate at time T_t. It is the probability of surviving to the start of the interval and then through the interval.

Hazard Rate

This is the estimated hazard rate function, $h(T_{mt})$. It is an estimate of the number of deaths per unit time divided by the average number of survivors computed at the interval midpoint.

Life-Table Analysis Summary

Time	Cumulative Proportion Surviving	Cumulative Hazard Function	Hazard Rate	Death Density Function	Median Remaining Lifetime	Number Starting Interval
0.0	1.00000	0.18859	0.20822	0.18859	5.3	2418
1.0	0.81141	0.30493	0.12353	0.09440	6.2	1962
2.0	0.71701	0.39508	0.09441	0.06464	6.3	1697
3.0	0.65237	0.50822	0.11992	0.07380	6.2	1523
4.0	0.57856	0.61072	0.10804	0.05931	6.2	1329
5.0	0.51926	0.72268	0.11860	0.05813	5.9	1170
6.0	0.46112	0.81792	0.10000	0.04392	5.6	938
7.0	0.41721	0.92820	0.11672	0.04601	5.2	722
8.0	0.37120	1.02781	0.10483	0.03697	4.9	546
9.0	0.33422	1.13414	0.11230	0.03554	4.8	427
10.0	0.29868	1.27819	0.15523	0.04303	4.7	321
11.0	0.25566	1.44284	0.17942	0.04209		233
12.0	0.21356	1.58184	0.14938	0.02968		146
13.0	0.18388	1.69227	0.11688	0.02031		95
14.0	0.16357	1.81858	0.13483	0.02066		59
15.0	0.14291	1.81858				30

This report displays the most interesting quantities from a life table. The formulas used were presented earlier.

Time

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \leq T < T_{t+1}$.

Cumulative Survival

This is the estimate of the survivorship function, $S\left(T_t\right)$. It is also called the cumulative survival rate at time T_t. It is the probability of surviving to the start of the interval and then through the interval.

Cumulative Hazard Function

This is the estimate of the cumulative hazard function, $H\left(T_t\right)$.

Hazard Rate

This is the estimated hazard rate function, $h\left(T_{mt}\right)$. It is an estimate of the number of deaths per unit time divided by the average number of survivors computed at the interval midpoint.

Death Density Function

This is the estimated death density function, $f\left(T_{mt}\right)$. It is an estimate of the probability of dying at the interval midpoint.

Median Remaining Lifetime

This is the median remaining lifetime, MRT_t. It is the time value at which exactly one-half of those who survived until T_t are still alive.

Number Starting Interval

This is the number entering the *t*th interval. In the first interval, it is the total sample size.

Survival Analysis

Time	Cumulative Survival	Standard Error	Lower 95% C.L.	Upper 95% C.L.
0.0	1.00000	0.00000	1.00000	1.00000
1.0	0.81141	0.00796	0.79582	0.82701
2.0	0.71701	0.00918	0.69902	0.73500
3.0	0.65237	0.00973	0.63329	0.67145
4.0	0.57856	0.01014	0.55869	0.59844
5.0	0.51926	0.01030	0.49906	0.53945
6.0	0.46112	0.01038	0.44078	0.48147
7.0	0.41721	0.01045	0.39672	0.43769
8.0	0.37120	0.01058	0.35046	0.39193
9.0	0.33422	0.01072	0.31322	0.35523
10.0	0.29868	0.01089	0.27734	0.32003
11.0	0.25566	0.01112	0.23385	0.27746
12.0	0.21356	0.01140	0.19123	0.23590
13.0	0.18388	0.01177	0.16082	0.20694
14.0	0.16357	0.01226	0.13954	0.18760
15.0	0.14291	0.01330	0.11684	0.16898

This report displays the life-table survival distribution along with confidence limits. The formulas used were presented earlier.

Time

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \leq T < T_{t+1}$.

Cumulative Survival

This is the estimate of the survivorship function, $S\left(T_t\right)$. It is also called the cumulative survival rate at time T_t. It is the probability of surviving to the start of the interval and then through the interval.

Standard Error

This is the large-sample estimate of standard error of the survival function. It is a measure of the precision of the survival estimate.

Lower and Upper Confidence Limits

The lower and upper confidence limits provide a pointwise confidence interval for the survival function. These limit are constructed so that the probability that the true survival probability lies between them is $1-\alpha$. Note that these limits are constructed for a single time point. Several of them cannot be used together to form a confidence band such that the entire survival function lies within the band.

Three difference confidence intervals are available. All three confidence intervals perform about the same in large samples. The linear (Greenwood) interval is the most commonly used. However, the log-transformed and the arcsine-square intervals behave better in small to moderate samples, so they are recommended. The formulas for these limits were given at the beginning of the chapter and are not repeated here.

Cumulative Hazard

Time	Cumulative Hazard	Standard Error	Lower 95% C.L.	Upper 95% C.L.
0.0	0.18859	0.00883	0.17128	0.20589
1.0	0.30493	0.01174	0.28192	0.32795
2.0	0.39508	0.01383	0.36797	0.42220
3.0	0.50822	0.01632	0.47624	0.54020
4.0	0.61072	0.01855	0.57437	0.64708
5.0	0.72268	0.02108	0.68137	0.76399
6.0	0.81792	0.02353	0.77180	0.86403
7.0	0.92820	0.02679	0.87568	0.98072
8.0	1.02781	0.03021	0.96860	1.08702
9.0	1.13414	0.03438	1.06676	1.20151
10.0	1.27819	0.04080	1.19824	1.35815
11.0	1.44284	0.04961	1.34560	1.54009
12.0	1.58184	0.05946	1.46531	1.69837
13.0	1.69227	0.06993	1.55521	1.82932
14.0	1.81858	0.08689	1.64829	1.98888
15.0	1.81858	0.08689	1.64829	1.98888

This report displays estimates of the cumulative hazard function. The formulas used were presented earlier.

Time

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \leq T < T_{t+1}$.

Cumulative Hazard

This is the Nelson-Aalen estimate of the cumulative hazard function, $H(T_t)$.

Standard Error

This is the estimated standard error of the above cumulative hazard function. The formula used was specified under the Variables tab in the Variance box. The standard error is the square root of this variance.

Lower and Upper Confidence Limits

The lower and upper confidence limits provide a pointwise confidence interval for the cumulative hazard at each time point. These limits are constructed so that the probability that the true cumulative hazard lies between them is $1-\alpha$. Note that these limits are constructed for a single time point. Several of them cannot be used together to form a confidence band such that the entire cumulative hazard function lies within the band.

Three difference confidence intervals are available. All three confidence intervals perform about the same in large samples. The linear (Greenwood) interval is the most commonly used. However, the log-transformed and the arcsine-square intervals behave better in small to moderate samples, so they are recommended. The formulas for these limits were given at the beginning of the chapter and are not repeated here.

Hazard Rate

Time	Hazard Rate	Standard Error	Lower 95% C.L.	Upper 95% C.L.
0.0	0.20822	0.00970	0.18921	0.22723
1.0	0.12353	0.00820	0.10746	0.13961
2.0	0.09441	0.00765	0.07942	0.10940
3.0	0.11992	0.00915	0.10197	0.13786
4.0	0.10804	0.00929	0.08984	0.12624
5.0	0.11860	0.01059	0.09784	0.13935
6.0	0.10000	0.01096	0.07851	0.12149
7.0	0.11672	0.01355	0.09017	0.14327
8.0	0.10483	0.01466	0.07610	0.13356
9.0	0.11230	0.01730	0.07839	0.14621
10.0	0.15523	0.02360	0.10898	0.20149
11.0	0.17942	0.03065	0.11935	0.23948
12.0	0.14938	0.03511	0.08056	0.21819
13.0	0.11688	0.03889	0.04065	0.19311
14.0	0.13483	0.05492	0.02719	0.24247
15.0				

This report displays estimates of the hazard rates at the midpoints of each of the time intervals. The formulas used were presented earlier.

Time

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \leq T < T_{t+1}$. Note that the hazard rate is actually computed at the midpoint of each interval.

Cumulative Hazard

This is the estimate of the hazard rate, $h\left(T_{mt}\right)$.

Standard Error

This is the estimated standard error of the above hazard rate. The formula used was given earlier. The standard error is the square root of this variance.

Lower and Upper Confidence Limits

The lower and upper confidence limits provide a pointwise confidence interval for the hazard rate at the midpoint of the time interval. These limits are constructed so that the probability that the true hazard rate lies between them is $1-\alpha$. Note that these limits are constructed for a single time point. Several of them cannot be used together to form a confidence band such that the entire hazard rate lies within the band.

Three difference confidence intervals are available. All three confidence intervals perform about the same in large samples. The linear (Greenwood) interval is the most commonly used. However, the log-transformed and the arcsine-square intervals behave better in small to moderate samples, so they are recommended. The formulas for these limits were given at the beginning of the chapter and are not repeated here.

Death Density Function

Time	Death Density	Standard Error	Lower 95% C.L.	Upper 95% C.L.
0.0	0.18859	0.00796	0.17299	0.20418
1.0	0.09440	0.00598	0.08269	0.10612
2.0	0.06464	0.00507	0.05471	0.07458
3.0	0.07380	0.00543	0.06317	0.08444
4.0	0.05931	0.00495	0.04961	0.06900
5.0	0.05813	0.00503	0.04827	0.06800
6.0	0.04392	0.00469	0.03472	0.05311
7.0	0.04601	0.00518	0.03587	0.05615
8.0	0.03697	0.00502	0.02713	0.04682
9.0	0.03554	0.00531	0.02513	0.04594
10.0	0.04303	0.00627	0.03074	0.05532
11.0	0.04209	0.00685	0.02867	0.05551
12.0	0.02968	0.00668	0.01659	0.04278
13.0	0.02031	0.00651	0.00754	0.03307
14.0	0.02066	0.00804	0.00491	0.03641
15.0				

This report displays estimates of the hazard rates at the midpoints of each of the time intervals. The formulas used were presented earlier.

Time

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \le T < T_{t+1}$. Note that the hazard rate is actually computed at the midpoint of each interval.

Death Density

This is the estimate of the death density, $f\left(T_{mt}\right)$.

Standard Error

This is the estimated standard error of the above density. The formula used was given earlier. The standard error is the square root of this variance.

Lower and Upper Confidence Limits

The lower and upper confidence limits provide a pointwise confidence interval for the death density at the midpoint of the time interval. These limits are constructed so that the probability that the true density lies between them is $1-\alpha$.

Three difference confidence intervals are available. All three confidence intervals perform about the same in large samples. The linear (Greenwood) interval is the most commonly used. However, the log-transformed and the arcsine-square intervals behave better in small to moderate samples, so they are recommended. The formulas for these limits were given at the beginning of the chapter and are not repeated here.

Median Remaining Lifetime

Time	Median Remaining Lifetime	Standard Error	Lower 95% C.L.	Upper 95% C.L.
0.0	5.3	0.17491	5.0	5.7
1.0	6.2	0.20006	5.9	6.6
2.0	6.3	0.23614	5.9	6.8
3.0	6.2	0.23609	5.8	6.7
4.0	6.2	0.18526	5.9	6.6
5.0	5.9	0.18059	5.6	6.3
6.0	5.6	0.18554	5.2	6.0
7.0	5.2	0.27129	4.6	5.7
8.0	4.9	0.27632	4.4	5.5
9.0	4.8	0.41408	4.0	5.6
10.0	4.7	0.41835	3.9	5.5
11.0				
12.0				
13.0				
14.0				
15.0				

This report displays estimates of the median remaining lifetime of those who are alive at the beginning of the interval. The formulas used were presented earlier.

Time

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \leq T < T_{t+1}$.

Median Remaining Lifetime

This is the estimate of the median remaining lifetime of an individual who survives to the beginning of this interval.

Standard Error

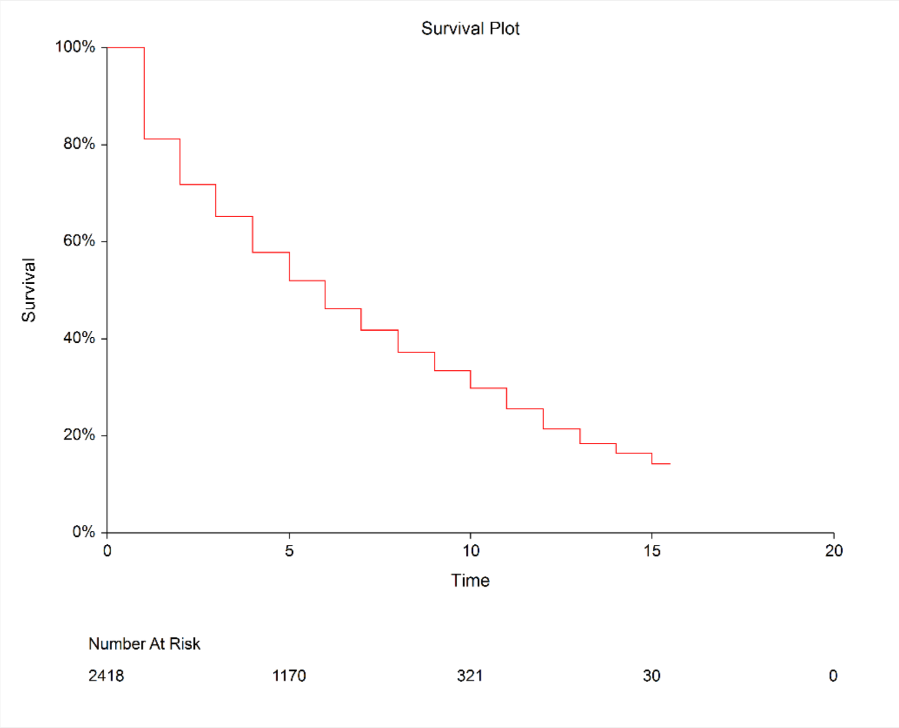
This is the estimated standard error of the above lifetime. The formula used was given earlier.

Lower and Upper Confidence Limits

The lower and upper confidence limits provide a pointwise confidence interval for the hazard rate at the midpoint of the time interval. These limits are constructed so that the probability that the true remaining lifetime lies between them is $1-\alpha$.

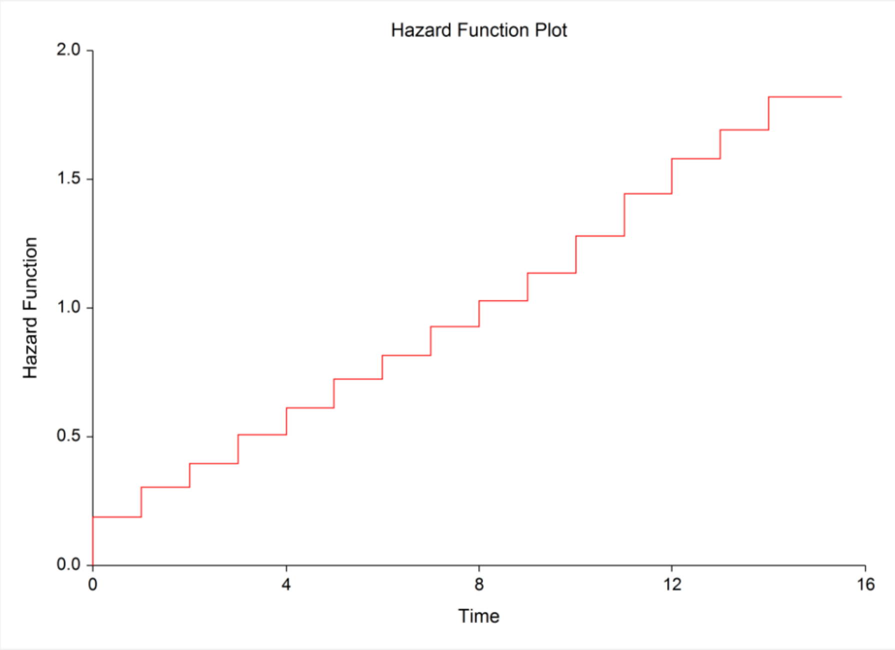
These confidence intervals are highly approximate. They depend on the assumption that the median remaining lifetime is normally distributed. This may not be true even in large samples.

Survival Plot(s)



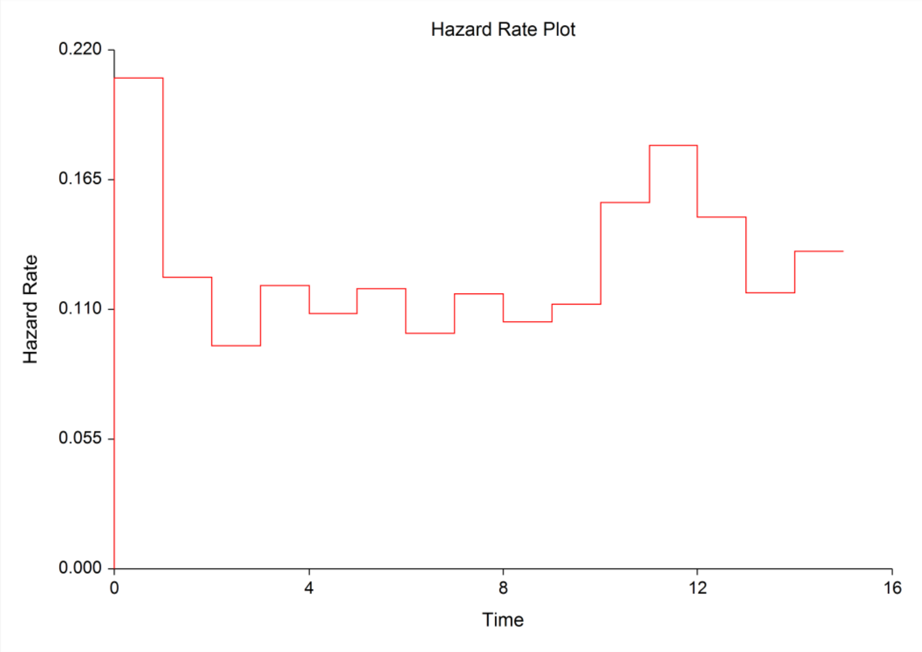
This plot shows the survivorship function. If there are several groups, a separate line is drawn for each group. The plot includes the number at risk at several time points.

Cumulative Hazard Function Plot(s)



This plot shows the Nelson-Aalen cumulative hazard function. If you have several groups, then a separate line is drawn for each group.

Hazard Rate Plot(s)



This plot shows the hazard rates. Note the unusual step-like appearance of the plot because the hazard rates are assumed constant for the duration of the interval.

Validation of Life-Table Estimator using Lee (1992)

This section presents validation of our life-table estimator. Lee (1992) presents an example on page 91 of a calculated life table. We will include the results of one line of that table so that you can compare those results with those produced by this program. If you compare these values with those shown above, you can validate that **NCSS** provides the correct results.

Parameter	Value
Time	3
Lost	23
Dying	171
Entering	1523
Exposed	1511.5
Proportion Dying	0.1131
Proportion Surviving	0.8869
S(T)	0.6524
S.E. S(T)	0.0097
h(T)	0.1199
S.E. h(T)	0.0092
Median Rem. Lifetime	6.23
S.E. MRL(T)	0.9
f(T)	0.0738
S.E. f(T)	0.0054