Chapter 570 Life-Table Analysis

Introduction

A life table presents the proportion surviving, the cumulative hazard function, and the hazard rates of a large group of subjects followed over time. The analysis accounts for subjects who die (fail) as well as subjects who are censored (withdrawn). The life-table method competes with the Kaplan-Meier product-limit method as a technique for survival analysis. The life-table method was developed first, but the Kaplan-Meier method has been shown to be superior and with the advent of computers is now the method of choice. However, for large samples, the life-table method is still popular in that it provides a simple summary of a large set of data.

Construction of a Life Table

We will give a brief introduction to the subject in this section. For a complete account of life-table analysis, we suggest the books by Lee (1992) and Elandt-Johnson and Johnson (1980). We will use the same terminology as in the Kaplan-Meier Survival Curves chapter. We suggest that you read the introduction to survival analysis given in that chapter if you are not familiar with common survival analysis terms such as *cumulative survival distribution, cumulative hazard function*, and *hazard rates*.

A life table is constructed from a set of grouped or ungrouped failure data. The columns of the table are created using a set of formulas. The rows of the table represent various time intervals. We will now define each of the columns in the life table. Note, however, that because of the large number of columns required to display all of the items, there will be several output reports produced.

Time Interval

Each time interval is represented by $T_t \le T < T_{t+1}$ or $[T_t, T_{t+1})$, where t = 1, ..., s. The interval is from T_t up to but not including T_{t+1} . The intervals are assumed to be fixed. The intervals do not have to be of equal length, but it is often convenient to make them so.

The midpoint of the interval, T_{mt} , is defined as halfway through the interval.

The width of the interval is b_t where $b_t = T_{t+1} - T_t$. The width of the last interval, b_s , is theoretically infinite, so items requiring this value will be left blank.

Number Lost to Follow-Up

The number lost to follow-up, l_t , is the number of individuals who were loss to observation during this interval, so their survival status is unknown.

Number Withdrawn Alive

The number withdrawn alive, w_t , is the number of individuals who had not died (failed) by the end of the study.

Number Dying

The number dying, d_t , is the number of individuals who die (fail) during the interval.

Number Entering the tth Interval

The number entering the t^{th} interval, n'_t , is computed as follows. In the first interval, it is the total sample size. In the remaining intervals, it is computed using the formula

$$n'_t = n'_{t-1} - l_{t-1} - w_{t-1} - d_{t-1}$$

Number Exposed to Risk

The number exposed to risk, n_t , is computed using the formula

$$n_t = n_{t-1}' - \frac{1}{2}(l_{t-1} + w_{t-1})$$

This formula assumes that times to loss or withdrawal are distributed uniformly across the interval.

Conditional Proportion Dying

The conditional proportion dying, q_t , is an estimate of the conditional probability of death in the interval given exposure to the risk of death in the interval. It is computed using the formula

$$q_t = \frac{d_t}{n_t}$$

Conditional Proportion Surviving

The conditional proportion surviving, p_t , is an estimate of the conditional probability of surviving through the interval. It is computed using the formula

$$p_t = 1 - q_t$$

Cumulative Proportion Surviving

The cumulative proportion surviving, $S(T_t)$, is an estimate of cumulative survival rate at time T_t . It is computed using the formula

$$S(T_t) = S(T_{t-1})p_{t-1}$$

where

 $S(T_1) = 1$

The variance of this estimate is itself estimated using the formula

$$V[S(T_t)] = S(T_t)^2 \sum_{j=1}^{t-1} \frac{q_j}{n_j p_j}$$

Using these estimates, pointwise confidence intervals are given using the Kaplan-Meier product-limit formulas given in the Kaplan-Meier chapter.

Estimated Death Density Function

The estimated death density function, $f(T_{mt})$, is an estimate of the probability of dying in this interval per unit width. At the interval midpoint, it is computed using the formula

$$f(T_{mt}) = \frac{S(T_t) - S(T_{t+1})}{b_t}$$
$$= \frac{S(T_t)q_t}{b_t}$$

The variance of this estimate is itself estimated using the formula

$$V[f(T_{mt})] = \frac{S(T_t)^2 q_t^2}{b_t} \sum_{j=1}^{t-1} \left(\frac{q_j}{n_j p_j} + \frac{p_j}{n_j q_j} \right)$$

Hazard Rate Function

The estimated hazard rate function, $h(T_{mt})$, is an estimate of the number of deaths per unit time divided by the average number of survivors at the interval midpoint. It is computed using the formula

$$h(T_{mt}) = \frac{f(T_{mt})}{S(T_{mt})}$$
$$= \frac{d_t}{b_t \left(n_t - \frac{1}{2}d_t\right)}$$
$$= \frac{2q_t}{b_t (1 + p_t)}$$

The variance of this estimate is itself estimated using the formula

$$V[h(T_{mt})] = \frac{h(T_{mt})^2}{n_t q_t} \left(1 - \left[\frac{h(T_{mt})b_t}{2_t} \right]^2 \right)$$

Using these estimates, pointwise confidence intervals are given using the cumulative hazard confidence interval formulas given in the Kaplan-Meier chapter.

Cumulative Hazard Function

The cumulative hazard function, $H(T_t)$, is estimated using the Nelson-Aalen method. It is computed using the formula

$$\widetilde{H}(T_t) = \sum_{j=1}^t \frac{d_j}{n_j}$$

The variance of this estimate is itself estimated using the formula

$$V\big[\widetilde{H}(T_t)\big] = \sum_{j=1}^t \frac{d_j}{n_j^2}$$

Using these estimates, pointwise confidence intervals are given using the Nelson-Aalen formulas given in the Kaplan-Meier chapter.

Median Remaining Lifetime

The median remaining lifetime, MRT_t , is the time value at which exactly one-half of those who survived until T_t are still alive.

To compute this value, find the value *j* such that $S(T_j) \ge \frac{1}{2}S(T_t)$ and $S(T_{j+1}) < \frac{1}{2}S(T_t)$. Next, computed the median remaining lifetime using the formula

$$MRT_t = (T_j - T_t) + \frac{b_j \left(S(T_j) - \frac{1}{2}S(T_t)\right)}{S(T_j) - S(T_{j+1})}$$

The variance of this estimate is itself estimated using the formula

$$V(MRT_t) = \frac{S(T_t)^2}{4n_i [f(T_{mj})]^2}$$

Using these estimates, pointwise confidence intervals are given using the Nelson-Aalen formulas given in the Kaplan-Meier chapter. Note that in this case, the confidence intervals are very crude since the MRT_t are not necessarily distributed normally, even in large samples.

Data Structure

Survival datasets require the ending survival time and an indicator of whether an observation was censored or failed. Additionally, you may also include a frequency variable the gives the count for each row.

The table below shows a dataset from which Lee (1992) constructs a life table. The survival experience of 2418 males with angina pectoris is recorded in years. The life table will use 16 intervals of one year each. These data are contained in the Lee91 dataset. Note that two rows are required for each data value, one for the failed individuals and another for the censored individuals.

Lee91 Dataset (Subset)

| Time | Censor | Count |
|------|--------|-------|
| 0.5 | 1 | 456 |
| 1.5 | 1 | 226 |
| 2.5 | 1 | 152 |
| 3.5 | 1 | 171 |
| 4.5 | 1 | 135 |
| 5.5 | 1 | 125 |
| 6.5 | 1 | 83 |
| 7.5 | 1 | 74 |
| 8.5 | 1 | 51 |
| 9.5 | 1 | 42 |
| 10.5 | 1 | 43 |
| 11.5 | 1 | 34 |
| 12.5 | 1 | 18 |
| 13.5 | 1 | 9 |
| 14.5 | 1 | 6 |
| 15.5 | 1 | 0 |
| 0.5 | 0 | 0 |
| 1.5 | 0 | 39 |
| 2.5 | 0 | 22 |
| 3.5 | 0 | 23 |
| 4.5 | 0 | 24 |
| 5.5 | 0 | 107 |

Example 1 – Creating a Life Table

This section presents an example of how to create a life table. This example will use the survival data contained in the Lee91 dataset.

Setup

To run this example, complete the following steps:

1 Open the Lee91 example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select Lee91 and click OK.

2 Specify the Life-Table Analysis procedure options

- Find and open the **Life-Table Analysis** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

| Variables Tab | |
|-------------------------------------------------|--------------------|
| Time Variable | Time |
| Time Interval Boundaries | .0:15(1) |
| Frequency Variable | Count |
| Censor Variable | Censor |
| Plots Tab | |
| Kaplan-Meier Survival/Reliability Plot | Checked |
| Hazard Rate Plot | Checked |
| Cumulative Hazard Function Plot | Checked |
| Kaplan-Meier Survival/Reliability Plot Format (| Click the Button |
| | Concil and Ballony |
| At-Risk Table Tab | |

3 Run the procedure

• Click the **Run** button to perform the calculations and generate the output.

Data Summary

| Data Summary | | | | | | | | | |
|--------------|------|-------|-------------|---------|---------|--|--|--|--|
| Туре | Rows | Count | Percent (%) | Minimum | Maximum | | | | |
| Failed | 15 | 1625 | 67.2% | 0.5 | 14.5 | | | | |
| Censored | 15 | 793 | 32.8% | 1.5 | 15.5 | | | | |
| Total | 30 | 2418 | 100.0% | 0.5 | 15.5 | | | | |

This report displays a summary of the amount of data that were analyzed. Scan this report to determine if there were any obvious data errors by double checking the counts and the minimum and maximum times.

Life-Table Analysis Detail

Life-Table Analysis Detail

| Time | Number Starting Interval | Number Lost | Number Died | Number Exposed to Risk | Conditonal Proportion Surviving | Cumulative Proportion Surviving | Hazard Rate |
|------|--------------------------------|----------------|----------------|------------------------------|---------------------------------------|---------------------------------------|----------------|
| 0.0 | 2418 | 0 | 456 | 2418.0 | 0.81141 | 1.00000 | 0.20822 |
| 1.0 | 1962 | 39 | 226 | 1942.5 | 0.88366 | 0.81141 | 0.12353 |
| 2.0 | 1697 | 22 | 152 | 1686.0 | 0.90985 | 0.71701 | 0.09441 |
| 3.0 | 1523 | 23 | 171 | 1511.5 | 0.88687 | 0.65237 | 0.11992 |
| 4.0 | 1329 | 24 | 135 | 1317.0 | 0.89749 | 0.57856 | 0.10804 |
| 5.0 | 1170 | 107 | 125 | 1116.5 | 0.88804 | 0.51926 | 0.11860 |
| 6.0 | 938 | 133 | 83 | 871.5 | 0.90476 | 0.46112 | 0.10000 |
| 7.0 | 722 | 102 | 74 | 671.0 | 0.88972 | 0.41721 | 0.11672 |
| 8.0 | 546 | 68 | 51 | 512.0 | 0.90039 | 0.37120 | 0.10483 |
| 9.0 | 427 | 64 | 42 | 395.0 | 0.89367 | 0.33422 | 0.11230 |
| 10.0 | 321 | 45 | 43 | 298.5 | 0.85595 | 0.29868 | 0.15523 |
| 11.0 | 233 | 53 | 34 | 206.5 | 0.83535 | 0.25566 | 0.17942 |
| 12.0 | 146 | 33 | 18 | 129.5 | 0.86100 | 0.21356 | 0.14938 |
| 13.0 | 95 | 27 | 9 | 81.5 | 0.88957 | 0.18388 | 0.11688 |
| 14.0 | 59 | 23 | 6 | 47.5 | 0.87368 | 0.16357 | 0.13483 |
| 15.0 | 30 | 30 | 0 | 15.0 | 1.00000 | 0.14291 | |

This report displays the standard life table. The formulas used were presented earlier.

Time

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \le T < T_{t+1}$.

Number Starting Interval

This is the number entering the t^{th} interval. In the first interval, it is the total sample size.

Number Lost

This is the number lost to follow-up and the number withdrawn from the study alive.

Number Died

This is the number of individuals who died (failed) during the interval.

Number Exposed to Risk

This is the average number exposed to risk in the interval. It is calculated under the assumption that losses and withdrawals are distributed uniformly across the interval.

Conditional Proportion Surviving

This is the conditional proportion surviving through the interval.

Cumulative Proportion Surviving

This is the estimate of the survivorship function, $S(T_t)$. It is also called the cumulative survival rate at time T_t . It is the probability of surviving to the start of the interval and then through the interval.

Hazard Rate

This is the estimated hazard rate function, $h(T_{mt})$. It is an estimate of the number of deaths per unit time divided by the average number of survivors computed at the interval midpoint.

Life-Table Analysis Summary

Life-Table Analysis Summary

| | Cumulative | Cumulative | | Death | Median | Number |
|------|------------|------------|---------|----------|-----------|----------|
| | Proportion | Hazard | Hazard | Density | Remaining | Starting |
| Time | Surviving | Function | Rate | Function | Lifetime | Interval |
| 0.0 | 1.00000 | 0.18859 | 0.20822 | 0.18859 | 5.3 | 2418 |
| 1.0 | 0.81141 | 0.30493 | 0.12353 | 0.09440 | 6.2 | 1962 |
| 2.0 | 0.71701 | 0.39508 | 0.09441 | 0.06464 | 6.3 | 1697 |
| 3.0 | 0.65237 | 0.50822 | 0.11992 | 0.07380 | 6.2 | 1523 |
| 4.0 | 0.57856 | 0.61072 | 0.10804 | 0.05931 | 6.2 | 1329 |
| 5.0 | 0.51926 | 0.72268 | 0.11860 | 0.05813 | 5.9 | 1170 |
| 6.0 | 0.46112 | 0.81792 | 0.10000 | 0.04392 | 5.6 | 938 |
| 7.0 | 0.41721 | 0.92820 | 0.11672 | 0.04601 | 5.2 | 722 |
| 8.0 | 0.37120 | 1.02781 | 0.10483 | 0.03697 | 4.9 | 546 |
| 9.0 | 0.33422 | 1.13414 | 0.11230 | 0.03554 | 4.8 | 427 |
| 10.0 | 0.29868 | 1.27819 | 0.15523 | 0.04303 | 4.7 | 321 |
| 11.0 | 0.25566 | 1.44284 | 0.17942 | 0.04209 | | 233 |
| 12.0 | 0.21356 | 1.58184 | 0.14938 | 0.02968 | | 146 |
| 13.0 | 0.18388 | 1.69227 | 0.11688 | 0.02031 | | 95 |
| 14.0 | 0.16357 | 1.81858 | 0.13483 | 0.02066 | | 59 |
| 15.0 | 0.14291 | 1.81858 | | | | 30 |

This report displays the most interesting quantities from a life table. The formulas used were presented earlier.

Time

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \le T < T_{t+1}$.

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Cumulative Survival

This is the estimate of the survivorship function, $S(T_t)$. It is also called the cumulative survival rate at time T_t . It is the probability of surviving to the start of the interval and then through the interval.

Cumulative Hazard Function

This is the estimate of the cumulative hazard function, $H(T_t)$.

Hazard Rate

This is the estimated hazard rate function, $h(T_{mt})$. It is an estimate of the number of deaths per unit time divided by the average number of survivors computed at the interval midpoint.

Death Density Function

This is the estimated death density function, $f(T_{mt})$. It is an estimate of the probability of dying at the interval midpoint.

Median Remaining Lifetime

This is the median remaining lifetime, MRT_t . It is the time value at which exactly one-half of those who survived until T_t are still alive.

Number Starting Interval

This is the number entering the t^{th} interval. In the first interval, it is the total sample size.

Survival Analysis

| Surviva | Survival Analysis | | | | | | | | | |
|---------|------------------------|-------------------|-------------------|-------------------|--|--|--|--|--|--|
| Time | Cumulative Survival | Standard Error | Lower 95% C.L. | Upper 95% C.L. | | | | | | |
| 0.0 | 1.00000 | 0.00000 | 1.00000 | 1.00000 | | | | | | |
| 1.0 | 0.81141 | 0.00796 | 0.79582 | 0.82701 | | | | | | |
| 2.0 | 0.71701 | 0.00918 | 0.69902 | 0.73500 | | | | | | |
| 3.0 | 0.65237 | 0.00973 | 0.63329 | 0.67145 | | | | | | |
| 4.0 | 0.57856 | 0.01014 | 0.55869 | 0.59844 | | | | | | |
| 5.0 | 0.51926 | 0.01030 | 0.49906 | 0.53945 | | | | | | |
| 6.0 | 0.46112 | 0.01038 | 0.44078 | 0.48147 | | | | | | |
| 7.0 | 0.41721 | 0.01045 | 0.39672 | 0.43769 | | | | | | |
| 8.0 | 0.37120 | 0.01058 | 0.35046 | 0.39193 | | | | | | |
| 9.0 | 0.33422 | 0.01072 | 0.31322 | 0.35523 | | | | | | |
| 10.0 | 0.29868 | 0.01089 | 0.27734 | 0.32003 | | | | | | |
| 11.0 | 0.25566 | 0.01112 | 0.23385 | 0.27746 | | | | | | |
| 12.0 | 0.21356 | 0.01140 | 0.19123 | 0.23590 | | | | | | |
| 13.0 | 0.18388 | 0.01177 | 0.16082 | 0.20694 | | | | | | |
| 14.0 | 0.16357 | 0.01226 | 0.13954 | 0.18760 | | | | | | |
| 15.0 | 0.14291 | 0.01330 | 0.11684 | 0.16898 | | | | | | |

This report displays the life-table survival distribution along with confidence limits. The formulas used were presented earlier.

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \le T < T_{t+1}$.

Cumulative Survival

This is the estimate of the survivorship function, $S(T_t)$. It is also called the cumulative survival rate at time T_t . It is the probability of surviving to the start of the interval and then through the interval.

Standard Error

This is the large-sample estimate of standard error of the survival function. It is a measure of the precision of the survival estimate.

Lower and Upper Confidence Limits

The lower and upper confidence limits provide a pointwise confidence interval for the survival function. These limits are constructed so that the probability that the true survival probability lies between them is $1 - \alpha$. Note that these limits are constructed for a single time point. Several of them cannot be used together to form a confidence band such that the entire survival function lies within the band.

Three difference confidence intervals are available. All three confidence intervals perform about the same in large samples. The linear (Greenwood) interval is the most commonly used. However, the log-transformed and the arcsine-square intervals behave better in small to moderate samples, so they are recommended. The formulas for these limits were given at the beginning of the chapter and are not repeated here.

| Cumula | Cumulative Hazard | | | | | | | | |
|--------|----------------------|-------------------|-------------------|-------------------|--|--|--|--|--|
| Time | Cumulative Hazard | Standard Error | Lower 95% C.L. | Upper 95% C.L. | | | | | |
| 0.0 | 0.18859 | 0.00883 | 0.17128 | 0.20589 | | | | | |
| 1.0 | 0.30493 | 0.01174 | 0.28192 | 0.32795 | | | | | |
| 2.0 | 0.39508 | 0.01383 | 0.36797 | 0.42220 | | | | | |
| 3.0 | 0.50822 | 0.01632 | 0.47624 | 0.54020 | | | | | |
| 4.0 | 0.61072 | 0.01855 | 0.57437 | 0.64708 | | | | | |
| 5.0 | 0.72268 | 0.02108 | 0.68137 | 0.76399 | | | | | |
| 6.0 | 0.81792 | 0.02353 | 0.77180 | 0.86403 | | | | | |
| 7.0 | 0.92820 | 0.02679 | 0.87568 | 0.98072 | | | | | |
| 8.0 | 1.02781 | 0.03021 | 0.96860 | 1.08702 | | | | | |
| 9.0 | 1.13414 | 0.03438 | 1.06676 | 1.20151 | | | | | |
| 10.0 | 1.27819 | 0.04080 | 1.19824 | 1.35815 | | | | | |
| 11.0 | 1.44284 | 0.04961 | 1.34560 | 1.54009 | | | | | |
| 12.0 | 1.58184 | 0.05946 | 1.46531 | 1.69837 | | | | | |
| 13.0 | 1.69227 | 0.06993 | 1.55521 | 1.82932 | | | | | |
| 14.0 | 1.81858 | 0.08689 | 1.64829 | 1.98888 | | | | | |
| 15.0 | 1.81858 | 0.08689 | 1.64829 | 1.98888 | | | | | |

Cumulative Hazard

This report displays estimates of the cumulative hazard function. The formulas used were presented earlier.

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This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \le T < T_{t+1}$.

Cumulative Hazard

This is the Nelson-Aalen estimate of the cumulative hazard function, $H(T_t)$.

Standard Error

This is the estimated standard error of the above cumulative hazard function. The formula used was specified under the Variables tab in the Variance box. The standard error is the square root of this variance.

Lower and Upper Confidence Limits

The lower and upper confidence limits provide a pointwise confidence interval for the cumulative hazard at each time point. These limits are constructed so that the probability that the true cumulative hazard lies between them is $1 - \alpha$. Note that these limits are constructed for a single time point. Several of them cannot be used together to form a confidence band such that the entire cumulative hazard function lies within the band.

Three difference confidence intervals are available. All three confidence intervals perform about the same in large samples. The linear (Greenwood) interval is the most commonly used. However, the log-transformed and the arcsine-square intervals behave better in small to moderate samples, so they are recommended. The formulas for these limits were given at the beginning of the chapter and are not repeated here.

| | Hazard | Standard | Lower | Upper |
|------|---------|----------|----------|----------|
| Time | Rate | Error | 95% C.L. | 95% C.L. |
| 0.0 | 0.20822 | 0.00970 | 0.18921 | 0.22723 |
| 1.0 | 0.12353 | 0.00820 | 0.10746 | 0.13961 |
| 2.0 | 0.09441 | 0.00765 | 0.07942 | 0.10940 |
| 3.0 | 0.11992 | 0.00915 | 0.10197 | 0.13786 |
| 4.0 | 0.10804 | 0.00929 | 0.08984 | 0.12624 |
| 5.0 | 0.11860 | 0.01059 | 0.09784 | 0.13935 |
| 6.0 | 0.10000 | 0.01096 | 0.07851 | 0.12149 |
| 7.0 | 0.11672 | 0.01355 | 0.09017 | 0.14327 |
| 8.0 | 0.10483 | 0.01466 | 0.07610 | 0.13356 |
| 9.0 | 0.11230 | 0.01730 | 0.07839 | 0.14621 |
| 10.0 | 0.15523 | 0.02360 | 0.10898 | 0.20149 |
| 11.0 | 0.17942 | 0.03065 | 0.11935 | 0.23948 |
| 12.0 | 0.14938 | 0.03511 | 0.08056 | 0.21819 |
| 13.0 | 0.11688 | 0.03889 | 0.04065 | 0.19311 |
| 14.0 | 0.13483 | 0.05492 | 0.02719 | 0.24247 |
| 15.0 | | | | |

Hazard Rate

This report displays estimates of the hazard rates at the midpoints of each of the time intervals. The formulas used were presented earlier.

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This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \leq T < T_{t+1}$. Note that the hazard rate is actually computed at the midpoint of each interval.

Cumulative Hazard

This is the estimate of the hazard rate, $h(T_{mt})$.

Standard Error

This is the estimated standard error of the above hazard rate. The formula used was given earlier. The standard error is the square root of this variance.

Lower and Upper Confidence Limits

The lower and upper confidence limits provide a pointwise confidence interval for the hazard rate at the midpoint of the time interval. These limits are constructed so that the probability that the true hazard rate lies between them is $1 - \alpha$. Note that these limits are constructed for a single time point. Several of them cannot be used together to form a confidence band such that the entire hazard rate lies within the band.

Three difference confidence intervals are available. All three confidence intervals perform about the same in large samples. The linear (Greenwood) interval is the most commonly used. However, the log-transformed and the arcsine-square intervals behave better in small to moderate samples, so they are recommended. The formulas for these limits were given at the beginning of the chapter and are not repeated here.

| | Death | Standard | Lower | Upper |
|------|---------|----------|----------|----------|
| Time | Density | Error | 95% C.L. | 95% C.L. |
| 0.0 | 0.18859 | 0.00796 | 0.17299 | 0.20418 |
| 1.0 | 0.09440 | 0.00598 | 0.08269 | 0.10612 |
| 2.0 | 0.06464 | 0.00507 | 0.05471 | 0.07458 |
| 3.0 | 0.07380 | 0.00543 | 0.06317 | 0.08444 |
| 4.0 | 0.05931 | 0.00495 | 0.04961 | 0.06900 |
| 5.0 | 0.05813 | 0.00503 | 0.04827 | 0.06800 |
| 6.0 | 0.04392 | 0.00469 | 0.03472 | 0.05311 |
| 7.0 | 0.04601 | 0.00518 | 0.03587 | 0.05615 |
| 8.0 | 0.03697 | 0.00502 | 0.02713 | 0.04682 |
| 9.0 | 0.03554 | 0.00531 | 0.02513 | 0.04594 |
| 10.0 | 0.04303 | 0.00627 | 0.03074 | 0.05532 |
| 11.0 | 0.04209 | 0.00685 | 0.02867 | 0.05551 |
| 12.0 | 0.02968 | 0.00668 | 0.01659 | 0.04278 |
| 13.0 | 0.02031 | 0.00651 | 0.00754 | 0.03307 |
| 14.0 | 0.02066 | 0.00804 | 0.00491 | 0.03641 |
| 15.0 | | | | |

Death Density Function

This report displays estimates of the hazard rates at the midpoints of each of the time intervals. The formulas used were presented earlier.

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \leq T < T_{t+1}$. Note that the hazard rate is actually computed at the midpoint of each interval.

Death Density

This is the estimate of the death density, $f(T_{mt})$.

Standard Error

This is the estimated standard error of the above density. The formula used was given earlier. The standard error is the square root of this variance.

Lower and Upper Confidence Limits

The lower and upper confidence limits provide a pointwise confidence interval for the death density at the midpoint of the time interval. These limits are constructed so that the probability that the true density lies between them is $1 - \alpha$.

Three difference confidence intervals are available. All three confidence intervals perform about the same in large samples. The linear (Greenwood) interval is the most commonly used. However, the log-transformed and the arcsine-square intervals behave better in small to moderate samples, so they are recommended. The formulas for these limits were given at the beginning of the chapter and are not repeated here.

Median Remaining Lifetime

Median Remaining Lifetime

| | Median | . | _ | |
|------|-----------------------|-------------------|-------------------|-------------------|
| Time | Remaining Lifetime | Standard Error | Lower 95% C.L. | Upper 95% C.L. |
| 0.0 | 5.3 | 0.17491 | 5.0 | 5.7 |
| 1.0 | 6.2 | 0.20006 | 5.9 | 6.6 |
| 2.0 | 6.3 | 0.23614 | 5.9 | 6.8 |
| 3.0 | 6.2 | 0.23609 | 5.8 | 6.7 |
| 4.0 | 6.2 | 0.18526 | 5.9 | 6.6 |
| 5.0 | 5.9 | 0.18059 | 5.6 | 6.3 |
| 6.0 | 5.6 | 0.18554 | 5.2 | 6.0 |
| 7.0 | 5.2 | 0.27129 | 4.6 | 5.7 |
| 8.0 | 4.9 | 0.27632 | 4.4 | 5.5 |
| 9.0 | 4.8 | 0.41408 | 4.0 | 5.6 |
| 10.0 | 4.7 | 0.41835 | 3.9 | 5.5 |
| 11.0 | | | | |
| 12.0 | | | | |
| 13.0 | | | | |
| 14.0 | | | | |
| 15.0 | | | | |

This report displays estimates of the median remaining lifetime of those who are alive at the beginning of the interval. The formulas used were presented earlier.

This is the left boundary of the time interval reported on this line. The right boundary is the entry on the following line. Each interval is represented by $T_t \le T < T_{t+1}$.

Median Remaining Lifetime

This is the estimate of the median remaining lifetime of an individual who survives to the beginning of this interval.

Standard Error

This is the estimated standard error of the above lifetime. The formula used was given earlier.

Lower and Upper Confidence Limits

The lower and upper confidence limits provide a pointwise confidence interval for the hazard rate at the midpoint of the time interval. These limits are constructed so that the probability that the true remaining lifetime lies between them is $1 - \alpha$.

These confidence intervals are highly approximate. They depend on the assumption that the median remaining lifetime is normally distributed. This may not be true even in large samples.

Survival Plot(s)



This plot shows the survivorship function. If there are several groups, a separate line is drawn for each group. The plot includes the number at risk at several time points.

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Cumulative Hazard Function Plot(s)

Cumulative Hazard Function Plot(s)



This plot shows the Nelson-Aalen cumulative hazard function. If you have several groups, then a separate line is drawn for each group.

Hazard Rate Plot(s)





This plot shows the hazard rates. Note the unusual step-like appearance of the plot because the hazard rates are assumed constant for the duration of the interval.

Validation of Life-Table Estimator using Lee (1992)

This section presents validation of our life-table estimator. Lee (1992) presents an example on page 91 of a calculated life table. We will include the results of one line of that table so that you can compare those results with those produced by this program. If you compare these values with those shown above, you can validate that **NCSS** provides the correct results.

| <u>Parameter</u> | <u>Value</u> |
|----------------------|--------------|
| Time | 3 |
| Lost | 23 |
| Dying | 171 |
| Entering | 1523 |
| Exposed | 1511.5 |
| Proportion Dying | 0.1131 |
| Proportion Surviving | 0.8869 |
| S(T) | 0.6524 |
| S.E. S(T) | 0.0097 |
| h(T) | 0.1199 |
| S.E. h(T) | 0.0092 |
| Median Rem. Lifetime | 6.23 |
| S.E. MRL(T) | 0.9 |
| f(T) | 0.0738 |
| S.E. f(T) | 0.0054 |
| S.E. f(T) | 0.0054 |