Chapter 248

Moving Average Charts

Introduction

This procedure generates moving average control charts for variables. The format of the control charts is fully customizable. The data for the subgroups can be in a single column or in multiple columns. This procedure permits the defining of stages. The target value can be entered directly or estimated from the data, or a sub-set of the data. Sigma may be estimated from the data, or a standard sigma value may be entered. Means can be stored to the spreadsheet.

Moving Average Control Charts

The moving average chart is control chart for the mean that uses the average of the current mean and a handful of previous means to produce each moving average. Moving average charts are used to monitor the mean of a process based on samples taken from the process at given times (hours, shifts, days, weeks, months, etc.). The measurements of the samples at a given time constitute a subgroup.

The moving average chart relies on the specification of a target value and a known or reliable estimate of the standard deviation. For this reason, the moving average chart is better used after process control has been established.
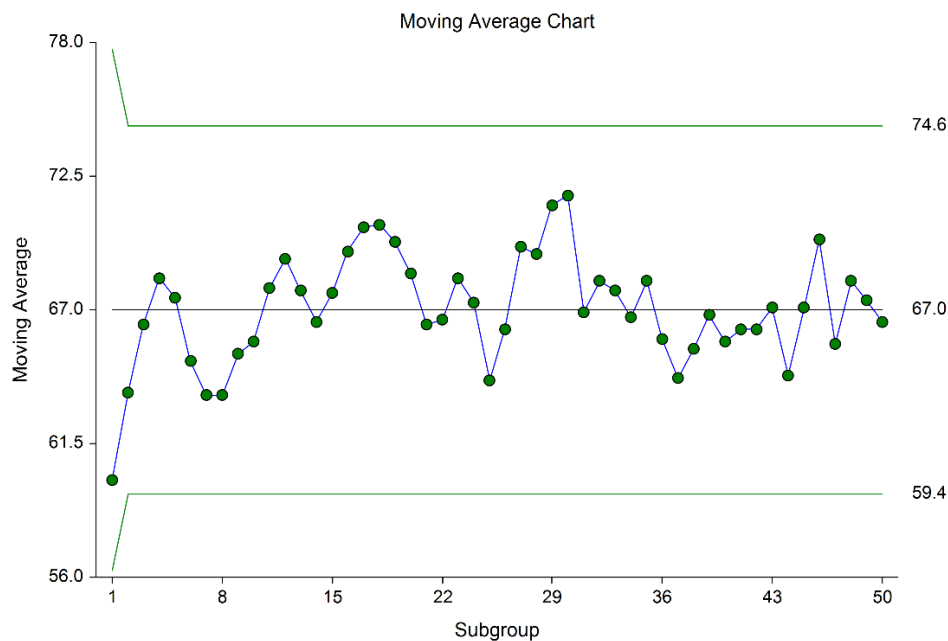
Other Control Charts for the Mean and Variation of a Process

When monitoring the mean, the process variation is usually monitored as well using either the range or the standard deviation. Two procedures for monitoring both the mean and the variation are the X-bar and R, and X-bar and s charts.

Two additional control charts available for monitoring the process mean are the cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts. The CUSUM and EWMA charts are somewhat similar to the moving average charts in that they take into account the information of previous means at each point. CUSUM and EWMA methods also assume a reliable estimate or known value for the true standard deviation is available.

The moving average chart may also be used when only a single response is available at each time point. Another option for single responses is the individuals and moving range (I-MR) control charts. CUSUM and EWMA charts may also be used for single responses and are useful when small changes in the mean need to be detected.

Control Chart Formulas

Suppose we have k subgroups, each of size n. Let x_{ij} represent the measurement in the jth sample of the ith subgroup.

Formulas for the Points on the Chart

The ith subgroup mean is calculated using

$$\bar{x}_i = \frac{\sum_{j=1}^n x_{ij}}{n}$$

The points of the chart are obtained from the $\bar{x}_i$'s by taking the average of the last w_i subgroups (including the current subgroup), where w_i is the user-specified moving average width.

$$M_i = \frac{\bar{x}_i + \bar{x}_{i-1} + \cdots + \bar{x}_{i-w_i+1}}{w_i}$$

Note that the value of w_i changes during the first few subgroups and then stays constant at the value set by the user.

Estimating the Moving Average Chart Center Line (Grand Mean)

In the Moving Average Charts procedure, the target mean may be input directly, or it may be estimated from a series of subgroups. If it is estimated from the subgroups the formula for the grand average is

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij}}{\sum_{i=1}^{k} n_i}.$$

If the subgroups are of equal size, the above equation for the grand mean reduces to

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{k} \bar{x}_i}{k} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k}.$$

Estimating Sigma – Sample Ranges

Either the range or the standard deviation of the subgroups may be used to estimate sigma, or a known (standard) sigma value may be entered directly. If the standard deviation (sigma) is to be estimated from the ranges, it is estimated as

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

where

$$\bar{R} = \frac{\sum_{i=1}^{k} R_i}{k}$$

$$d_2 = \frac{E(R)}{\sigma} = \frac{\mu_R}{\sigma}$$

The calculation of $E(R)$ requires the knowledge of the underlying distribution of the x_{ij}'s. Making the assumption that the x_{ij}'s follow the normal distribution with constant mean and variance, the values for d_2 are derived through the use of numerical integration. It is important to note that the normality assumption is used and that the accuracy of this estimate requires that this assumption be valid.

When n is one, we cannot calculate R_i since it requires at least two measurements. The procedure in this case is to use the ranges of successive pairs of observations. Hence, the range of the first and second observation is computed, the range of the second and third is computed, and so on. The average of these approximate ranges is used to estimate σ.

Estimating Sigma – Sample Standard Deviations

If the standard deviation (sigma) is to be estimated from the standard deviations, it is estimated as

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

where

$$\bar{s} = \frac{\sum_{i=1}^{k} s_i}{k}$$

$$c_4 = \frac{E(s)}{\sigma} = \frac{\mu_s}{\sigma}$$

The calculation of *E(s)* requires the knowledge of the underlying distribution of the x_{ij}'s. Assuming that the x_{ij}'s follow the normal distribution with constant mean and variance, the values for c_4 are obtained from

$$c_4 = \sqrt{\frac{2}{n-1}} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$$

Estimating Sigma – Weighted Approach

When the sample size is variable across subgroups, a weighted approach is recommended for estimating sigma (Montgomery, 2013):

$$\hat{\sigma} = \bar{s} = \left[\frac{\sum_{i=1}^{k}(n_i - 1)s_i^2}{\sum_{i=1}^{k} n_i - k}\right]^{1/2}$$

Moving Average Chart Limits

The lower and upper control limits for the moving-average chart are calculated using the formula

$$LCL_i = \mu_0 - m\left(\frac{\hat{\sigma}}{\sqrt{n_i w_i}}\right)$$

$$UCL_i = \mu_0 + m\left(\frac{\hat{\sigma}}{\sqrt{n_i w_i}}\right)$$

where m is a multiplier (usually set to three) and w_i is the number of rows used in this average. Note that the value of w_i changes during the first few subgroups and then stays constant at the value set by the user.

Data Structure

In this procedure, the data may be in either of two formats. The first data structure option is to have the data in several columns, with one subgroup per row.

Example Dataset

S1	S2	S3	S4	S5
2	6	3	8	5
8	8	7	7	9
6	2	2	4	3
5	6	7	6	10
48	2	6	5	0
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

The second data structure option uses one column for the response data, and either a subgroup size or a second column defining the subgroups.

Alternative Example Dataset

Response	Subgroup
2	1
6	1
3	1
8	1
5	1
8	2
8	2
7	2
7	2
9	2
6	3
2	3
.	.
.	.
.	.

In the alternative example dataset, the Subgroup column is not needed if every subgroup is of size 5 and the user specifies 5 as the subgroup size. If there are missing values, the Subgroup column should be used, or the structure of the first example dataset.

# Quality Control Chart Format Window Options

This section describes a few of the specific options available on the first tab of the control chart format window, which is displayed when a quality control chart format button is pressed. Common options, such as axes, labels, legends, and titles are documented in the Graphics Components chapter.
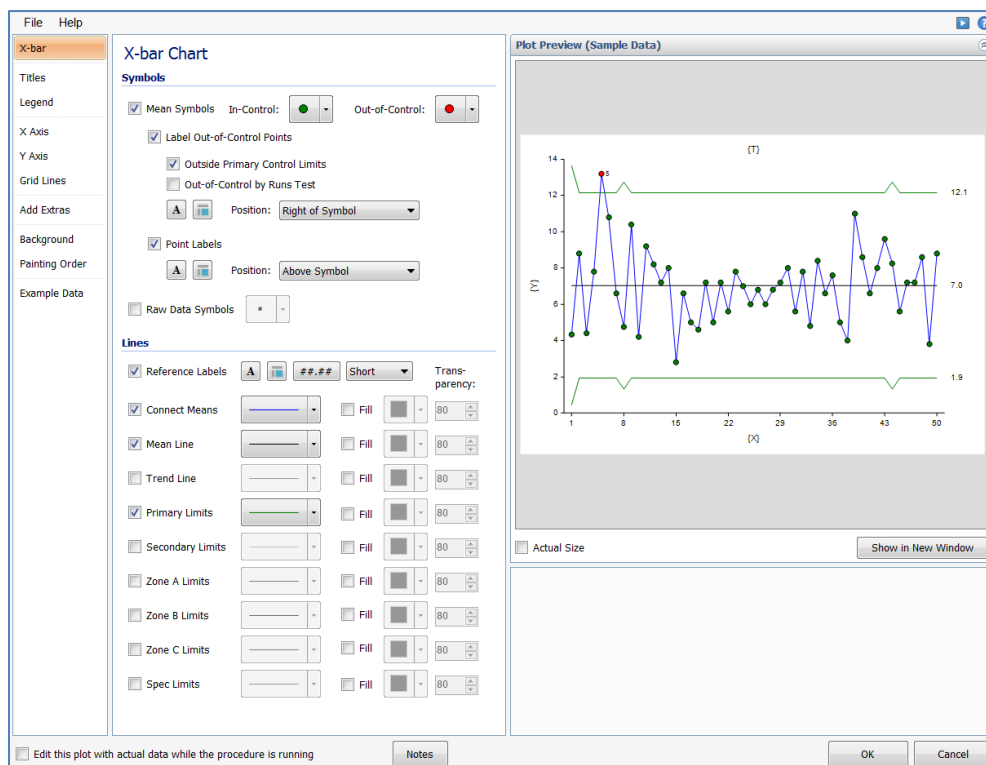
[Xbar] / [Range] Chart Tab

Symbols Section

You can modify the attributes of the symbols using the options in this section.
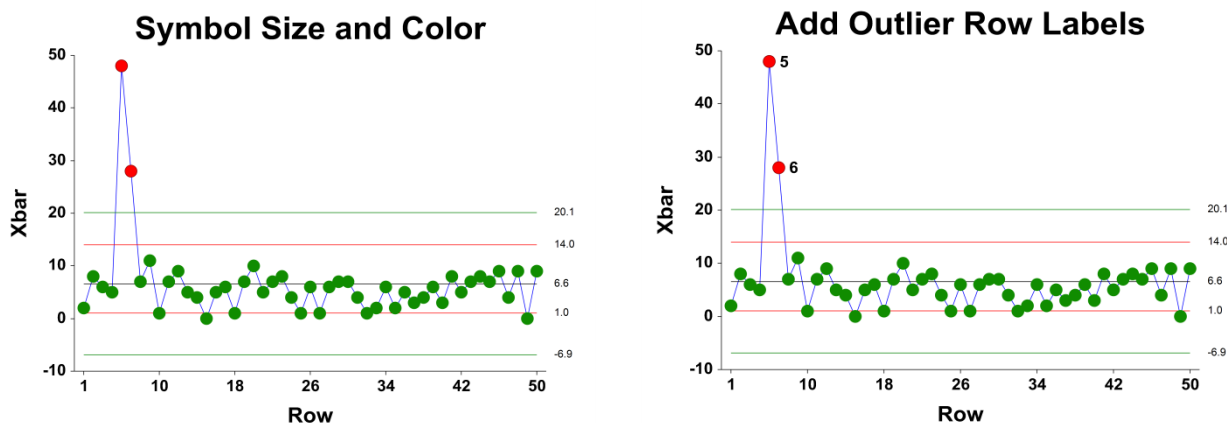
A wide variety of sizes, shapes, and colors are available for the symbols. The symbols for in-control and out-of-control points are specified independently. There are additional options to label out-of-control points. The label for points outside the primary control limits is the subgroup number. The label for points that are out-of-control based on the runs test is the number of the first runs test that is signaled by this point.

The user may also specify a column of point labels on the procedure variables tab, to be used to label all or some of the points of the chart. The raw data may also be shown, based on customizable raw data symbols.

Lines Section

You can specify the format of the various lines using the options in this section. Note that when shading is desired, the fill will be to the bottom for single lines (such as the mean line), and between the lines for pairs of lines (such as primary limits).
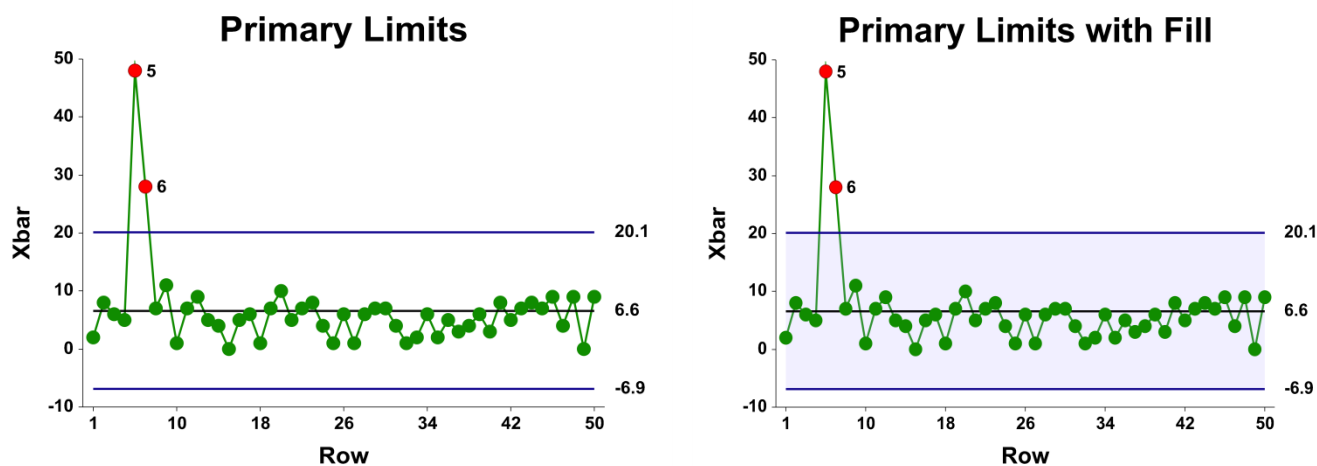
Lines for the zones, secondary limits, and specification limits are also specified here.

Titles, Legend, Numeric Axis, Group Axis, Grid Lines, and Background Tabs

Details on setting the options in these tabs are given in the Graphics Components chapter. The legend does not show by default but can easily be included by going to the Legend tab and clicking the Show Legend checkbox.

Example 1 – Moving Average Chart

This section presents an example of how to produce a moving average chart with a width of 5. The data represent 50 subgroups of size 5. The data used are in the QC dataset. We will analyze the variables D1 through D5 of this dataset.

Setup

To run this example, complete the following steps:

1   Open the QC example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **QC** and click **OK**.

2   Specify the Moving Average Charts procedure options

- Find and open the **Moving Average Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Data Variables **D1-D5**

Target & Sigma Tab

Target Value Specification...............................**Enter Target Value(s)**
Target Value(s) **67**
Sigma Specification ....**Enter Sigma Value(s)**
Sigma Value(s) **8**

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3   Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Target Value

Target Value (μ0) for Subgroups 1 to 50

Number of Subgroups: 50

Target Value Specification	Value
User-Specified Target Value	67

This section displays the target value that is used in the moving average chart.

Target Value Specification

This is the method by which the target value is obtained.

Value

This is the target value used to create the moving average chart.

Sigma Estimation

Sigma Estimation for Subgroups 1 to 50
_____

Number of Subgroups: 50
_____

Sigma Specification	Value
User-Specified Sigma	8
_____

This section shows the specification of the standard deviation used in the moving average chart.
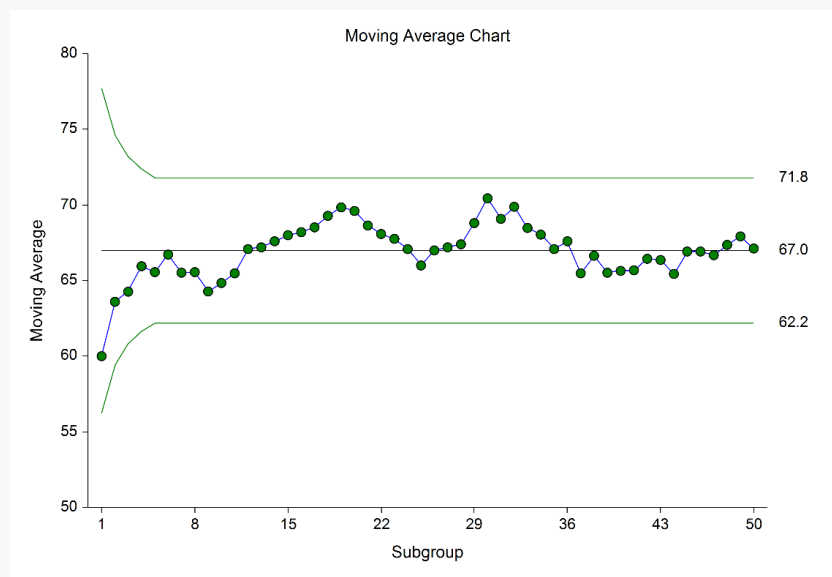
Sigma Specification

This is the method by which the sigma value is obtained.

Value

This is the value of sigma entered by the user and is the value used to create the moving average chart.

Moving Average Chart

Charts for Subgroups 1 to 50
_____

This moving average plot shows the progress of the moving averages across the 50 subgroups. There does not appear to be an indication of a change in the process mean.

Example 2 – Moving Average Chart with Additional Formatting

This example uses the same setup as Example 1, except that a variety of improvements are made in the plot format. These improvements are made by clicking the Moving Average Chart format button on the MA Chart tab.

The settings for this example are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu of the Moving Range Charts procedure window.

Moving Average Chart

Charts for Subgroups 1 to 50
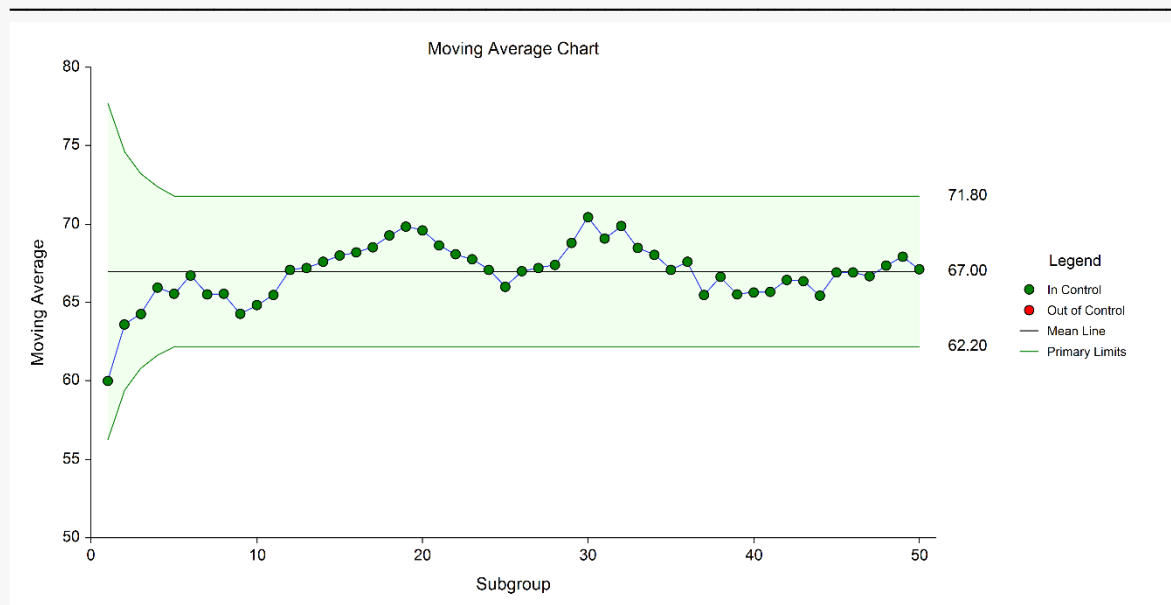
As shown here, a variety of enhancements can be made to the formatting of the control charts to make the chart as easy to read as possible.