Chapter 480

Linear Programming with Tableau

Introduction

Linear programming maximizes a linear objective function subject to one or more constraints. The technique finds broad use in operations research and is included here because it is occasionally of use in statistical work.

The mathematical representation of the linear programming (LP) problem is

Maximize

$$Z = C_1 X_1 + C_2 X_2 + \cdots + C_n X_n$$

Subject to

$$X_1 \geq 0, X_1 \geq 0, \cdots, X_n \geq 0$$

$$a_{i1} X_1 + a_{i2} X_2 + \cdots + a_{in} X_n \{\leq, =, \geq\}\, b_i \geq 0 \quad i = 1, \cdots, m$$

The X's are called *decision variables* (the unknowns), the first equation is called the *objective function* and the m inequalities (and equalities) are called *constraints*. The b_i's are often called *right-hand sides* (RHS).

The *simplex* algorithm, which solves this problem, was discovered by George Dantzig in 1947. We use a modified version of the revised simplex algorithm given by Press, Teukolsky, Vetterling, and Flannery (1992).

Example

We will solve the following problem using **NCSS**:

Maximize

$$Z = X_1 + X_2 + 2X_3 - 2X_4$$

subject to

$$X_1 + 2X_3 \leq 700$$
$$2X_2 - 8X_3 \leq 0$$
$$X_2 - 2X_3 - X_4 \geq 1$$
$$X_1 + X_2 + X_3 + X_4 = 10$$

The solution is $X_1 = 9$, $X_2 = 8$, $X_3 = 0$, and $X_4 = 0.2$ which results in $Z = 9.4$.

Data Structure

This technique requires a special data format. The coefficients of the object function are stored in one (usually the first) row. The constraints are stored one to a row. The type of constraint (less than, greater than, or equal to) is stored in a column. The following is an example of how to store the above example in an **NCSS** dataset. This particular dataset is called LP.

LP Dataset

X1	X2	X3	X4	Logic	RHS
1	1	2	-2	O	
1		2		LT	700
	2		-8	LT	0
	1	-2	1	GT	1
1	1	1	1	EQ	10

Example 1 – Linear Programming

This section presents an example of how to run the data presented in the example given above. The data are contained in the LP database.

Setup

To run this example, complete the following steps:

1   Open the LP example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **LP** and click **OK**.

2   Specify the Linear Programming with Tableau procedure options

- Find and open the **Linear Programming with Tableau** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Constraint (A) Variables...................................**X1-X4**

Logic Variable...**Logic** (Select Logic as the Logic Variable since this variable contains the logical sign of each constraint.)

Bounds (R.H.S.) Variable ....**RHS** (Select RHS as the Bounds (R.H.S.) Variable since this variable contains the logical sign of each constraint.)

3   Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Initial Tableau

Initial Tableau

Row	X1	X2	X3	X4	RHS
1 Objective Function	1	1	2	-2	0
2 <=	1	0	2	0	700
3 <=	0	2	0	-8	0
4 >=	0	1	-2	1	1
5 =	1	1	1	1	10

This report lists the initial values so you can double check the input.

Optimal Solution

Optimal Solution

Variable	Optimal Value	Cost		Status
		Original	Reduced	
X1	9.0	1	0.0	Basis
X2	0.8	1	0.0	Basis
X3	0.0	2	-0.2	Non-Basis
X4	0.2	-2	0.0	Basis
Objective Function	9.4			

This report presents the solution. It shows the optimal value of each variable.

Variable

The variables that are being solved for.

Optimal Value

These are the values of the independent variables that result in a maximum value of the objective function. The maximum value of the objective function is given as the last line of the report.

Original Cost

These are the values of the coefficients of the objective functions. These are the C's.

Reduced Cost

The reduced costs are an additional output of the simplex method.

Status

This column gives the status of each independent variable in final solution. The solution is found by ignoring some variables (setting their values to zeros). When a variable is ignored, it is said to be a "non-basis" variable. When a variable is not ignored, it is said to be a "basis" variable.

Constraints

Constraints

Row Number	Type	RHS		Constraint
		Original	Optimal	
2	<=	700	9	X1 + 2X3
3	<=	0	0	2X2 - 8X4
4	>=	1	1	X2 - 2X3 + X4
5	=	10	10	X1 + X2 + X3 + X4

This report presents an analysis of each constraint when the variables are set to their optimal values.

Row Number

The row of the database from which this constraint comes.

Type

The type of constraint that this row represents.

Original RHS

The original value of the right-hand side of the constraint.

Optimal RHS

The value of this constraint at the optimal solution.

Constraint

The first forty characters of the constraint.

Final Tableau

Final Tableau

Variable	X3	Slack2	Art1	Slack3	RHS
Z	-0.2	-0.3	1	-0.6	9.4
Slack1	-1.0	0.0	1	-1.0	691.0
X2	-1.6	0.1	0	-0.8	0.8
X4	-0.4	-0.1	0	-0.2	0.2
X1	3.0	0.0	-1	1.0	9.0

This report presents the final values of the simplex tableau. The variables listed down the left side are the basis variables. These are the variables that are active in the solution. The variables listed across the top are the non-basis variables. These variables were not in the solution.

A slack variable is generated for each inequality constraint. An artificial variable is generated for each equality constraint. The values in the RHS column are the solution values.