

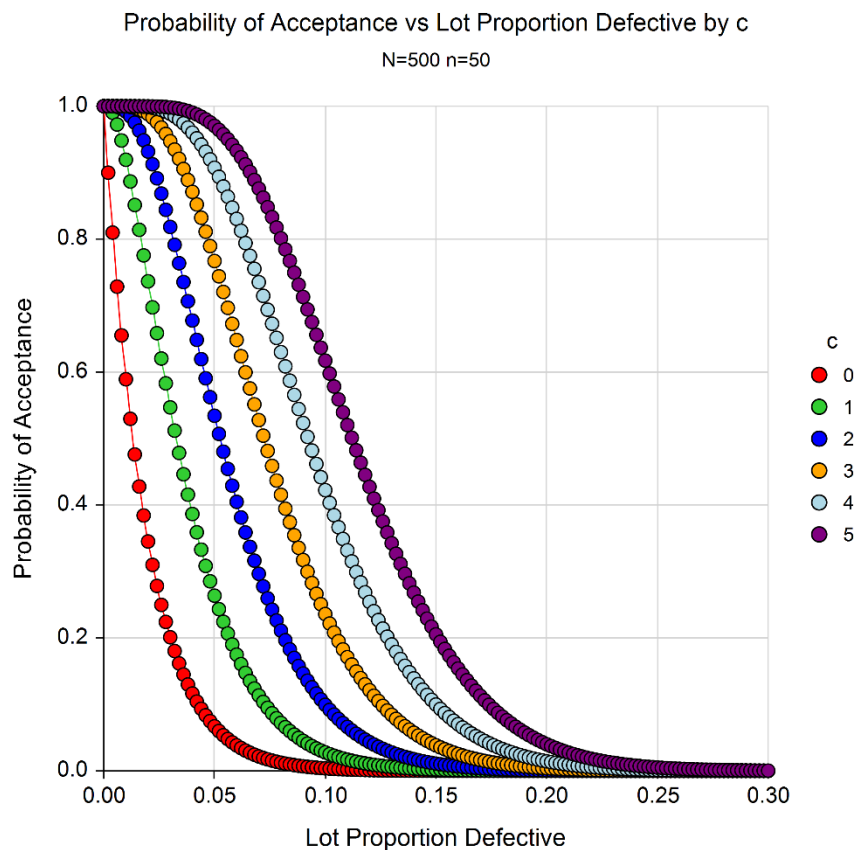
Chapter 239

Operating Characteristic Curves for Acceptance Sampling for Attributes

Introduction

The *Operating Characteristic Curves for Acceptance Sampling for Attributes* procedure is a companion procedure to the procedure *Acceptance Sampling for Attributes*. This procedure is used view graphically the probability of lot acceptance versus the lot proportion defective for a given sample size and acceptance number. Plots with multiple curves for multiple sample sizes or multiple acceptance numbers may be generated using this procedure.

In this procedure, the lot size can be assumed to be infinite (or continuous) and use the binomial distribution for calculations, or the lot can have a fixed size, whereupon the calculations are based on the hypergeometric distribution.



Technical details

Define N to be the lot size (possibly infinite), n as the size of the sample to be drawn, and c to be the acceptance number (the highest number of nonconforming units for which the lot will still be accepted). Let X denote the number of nonconforming units in the sample. If the lot size is assumed to be infinite, the probability of acceptance, P_a , for a given lot proportion defective, p , is given by

$$P_a = \sum_{X=0}^c \frac{n!}{X!(n-X)!} p^X (1-p)^{n-X}$$

If the lot size is finite, the hypergeometric distribution is used, where, for each X ,

$$h(X; N, M, n) = \frac{\binom{M}{X} \binom{N-M}{n-X}}{\binom{N}{n}}$$

where $M = [Np]$.

The probability of acceptance is given by

$$P_a = \sum_{X=0}^c h(X; N, M, n)$$

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains the parameters associated with this test such as the proportions, sample sizes, alpha, and power.

Lot Size

Lot Size (N)

Specify the total number of units in the lot that is subject to quality inspection. If the lot size is infinite or continuous, select Infinite for the Lot Size. In this case, the binomial distribution is used for calculations.

If the lot size is set to any value other than infinite, the hypergeometric distribution is used in the calculations. You can choose a value from the drop-down, or you can enter a value directly.

Operating Characteristic Curves for Acceptance Sampling for Attributes

Sample Size and Acceptance Number

Sample Size (n)

Enter one or more values for the sample size (n). This is the number of elements from the lot that will be randomly sampled and examined. You can enter a single value such as *100* or a series of values such as *50 60 70 80 90 100* or *100 to 150 by 10*. The Sample Size should be less than the Lot Size.

Acceptance Number (c)

This cutoff value is the largest number of nonconforming/defective items from the sample of size n for which the lot should still be accepted. You can enter a single value such as *1* or a series of values such as *1 2 5 10* or *0 to 5 by 1*. The Acceptance Number should be less than the Sample Size.

Range and Increment of Proportion Defective

Proportion Defective Range

This is the series of values for the lot proportion defective. You can enter the range of values directly or you can use the button with the three arrows at the end of the box. The range is entered using the first and last values of the range followed by the amount to increment between each value. For example, you may enter *0 to 0.5 by 0.01*.

The full range of possible values is a series of values between 0 and 1. Typically, the Probability of Acceptance drops down to 0 long before the Proportion Defective reaches 1. We recommend that you run the procedure a few times until you have selected a Proportion Defective range that has a Probability of Acceptance near 0 at the end of the range. For example, you may try *0 to 0.5 by 0.01* on the first run, and then settle on *0 to 0.2 by 0.002* for the final run.

Example 1 – Generating an Operating Characteristic Curve

Suppose a quality engineer is to receive a lot of 500 units. The engineer is considering sampling 50 units and would like to examine the operating characteristic curve for an acceptance number of 3. To begin, the engineer will examine the full range of lot proportion defectives (0 to 1).

Setup

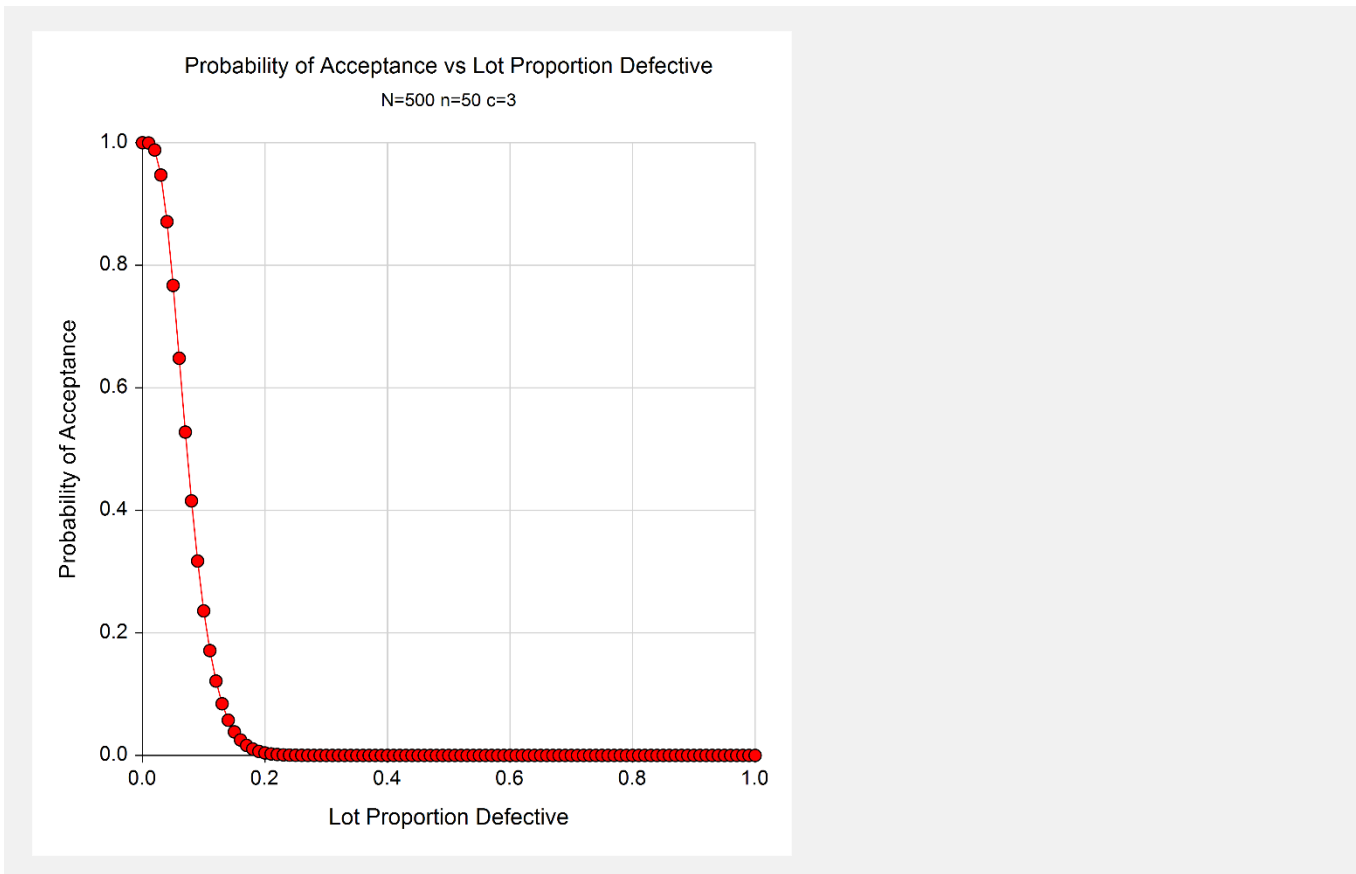
This section presents the values of each of the parameters needed to run this example. First, load the **Operating Characteristic Curves for Acceptance Sampling for Attributes** procedure window. You may then make the appropriate entries as listed below, or open **Example 1(a or b)** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Lot Size (N)	500
Sample Size (n)	50
Acceptance Number (c)	3
Proportion Defective Range	0 to 1 by 0.01

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Chart Section



Operating Characteristic Curves for Acceptance Sampling for Attributes

Numeric Results

Numeric Results for Operating Characteristic Curves for Acceptance Sampling for Attributes

Lot Size (N) = 500

Sample Size (n)	Acceptance Number (c)	Proportion Defective	Probability of Acceptance
50	3	0.0000	1.00000
50	3	0.0100	0.99959
50	3	0.0200	0.98814
50	3	0.0300	0.94723
50	3	0.0400	0.87112
50	3	0.0500	0.76708
50	3	0.0600	0.64824
50	3	0.0700	0.52769
50	3	0.0800	0.41537
50	3	0.0900	0.31727
50	3	0.1000	0.23586
50	3	0.1100	0.17108
50	3	0.1200	0.12134
50	3	0.1300	0.08430
50	3	0.1400	0.05744
50	3	0.1500	0.03844
50	3	0.1600	0.02528
50	3	0.1700	0.01636
.	.	.	.
.	.	.	.
.	.	.	.

Each line gives the probability of acceptance based on the corresponding proportion defective.

References and Report Definitions

References

Montgomery, D. C. 2013. Introduction to Statistical Quality Control, 7th Edition. John Wiley & Sons.
 Kenett, Ron S. and Zacks, Shelemyahu 2014. Modern Industrial Statistics, 2nd Edition. John Wiley & Sons. West Sussex, United Kingdom.

Report Definitions

Sample Size (n) is the size of the sample drawn from the lot.

c, the acceptance number, is the largest number of nonconforming items from the sample of size n for which the lot should still be accepted.

Proportion Defective is the proportion of the items of the lot that is assumed to be defective.

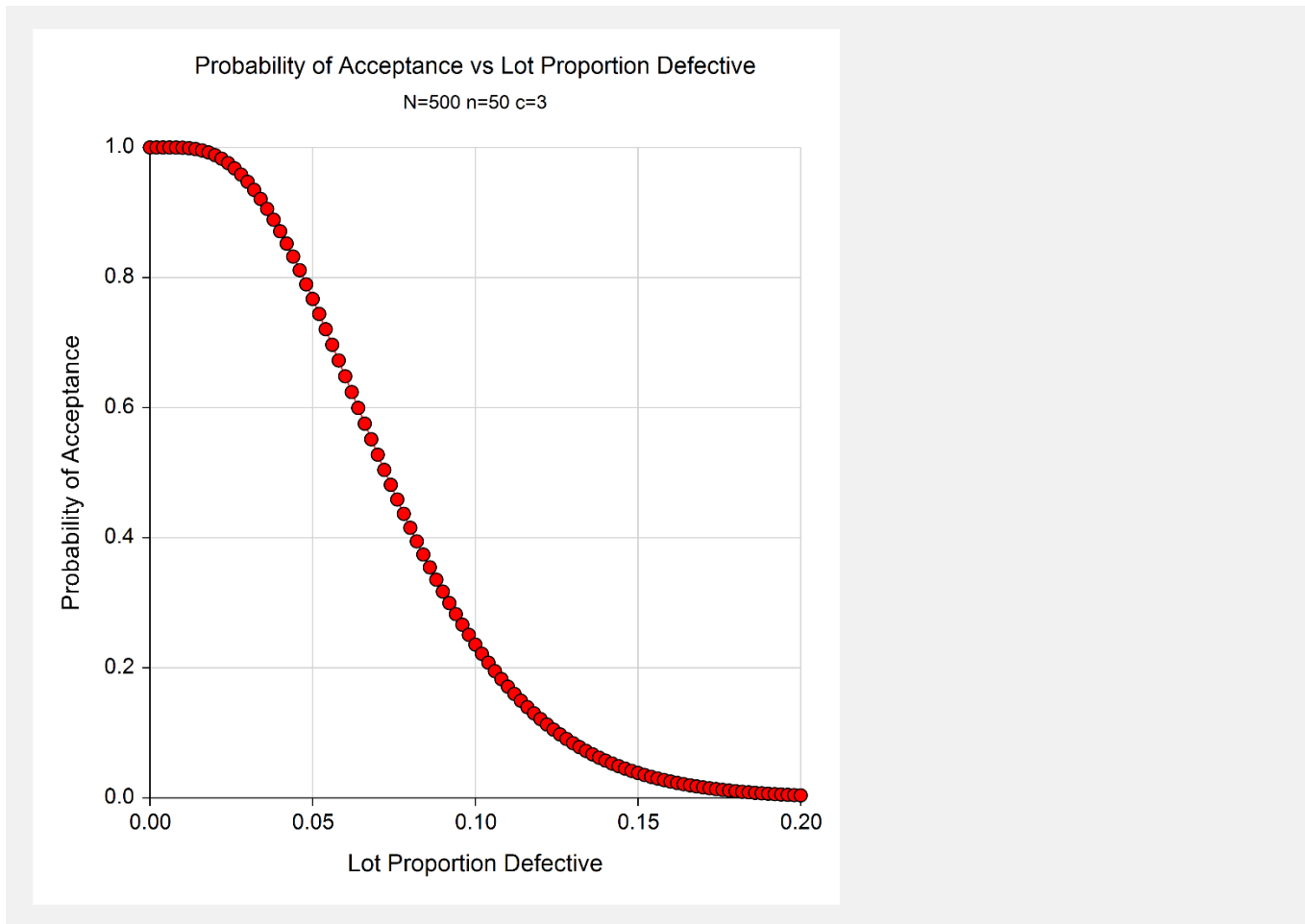
Probability of Acceptance is probability that the lot will be accepted, given the corresponding sample size, acceptance number, and assumed proportion defective.

These reports give some text references, as well as the definitions for each column.

Since the probability of acceptance is close to 0 when the lot proportion defective reaches 0.2, the Proportion Defective Range is changed to 0 to 0.2 by 0.002, and the procedure is run again.

Operating Characteristic Curves for Acceptance Sampling for Attributes

Chart Section



This second chart gives a nice range for the Lot Proportion Defective relative to the Probability of Acceptance.

Example 2 – OC Curves for Various Acceptance Numbers

Continuing with Example 1, suppose the quality engineer would like to see various curves for different values of the acceptance number. In particular, the engineer would like to examine acceptance numbers of 0, 1, 2, 3, 4, 5.

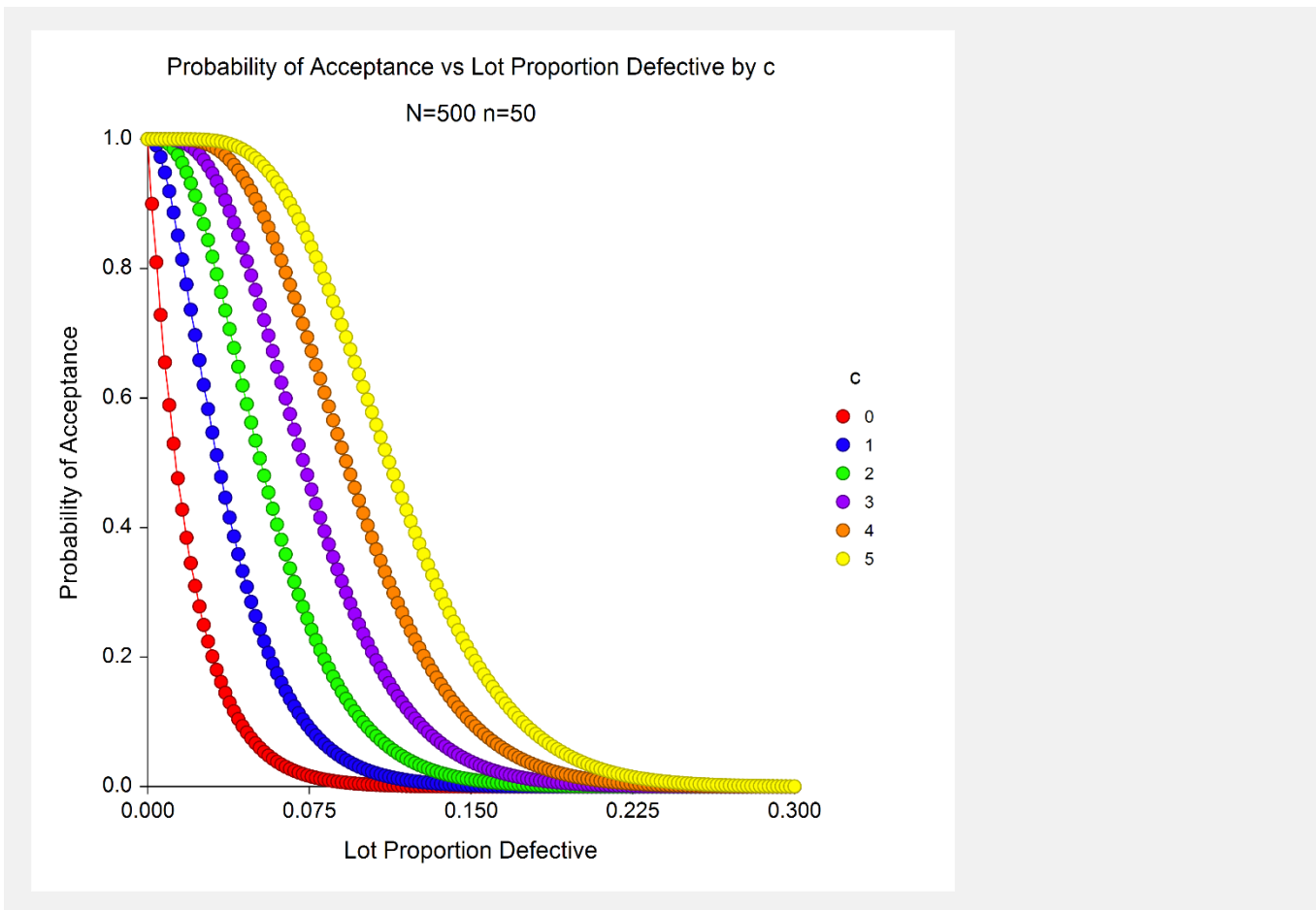
Setup

This section presents the values of each of the parameters needed to run this example. First, load the **Operating Characteristic Curves for Acceptance Sampling for Attributes** procedure window using the Analysis menu or the Procedure Navigator. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Lot Size (N)	500
Sample Size (n)	50
Acceptance Number (c)	0 1 2 3 4 5
Proportion Defective Range	0 to 0.3 by 0.002

Output

Click the Calculate button to perform the calculations and generate the following output.



This plot shows the operating characteristic curve for each acceptance number.

Example 3 – Validation using Kenett and Zacks (2014)

Kenett and Zacks (2014), pp. 260-261, give an operating characteristic curve example in which the sample size is 50, and the acceptance number is 1.

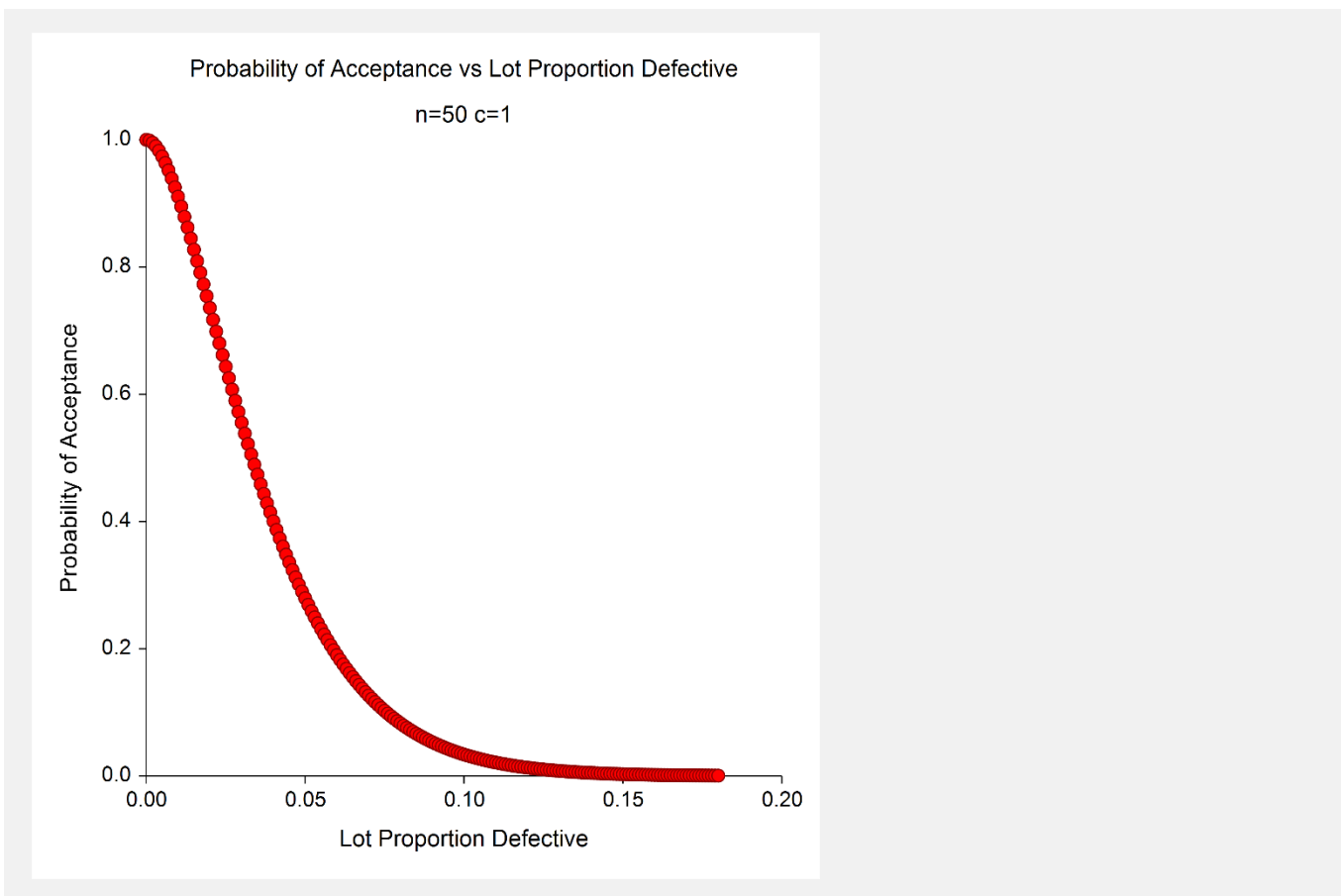
Setup

This section presents the values of each of the parameters needed to run this example. First, load the **Operating Characteristic Curves for Acceptance Sampling for Attributes** procedure window using the Analysis menu or the Procedure Navigator. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Lot Size (N).....	Infinite
Sample Size (n)	50
Acceptance Number (c).....	1
Proportion Defective Range	0 to 0.18 by 0.001

Output

Click the Calculate button to perform the calculations and generate the following output.



This plot matches the plot shown in Kenett and Zacks (2014).

Example 4 – Validation using Montgomery (2013)

Montgomery (2013), p. 657, gives an operating characteristic curve example in which the sample size is 89, and the acceptance numbers are 0, 1, and 2.

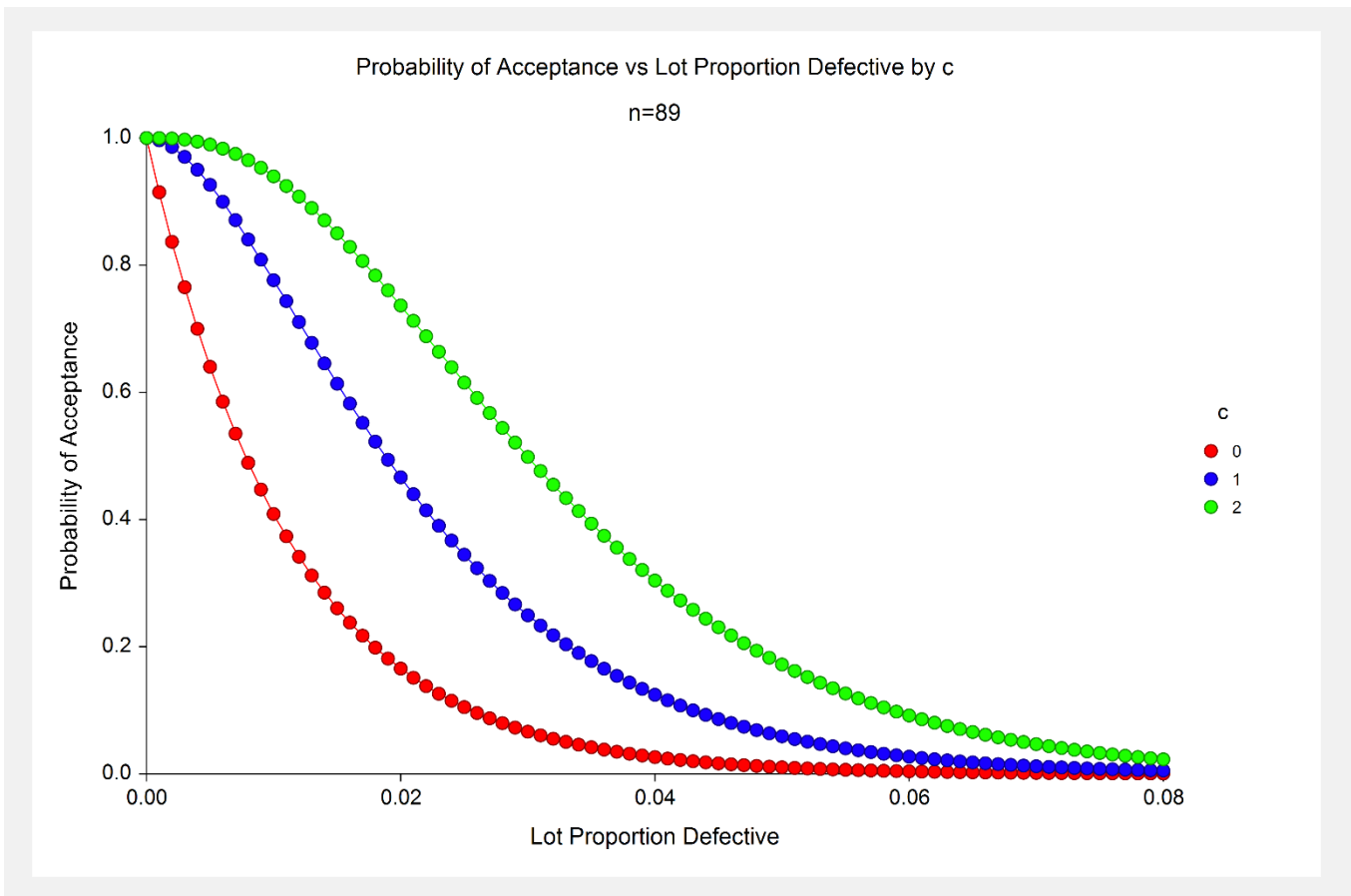
Setup

This section presents the values of each of the parameters needed to run this example. First, load the **Operating Characteristic Curves for Acceptance Sampling for Attributes** procedure window using the Analysis menu or the Procedure Navigator. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Lot Size (N).....	Infinite
Sample Size (n).....	89
Acceptance Number (c).....	0 1 2
Proportion Defective Range	0 to 0.08 by 0.001

Output

Click the Calculate button to perform the calculations and generate the following output.



This plot matches the plot shown in Montgomery (2013).