Chapter 311

Stepwise Regression

Introduction

Often, theory and experience give only general direction as to which of a pool of candidate variables (including transformed variables) should be included in the regression model. The actual set of predictor variables used in the final regression model must be determined by analysis of the data. Determining this subset is called the *variable selection* problem.

Finding this subset of regressor (independent) variables involves two opposing objectives. First, we want the regression model to be as complete and realistic as possible. We want every regressor that is even remotely related to the dependent variable to be included. Second, we want to include as few variables as possible because each irrelevant regressor decreases the precision of the estimated coefficients and predicted values. Also, the presence of extra variables increases the complexity of data collection and model maintenance. The goal of variable selection becomes one of parsimony: achieve a balance between simplicity (as few regressors as possible) and fit (as many regressors as needed).

There are many different strategies for selecting variables for a regression model. If there are no more than fifteen candidate variables, the *All Possible Regressions* procedure (discussed in the next chapter) should be used since it will always give as good or better models than the stepping procedures available in this procedure. On the other hand, when there are more than fifteen candidate variables, the four search procedures contained in this procedure may be of use.

These search procedures will often find very different models. Outliers and collinearity can cause this. If there is very little correlation among the candidate variables and no outlier problems, the four procedures should find the same model.

We will now briefly discuss each of these procedures.

Variable Selection Procedures

Forward (Step-Up) Selection

This method is often used to provide an initial screening of the candidate variables when a large group of variables exists. For example, suppose you have fifty to one hundred variables to choose from, way outside the realm of the all-possible regressions procedure. A reasonable approach would be to use this forward selection procedure to obtain the best ten to fifteen variables and then apply the all-possible algorithm to the variables in this subset. This procedure is also a good choice when multicollinearity is a problem.

The forward selection method is simple to define. You begin with no candidate variables in the model. Select the variable that has the highest R-Squared. At each step, select the candidate variable that increases R-Squared the most. Stop adding variables when none of the remaining variables are significant. Note that once a variable enters the model, it cannot be deleted.

Backward (Step-Down) Selection

This method is less popular because it begins with a model in which all candidate variables have been included. However, because it works its way down instead of up, you are always retaining a large value of R-Squared. The problem is that the models selected by this procedure may include variables that are not really necessary. The user sets the significance level at which variables can enter the model.

The backward selection model starts with all candidate variables in the model. At each step, the variable that is the least significant is removed. This process continues until no nonsignificant variables remain. The user sets the significance level at which variables can be removed from the model.

Stepwise Selection

Stepwise regression is a combination of the forward and backward selection techniques. It was very popular at one time, but the Multivariate Variable Selection procedure described in a later chapter will always do at least as well and usually better.

Stepwise regression is a modification of the forward selection so that after each step in which a variable was added, all candidate variables in the model are checked to see if their significance has been reduced below the specified tolerance level. If a nonsignificant variable is found, it is removed from the model.

Stepwise regression requires two significance levels: one for adding variables and one for removing variables. The cutoff probability for adding variables should be less than the cutoff probability for removing variables so that the procedure does not get into an infinite loop.

Min MSE

This procedure is similar to the Stepwise Selection search procedure. However, instead of using probabilities to add and remove, you specify a minimum change in the root mean square error. At each step, the variable whose status change (in or out of the model) will decrease the mean square error the most is selected and its status is reversed. If it is currently in the model, it is removed. If it is not in the model, it is added. This process continues until no variable can be found that will cause a change larger than the user-specified minimum change amount.

Assumptions and Limitations

The same assumptions and qualifications apply here as applied to multiple regression. Note that outliers can have a large impact on these stepping procedures, so you must make some attempt to remove outliers from consideration before applying these methods to your data.

The greatest limitation with these procedures is one of sample size. A good rule of thumb is that you have at least five observations for each variable in the candidate pool. If you have 50 variables, you should have 250 observations. With less data per variable, these search procedures may fit the randomness that is inherent in most datasets and spurious models will be obtained.

This point is critical. To see what can happen when sample sizes are too small, generate a set of random numbers for 20 variables with 30 observations. Run any of these procedures and see what a magnificent value of R-Squared is obtained, even though its theoretical value is zero!

Using This Procedure

This procedure performs one portion of a regression analysis: it obtains a set of independent variables from a pool of candidate variables. Once the subset of variables is obtained, you should proceed to the Multiple Regression procedure to estimate the regression coefficients, study the residuals, and so on.

Data Structure

An example of data appropriate for this procedure is shown in the table below. This data is from a study of the relationships of several variables with a person's IQ. Fifteen people were studied. Each person's IQ was recorded along with scores on five different personality tests. The data are contained in the IQ dataset. We suggest that you open this database now so that you can follow along with the example.

IQ Dataset

Test1	Test2	Test3	Test4	Test5	IQ
83	34	65	63	64	106
73	19	73	48	82	92
54	81	82	65	73	102
96	72	91	88	94	121
84	53	72	68	82	102
86	72	63	79	57	105
76	62	64	69	64	97
54	49	43	52	84	92
37	43	92	39	72	94
42	54	96	48	83	112
71	63	52	69	42	130
63	74	74	71	91	115
69	81	82	75	54	98
81	89	64	85	62	96
50	75	72	64	45	103

Missing Values

Rows with missing values in the active variables are ignored.

Example 1 – Stepwise Regression Analysis

This section presents an example of how to run a stepwise regression analysis of the data presented in the IQ dataset.

Setup

To run this example, complete the following steps:

1 Open the IQ example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **IQ** and click **OK**.

2 Specify the Stepwise Regression procedure options

- Find and open the **Stepwise Regression** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Y: Dependent Variable	**IQ**
X's: Independent Variables	**Test1-Test5**
Selection Method	**Backward**

Reports Tab

Report Format	**Verbose**
Descriptive Statistics	**Checked**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Descriptive Statistics

Descriptive Statistics

Variable	Count	Mean	Standard Deviation
Test1	15	67.93333	17.39239
Test2	15	61.4	19.39735
Test3	15	72.33334	14.73415
Test4	15	65.53333	13.95332
Test5	15	69.93333	16.15314
IQ	15	104.3333	11.0173

For each variable, the Count, Mean, and Standard Deviation are calculated. This report is especially useful for making certain that you have selected the right variables and that the appropriate number of rows was used.

Iteration Details (Verbose Version)

Iteration Details

Iteration 0: Unchanged

			R-Squared				Percent
In	Variable	Standardized Coefficient	Increment	vs. Other X's	T-Value	P-Value	Change in Sqrt(MSE)
Yes	Test1	-3.0524	0.235717	0.974701	-1.8789	0.092969	11.9387
Yes	Test2	-2.9224	0.241441	0.971730	-1.9016	0.089661	12.3210
Yes	Test3	0.1404	0.015210	0.227987	0.4773	0.644541	-3.9386
Yes	Test4	4.7853	0.283243	0.987631	2.0596	0.069522	15.0741
Yes	Test5	-0.0595	0.002715	0.232860	-0.2017	0.844669	-4.9176

R-Squared = 0.399068   Sqrt(MSE) = 10.65198

Iteration 1: Removed Test5 from the Model

			R-Squared				Percent
In	Variable	Standardized Coefficient	Increment	vs. Other X's	T-Value	P-Value	Change in Sqrt(MSE)
Yes	Test1	-3.0612	0.237250	0.974683	-1.9825	0.075558	12.5340
Yes	Test2	-2.9032	0.239195	0.971621	-1.9906	0.074546	12.6640
Yes	Test3	0.1163	0.012499	0.075203	0.4550	0.658798	-3.6717
Yes	Test4	4.7850	0.283206	0.987631	2.1660	0.055543	15.5681
No	Test5		0.002715	0.232860	0.2017	0.844669	5.1719

R-Squared = 0.396353   Sqrt(MSE) = 10.12816

```
Iteration 2: Removed Test3 from the Model
─────────────────────────────────────────────────────────────────────────────────────────────────
                                          R-Squared                                        Percent
                       Standardized   ─────────────────────                              Change in
In       Variable       Coefficient    Increment  vs. Other X's   T-Value      P-Value   Sqrt(MSE)
─────────────────────────────────────────────────────────────────────────────────────────────────
Yes      Test1              -3.1020     0.244443       0.974597   -2.0890     0.060743     13.1519
Yes      Test2              -2.9024     0.239064       0.971621   -2.0659     0.063218     12.7977
Yes      Test4               4.7988     0.284897       0.987628    2.2553     0.045468     15.7808
No       Test3                          0.012499       0.075203    0.4550     0.658798      3.8116
No       Test5                          0.000005       0.081040    0.0087     0.993205      4.8805
─────────────────────────────────────────────────────────────────────────────────────────────────
R-Squared = 0.383854   Sqrt(MSE) = 9.756291

Iteration 3: Unchanged
─────────────────────────────────────────────────────────────────────────────────────────────────
                                          R-Squared                                        Percent
                       Standardized   ─────────────────────                              Change in
In       Variable       Coefficient    Increment  vs. Other X's   T-Value      P-Value   Sqrt(MSE)
─────────────────────────────────────────────────────────────────────────────────────────────────
Yes      Test1              -3.1020     0.244443       0.974597   -2.0890     0.060743     13.1519
Yes      Test2              -2.9024     0.239064       0.971621   -2.0659     0.063218     12.7977
Yes      Test4               4.7988     0.284897       0.987628    2.2553     0.045468     15.7808
No       Test3                          0.012499       0.075203    0.4550     0.658798      3.8116
No       Test5                          0.000005       0.081040    0.0087     0.993205      4.8805
─────────────────────────────────────────────────────────────────────────────────────────────────
R-Squared = 0.383854   Sqrt(MSE) = 9.756291
```

This report presents information about each step of the search procedures. You can scan this report to see if you would have made the same choice. Each report shows the statistics after the specified action (entry or removal) was taken.

For each iteration, there are three possible actions:

1. *Unchanged*. No action was taken because of the scan in this step. Because of the "backward look" in the stepwise search method, this will show up a lot when this method is used. Otherwise, it will usually show up as the first and last steps.
2. *Removal.* A variable was removed from the model.
3. *Entry*. A variable was added to the model.

Individual definitions of the items on the report are as follows:

In

A *Yes* means the variable is in the model. A *No* means it is not.

Variable

This is the name of the candidate variable.

Standardized Coefficient

Standardized regression coefficients are the coefficients that would be obtained if you standardized each independent and dependent variable. Here *standardizing* is defined as subtracting the mean and dividing by the standard deviation of a variable. A regression analysis on these standardized variables would yield these standardized coefficients.

When there are vastly different units involved for the variables, this is a way of making comparisons between variables. The formula for the standardized regression coefficient is:

$$b_{j,std} = b_j \left(\frac{s_{x_j}}{s_y} \right)$$

where s_y and s_{x_j} are the standard deviations for the dependent variable and the corresponding *j*th independent variable.

R-Squared - Increment

This is the amount that R-Squared would be changed if the status of this variable were changed. If the variable is currently in the model, this is the amount the R-Squared value would be decreased if it were removed. If the variable is currently out of the model, this is the amount the overall R-Squared would be increased if it were added. Large values here indicate important independent variables.

You want to add variables that make a large contribution to R-Squared and to delete variables that make a small contribution to R-Squared.

R-Squared – vs. Other X's

This is a collinearity measure, which should be as small as possible. This is the R-Squared value that would result if this independent variable were regressed on all the other independent variables currently in the model.

T-Value

This is the t-value for testing the hypothesis that this variable should be added to, or deleted from, the model. The test is adjusted for the rest of the variables in the model. The larger this t-value is, the more important the variable.

P-Value

This is the two-tail p-value for the above t-value. The smaller this p-value, the more important the independent variable is. This is the significance value that is compared to the values of PIN and POUT (see *Stepwise Method* above).

Percent Change in Sqrt(MSE)

This is the percentage change in the square root of the mean square error that would occur if the specified variable were added to, or deleted from, the model. This is the value that is used by the *Min MSE* search procedure. This percentage change in root mean square error (RMSE) is computed as follows:

$$Percent\ change = \left[\frac{RMSE_{previous} - RMSE_{current}}{RMSE_{current}} \right] 100$$

R-Squared

This is the R-Squared value for the current model.

Sqrt(MSE)

This is the square root of the mean square error for the current model.

Iteration Details (Brief Version)

This report was not printed because the Report Format box on the Reports tab was set to *Verbose*. If Report Format is set to *Brief*, the following output is displayed:

```
Iteration Details
──────────────────────────────────────────────────────────────────────────────────────────
                                                                     Maximum
Iteration                                                            R-Squared
Number      Action       Variable    R-Squared    Sqrt(MSE)          vs. Other X's
────────────────────────────────────────────────────────────────────────────────
0           Unchanged                0.399068     10.65198           0.987631
1           Removed      Test5       0.396353     10.12816           0.987631
2           Removed      Test3       0.383854     9.756291           0.987628
3           Unchanged                0.383854     9.756291           0.987628
──────────────────────────────────────────────────────────────────────────────────────────
```

This is an abbreviated report summarizing the statistics at each iteration. Individual definitions of the items on the report are as follows:

Iteration Number

The number of this iteration.

Action

For each iteration, there are three possible actions:

1. *Unchanged*. No action was taken because of the scan in this step. Because of the "backward look" in the stepwise search method, this will show up a lot when this method is used. Otherwise, it will show up at the first and last steps.
2. *Removed*. A variable was removed from the model.
3. *Added*. A variable was added to the model.

Variable

This is the name of the variable whose status is being changed.

R-Squared

The value of R-Squared for the current model.

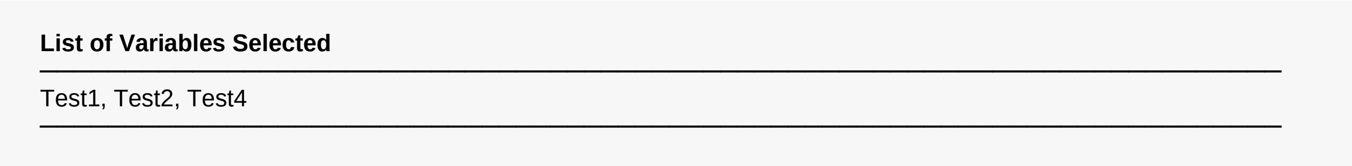
Sqrt(MSE)

This is the square root of the mean square error for the current model.

Maximum R-Squared vs. Other X's

This is the maximum value of R-Squared – vs. Other X's (see verbose report definitions) for all the variables in the model. This is a collinearity model. You want this value to be as small as possible. If it approaches 0.99, you should be concerned with the possibility that multicollinearity is distorting your results.

List of Variables Selected



List of Variables Selected

Test1, Test2, Test4

This report lists the variables selected by the stepwise regression procedure.