Chapter 492

Minimum Cost Capacitated Flow

Introduction

The *minimum cost capacitated flow* model is prominent among network flow models because so many other network models are special cases. The maximum flow, shortest-path, transportation, transshipment, and assignment models are all special cases of this model.

There are specialized algorithms that can be used to solve the minimum cost flow model. However, Hillier and Lieberman (2015) show that the model can be solved using linear programming. This is the approach we have taken in NCSS.

Data Structure

This procedure requires a special data format that includes two regions of the spreadsheet. These are the Arc Region which define the arcs, and the Node Region which defines the net flows at the nodes.

An Arc Region, defined using spreadsheet columns, gives the arcs (links, branches, or edges) of the network. Each row in this region represents an arc. The arc definition includes the name of starting node, ending node, incremental cost (cost of a single product), upper bound (if any), and lower bound (if any). The algorithm searches for the flows in the arcs. If flows can occur in both directions within an arc, two arcs must be included in the model.

A Node Region, defined using additional spreadsheet columns, gives the net flow through each node.

Example Model

The following dataset shows how to prepare the example on page 399 of Hillier and Lieberman (2015) for analysis. Suppose a company produces a product at two factories. It can ship product among factories, from factory to a warehouse, or from factory to two stores. Although not all factories, stores, and the warehouse are connected, many are. These connections are defined using columns Node1 and Node2. The cost of shipping a single product unit are given in the IncCost column. There are no special lower bounds on shipment amounts within the arcs. However, only up to 10 units can be sent from Factory1 to Factory2 and only up to 80 units can be sent from the warehouse to Store1.

The supply and demand values are defined in the Node Region. The amount produced by each factory (the supply) are shown as positive net flows. Thus, Factor1 produces 50 units and Factory2 produces 40 units. The amounts needed at the two stores are 30 and 60 units. These are shown as negative numbers since these units are leaving the network. Since the warehouse does not consume any product, its net flow is zero.

Note that the sum of the net flows is exactly zero. This is a requirement of the model.

These data are stored in the dataset *Min Cost Flow*.

Min Cost Flow dataset

Node1	Node2	IncCost	UpperBnd	LowerBnd	Node	NetFlow
Factory1	Factory2	2	10	0	A: Factory1	50
Factory1	Warehouse	4		0	B: Factory2	40
Factory1	Store1	9		0	C: Warehouse	0
Factory2	Warehouse	3		0	D: Store1	-30
Warehouse	Store1	1	80	0	E: Store2	-60
Store1	Store2	3		0		
Store2	Store1	2		0		

Linear Programming Formulation of the Minimum Cost Capacitated Flow Model

As stated earlier, we use a linear programming algorithm to solve for the maximum. Rather than present all the equations, we show how the above example is translated into a linear programming tableau. The LP tableau for the above data is

		X_{AB}	X_{AC}	X_{AD}	X_{BC}	X_{CE}	X_{DE}	X_{ED}	RHS
Maximize	z =	2	4	9	3	1	3	2	
Nodes									
A		1	1	1					= 50
B		-1			1				= 40
C			-1		-1	1			= 0
D				-1			1	-1	= -30
E						-1	-1	1	= -60
Capacity		10				80			

Procedure Options

This section describes the options available in this procedure.

Specifications Tab

Set the specifications for the analysis.

Optimum Type

Type of Optimum

Specify whether to find the minimum or the maximum of the objective function in the constrained region.

Arcs

Arc Node 1 (Starting) Column

Specify the column containing the starting node of each arc.

An arc connects two nodes. Each row specifies of a single, directed arc by giving its beginning node and ending node as well flow, cost, and bounds (if any). Because the flow is in only one direction, the order of the nodes is critical. If flows in either direction are possible, a separate arc (on a separate row) must be entered for each direction.

Each node may be specified by a number or a text value.

A directed arc is defined as a starting node followed by an ending node. Other names for arcs are edges, branches, and links.

Arc Node 2 (Ending) Column

Specify the column containing the ending node of each arc.

An arc connects two nodes. Each row specifies of a single, directed arc by giving its starting node and ending node. Because the flow is only defined in one direction, the order of the nodes is critical. If flows in either direction are possible, a separate arc must be entered for each direction.

Each node may be specified by a number or a text value.

A directed arc is defined as a starting node followed by an ending node along with the flow capacity between them. Other names for arcs are edges, branches, and links.

Arc Cost Column

Specify the column containing the per-unit cost associated with the flow through the arc defined on this row. Negative values are unusual, but can be used. Blanks are treated as zeros.

Arc Minimum Flow Column

Specify an optional column containing a lower bound on the arc's flow capacity. This is the minimum that must flow from node 1 to node 2. Usually, this value is assumed to be zero.

The values must be non-negative numbers. If the flow in a particular arc is zero, you can leave that cell blank.

Arc Flow Capacity Column

Specify the column containing an upper bound (capacity) of the arc's flow. This is the maximum that can flow from node 1 to node 2 through this arc.

The values must be positive numbers. If the flow in a particular arc is unlimited, leave that cell blank.

If flows can occur in both directions, a separate arc (row) must be entered for each flow.

Nodes

Node Name Column

This column contains the names of the nodes in the problem. The names can be text or numeric. Capitalization is ignored. The names used here must match the names used in the Node 1 and Node 2 columns. Any extra names entered here will be ignored. If names in Node 1 and Node 2 are omitted here, they will receive a net flow of zero.

This column is only used to identify the net flow at each node. It is not associated with the arc that may also be defined in other columns of this spreadsheet row.

Node Net Flow Column

This value defines the net flow through the node. The sign of the value defines the type of node this is:

If the net flow > 0, the node is a *source* (supply) node.

If the net flow < 0, the node is a *destination* (demand) node.

If the net flow = 0, the node is a *transshipment* (junction or distribution) node.

The net flow values must sum to zero in order for the problem to have a feasible solution. If they don't, you must adjust the net flow values or add a dummy source or destination node so that they do.

Reports Tab

Select Reports

Solution Reports, Model Network Arcs

Indicate which reports you want to view.

Report Options

Column Names

This option lets you select whether to display only variable names, variable labels, or both.

Example 1 – Minimum Cost Flow from Factory to Store

Example Model

The following dataset shows how to prepare the example on page 399 of Hillier and Lieberman (2015) for analysis. Suppose a company produces a product at two factories. It can ship product among factories, from factory to a warehouse, or from factory to two stores. Although not all factories, stores, and the warehouse are connected, many are. These connections are defined using columns Node1 and Node2. The cost of shipping a single product unit are given in the IncCost column. There are no special lower bounds on shipment amounts within the arcs. However, only up to 10 units can be sent from Factory1 to Factory2 and only up to 80 units can be sent from the warehouse to Store1.

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Min Cost Flow dataset

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Factory1	Warehouse	4		0	B: Factory2	40
Factory1	Store1	9		0	C: Warehouse	0
Factory2	Warehouse	3		0	D: Store1	-30
Warehouse	Store1	1	80	0	E: Store2	-60
Store1	Store2	3		0		
Store2	Store1	2		0		

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Minimum Cost Capacitated Flow window.

1 Open the Min Cost Flow dataset.
- From the **File** menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Min Cost Flow.NCSS**.
- Click **Open**.

2 Open the Minimum Cost Capacitated Flow window.
- Using the Analysis menu or the Procedure Navigator, find and select the **Minimum Cost Capacitated Flow** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the problem.
- On the **Minimum Cost Capacitated Flow** window, select the **Specifications tab**.
- Double-click in the **Arc Node 1 (Starting) Column** text box. This will bring up the column selection window.
- Select **Node1** from the list of columns and then click **Ok**. “Node1” will appear in this box.
- Double-click in the **Node 2 (Ending) Column** text box. This will bring up the column selection window.
- Select **Node2** from the list of columns and then click **Ok**. “Node2” will appear in this box.
- Double-click in the **Arc Cost Column** text box. This will bring up the column selection window.
- Select **IncCost** from the list of columns and then click **Ok**. “IncCost” will appear in this box.
- Double-click in the **Arc Flow Capacity Column** text box. This will bring up the column selection window.
- Select **UpperBnd** from the list of columns and then click **Ok**. “UpperBnd” will appear in this box.
- Double-click in the **Node Name Column** text box. This will bring up the column selection window.
- Select **Node** from the list of columns and then click **Ok**. “Node” will appear in this box.
- Double-click in the **Node Net Flow Column** text box. This will bring up the column selection window.
- Select **NetFlow** from the list of columns and then click **Ok**. “NetFlow” will appear in this box.

4 Run the procedure.
- From the **Run** menu, select **Run Procedure**. Alternatively, just click the green Run button.

Solution

Arc Row	Solution Flow	IncCost	Total Cost	UpperBnd	Node1	Node2
2	40	4	160		Factory1	Warehouse
3	10	9	90		Factory1	Store1
4	40	3	120		Factory2	Warehouse
5	80	1	80	80	Warehouse	Store1
6	60	3	180		Store1	Store2
Total	230		630			

The solution is optimal.

This report shows the flows through each of the arcs. Arcs not listed receive a flow of zero. The flow through the network is 230 units at a cost of 630.

Node Report

Type	Node	Flow In	Flow Out	Net Flow Out - In
Source	Factory1	0	50	50
Source	Factory2	0	40	40
Transshipment	Warehouse	80	80	0
Destination	Store1	90	60	-30
Destination	Store2	60	0	-60

This report shows the flows through each of the nodes. The *Flow In* is the sum of the flows of all arcs enter this node. The *Flow Out* is the sum of the flows of all arcs that leave this node.

Model Network Arcs

Arc Row	IncCost	UpperBnd	Node1	Node2
1	2	10	Factory1[1]	Factory2[1]
2	4		Factory1[1]	Warehouse
3	9		Factory1[1]	Store1[2]
4	3		Factory2[1]	Warehouse
5	1	80	Warehouse	Store1[2]
6	3		Store1[2]	Store2[2]
7	2		Store2[2]	Store1[2]

[1] Source (Supply) Node
[2] Destination (Demand) Node

This report lists all the arcs in the model as they were given on the dataset. The supply nodes are identified with a superscript of *1*. The demand nodes are identified with the superscript of *2*.