

## Chapter 491

# Maximum Flow

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## Introduction

Given a directed network defined by nodes, arcs, and flow capacities, this procedure finds the maximum flow that can occur between a source node and a sink node. An example of this is the flow of oil through a pipeline with several junctions.

There are specialized algorithms that can be used to solve for the maximum flow. Taha (2011) and Hillier and Lieberman (2015) show that the problem can be solved using a linear programming algorithm. This is the approach we have taken in NCSS.

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## Data Structure

This procedure requires a special data format in which the rows represent the arcs (branches) between two locations (nodes). Each arc is defined by two nodes and the positive flow amount. The flow is always from the first node to the second node.

If flow can occur in two directions, a separate row must be added for each flow. This occurs in the example below with nodes N3 and N4.

Note that by definition, you cannot have flow into the source node or out of the sink node. If you try to, an error will occur.

Consider the following material transportation example. Each row represents a pipe (arc) between two junctions (nodes). The junctions are identified by letters and the flow in gallons per unit time. We assume that N1 is the 'source' node and N5 is the 'sink' node.

These data are stored in the dataset *Maximum Flow*.

### Maximum Flow dataset

Node1	Node2	Flow
N1	N2	25
N1	N3	29
N1	N4	11
N2	N3	39
N2	N5	28
N3	N4	12
N3	N5	18
N4	N3	6
N4	N5	22

## Maximum Flow

## Linear Programming Formulation of the Maximum Flow Problem

As stated earlier, we use a linear programming algorithm to solve for the maximum. Rather than present all the equations, we show how the above example is translated into a linear programming tableau. The LP tableau for the above data is

		X <sub>12</sub>	X <sub>13</sub>	X <sub>14</sub>	X <sub>23</sub>	X <sub>25</sub>	X <sub>34</sub>	X <sub>35</sub>	X <sub>43</sub>	X <sub>45</sub>	RHS
Maximize	z =	1	1	1							
<b>Nodes</b>											
N2		1			-1	-1					= 0
N3			1		1		-1	-1	1		= 0
N4				1			1		-1	-1	= 0
Capacities (Bounds)		25	29	11	39	28	12	18	6	22	

## Procedure Options

This section describes the options available in this procedure.

### Specifications Tab

Set the specifications for the analysis.

### Starting and Ending Nodes of Each Arc

#### Node 1 (Starting) Column

Specify the column containing the starting node of each arc.

An arc connects two nodes. Each row specifies of a single, directed arc by giving its beginning node and ending node as well as the flow capacity from start to end. Because the flow can only be in one direction, the order of the arcs is critical. If flows in either direction are possible, a separate arc must be entered for each direction.

Each node may be specified by a number or a text value.

A directed arc is defined as a starting node followed by an ending node along with the flow capacity between them. Other names for arcs are edges, lines, and links.

A source and sink node must be entered. The source node cannot have flow into it, thus the source node can only occur in the Node 1 Column. A sink node cannot have flow leaving it.

#### Node 2 (Ending) Column

Specify the column containing the ending node of each arc.

An arc connects two nodes. Each row specifies of a single, directed arc by giving its starting node and ending node as well as the flow capacity from start to end. Because the flow can only be in one direction, the order of the arcs is critical. If flows in either direction are possible, a separate arc must be entered for each direction.

Each node may be specified by a number or a text value.

A directed arc is defined as a starting node followed by an ending node along with the flow capacity between them. Other names for arcs are edges, lines, and links.

A source and sink node must be entered. The source node cannot have flow into it. A sink node cannot have flow leaving it. Thus the sink node can only occur in the Node 2 Column.

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### Arc Capacities

#### Flow Capacity Column

Specify the column containing the arc's flow capacity. This is the upper maximum that can flow from node 1 to node 2.

If flows can occur in both directions, a separate row must be entered for each flow.

The values must be positive numbers.

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### Source and Sink Nodes

#### Node Values Input Type

Specify how you will enter the node names of the source and sink nodes. The possible selections are

*Use the first and last node names after sorting*

The names of the nodes will be sorted and the first node name will be designated as the starting node value and the last node name will be designated as the ending node value.

*Enter custom source and sink values*

Two text boxes will appear that will allow you to enter the names of the source and sink nodes.

#### Source (Starting) Node Value

Enter the value (number or text) of the source node. The procedure will find the maximum flow from this node to the sink node.

#### Sink (Ending) Node Value

Enter the value (number or text) of the sink node. The procedure will find the maximum flow from the source node to this node.

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## Reports Tab

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### Select Reports

#### Maximum Flow Solution, Possible Network Arcs

Indicate which reports you want to view.

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### Report Options

#### Column Names

This option lets you select whether to display only variable names, variable labels, or both.

## Example 1 – Maximum Flow through a Pipeline

This section presents an example of how to find the maximal flow between nodes N1 and N5 in the following network. As you use this example, you need to keep in mind that all flows are in only one direction.

### Maximum Flow dataset

Node1	Node2	Flow
N1	N2	25
N1	N3	29
N1	N4	11
N2	N3	39
N2	N5	28
N3	N4	12
N3	N5	18
N4	N3	6
N4	N5	22

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Maximum Flow window.

#### 1 Open the Max Flow dataset.

- From the **File** menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Max Flow.NCSS**.
- Click **Open**.

#### 2 Open the Maximum Flow window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Maximum Flow** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

#### 3 Specify the problem.

- On the **Maximum Flow** window, select the **Specifications tab**.
- Double-click in the **Node 1 (Starting) Column** text box. This will bring up the column selection window.
- Select **Node1** from the list of columns and then click **Ok**. “Node1” will appear in this box.
- Double-click in the **Node 2 (Ending) Column** text box. This will bring up the column selection window.
- Select **Node2** from the list of columns and then click **Ok**. “Node2” will appear in this box.
- Double-click in the **Flow Capacity Column** text box. This will bring up the column selection window.
- Select **Flow** from the list of columns and then click **Ok**. “Flow” will appear in this box.
- Set the **Node Value Input Type** to **Enter custom source and sink values**.
- Set **Source (Starting) Node Value** to **N1**.
- Set **Sink (Ending) Node Value** to **N5**.

#### 4 Run the procedure.

- From the **Run** menu, select **Run Procedure**. Alternatively, just click the green Run button.

## Maximum Flow

## Maximum Flow Solution

Maximum Flow				
Arc (Row)	Maximum Flow Through Arc	Flow (Capacity)	Node1 (Start)	Node2 (End)
1	25	25	N1 (Source)	N2
2	29	29	N1 (Source)	N3
3	11	11	N1 (Source)	N4
5	25	28	N2	N5 (Sink)
6	11	12	N3	N4
7	18	18	N3	N5 (Sink)
9	22	22	N4	N5 (Sink)
Total	65			

This report lists the solution by giving the arcs that form the maximal flow. The starting node is identified with the word *Source*. The ending node is identified with the word *Sink*.

The maximum flow through the network is 65.

## Possible Network Arcs

Possible Network Arcs			
Arc (Row)	Flow (Capacity)	Node1 (Start)	Node2 (End)
1	25*	N1 (Source)	N2
2	29*	N1 (Source)	N3
3	11*	N1 (Source)	N4
4	39	N2	N3
5	28*	N2	N5 (Sink)
6	12*	N3	N4
7	18*	N3	N5 (Sink)
8	6	N4	N3
9	22*	N4	N5 (Sink)

\* The starred links are part of the maximum flow.

This report lists all the arcs from which the maximal flow was derived. The starting node is identified with the word *Source*. The ending node is identified with the word *Sink*.