

## Chapter 475

# Theoretical ARMA

---

## Introduction

This procedure shows the theoretical characteristics of the autocorrelations, partial autocorrelations, and spectrum of user-specified ARMA models. Unlike the other time series programs, this one does not use data. Instead, it provides a theoretical analysis of various models. It also creates simulated series from these models.

We have found this program especially useful in training and model evaluation. While you are becoming familiar with the Box-Jenkins method, this program lets you study the characteristics of a large number of models. You will be able to see the sensitivity of the autocorrelation function to changes in the number of, and values of, parameters. You will be able to generate series from known models and see how difficult it is to identify the model that they came from.

For use in theoretical model evaluation, this program factors a model written as a polynomial in the backshift operator. This will let you compare several models that each seem adequate but appear quite different. It will let you study the characteristics of various models in detail.

It is useful in model identification, because it will let you generate a catalog of possible autocorrelation patterns from known theoretical models that you can compare sample autocorrelation functions with.

All of the models treated by this program come from the general class defined by the model:

$$\phi_p(B)\Phi_p(B)X_t = \theta_q(B)\Theta_q(B^s)a_t$$

(Refer to chapter on the Box-Jenkins method for more information on the interpretation of this equation.)

---

## Procedure Options

This section describes the options available in this procedure.

---

## Data Tab

The following options specify the model to be analyzed.

---

## ARMA Model Specification

### Regular AR (Phis)

The values of the autoregressive parameters, the phis.

This should be a list of values like **0.9 -0.2 0.3**. Note that the first value corresponds to lag one, the second corresponds to lag two, and so on.

## Theoretical ARMA

### Seasonal AR (Cap Phis)

The values of the seasonal autoregressive parameters, the capital phis. This should be a list of values like **0.9 -0.2 0.3**. Note that the first value corresponds to lag  $s$ , the second corresponds to lag  $2s$ , and so on.

### Regular MA (Thetas)

The values of the moving average parameters, the thetas.

This should be a list of values like **0.9 -0.2 0.3**. Note that the first value corresponds to lag one, the second corresponds to lag two, and so on.

### Seasonal MA (Cap Thetas)

The values of the seasonal moving average parameters, the capital thetas. This should be a list of values like **0.9 -0.2 0.3**. Note that the first value corresponds to lag  $s$ , the second corresponds to lag  $2s$ , and so on.

---

## Seasonality Options

### Seasons

Specify the number of seasons per year in the series. Use '4' for quarterly data or '12' for monthly data.

---

## Simulation Options

### Series Variance

This is the variance of the simulated series. It is the variance of the normal random numbers that are used in the simulation process.

### Number of Rows

Specifies the number of rows of data that are generated in the simulated series from this model. This is also the number of rows that will be stored on the current database.

---

## Reports Tab

The following options control which reports are displayed.

---

### Select Reports

#### Autocorrelation Report - Coefficient Report

Each of these options specifies whether the indicated report is displayed.

---

### Report / Plot Options

#### Number of Autocorrelations

Specifies the number of autocorrelations that are reported.

#### Number of Frequencies

Specifies the number of frequencies displayed in the spectral density.

#### Precision

Specify the precision of numbers in the report. Single precision will display seven-place accuracy, while the double precision will display thirteen-place accuracy. Note that all reports are formatted for single precision only.

---

## Autocorrelation Plot Tab

This section controls the inclusion and settings for the plots.

---

### Select Plots

#### Autocorrelation Plots - Data Plot

Each of these options specifies whether the indicated plot is displayed. Click the plot format button to change the plot settings.

---

## Storage Tab

The generated series may be stored on the current dataset for further analysis. These options lets you designate which column should receive this simulated series. The selected statistics are automatically stored to the current dataset when a column name is entered.

Note that existing data is replaced. Be careful that you do not specify columns that contain important data.

---

### Data Storage Columns

#### Data Series

The simulated series is stored in this column.

---

## Example 1 – Using the Theoretical ARMA Procedure

This section presents an example of how to use the theoretical ARMA program. You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Theoretical ARMA window.

### 1 Open the Theoretical ARMA window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Theoretical ARMA** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

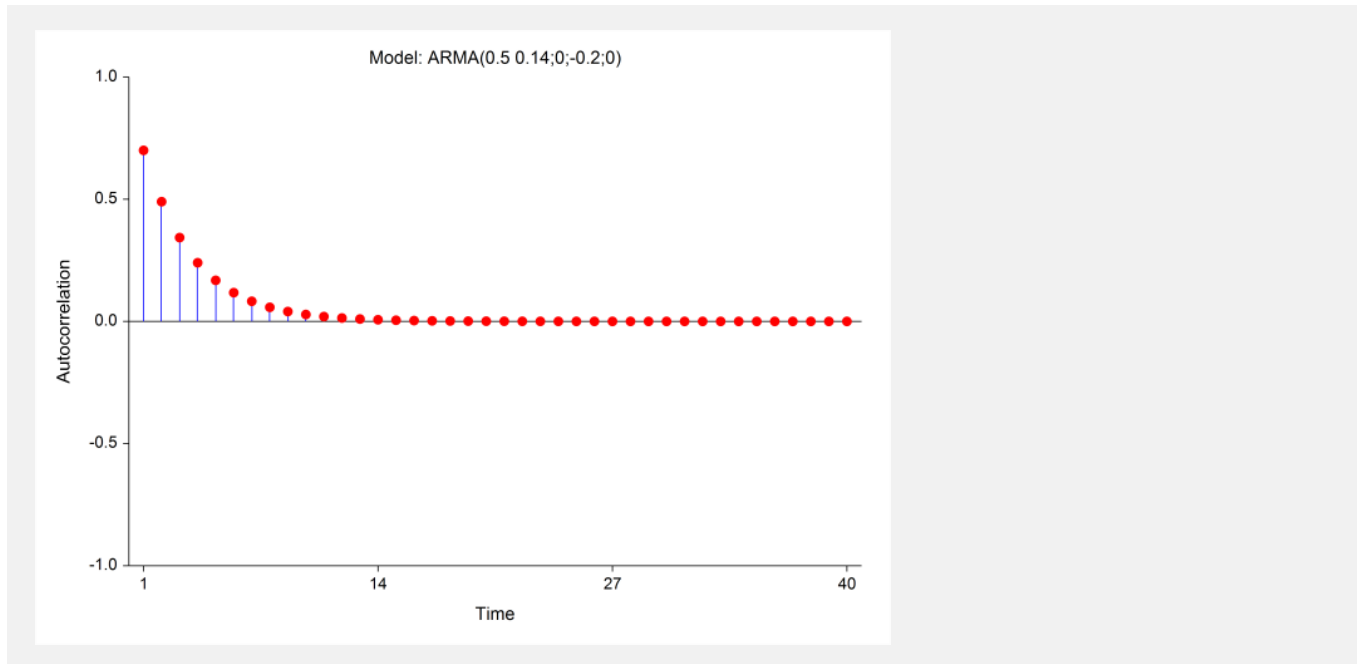
### 2 Specify the model.

- On the Theoretical ARMA window, select the **Data tab**.
- Enter **0.5 0.14** in the **Regular AR (Phis)** box.
- Enter **-0.2** in the **Regular MA (Thetas)** box.

### 3 Run the procedure.

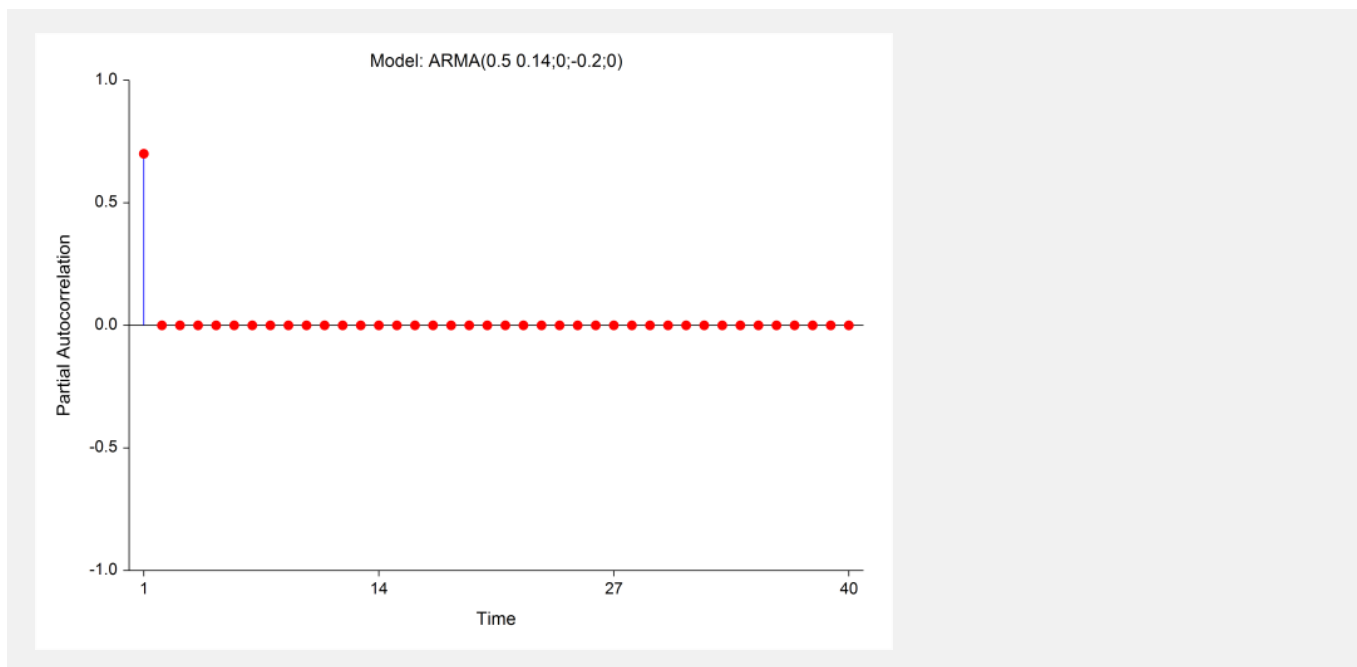
- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

## Autocorrelations Plot



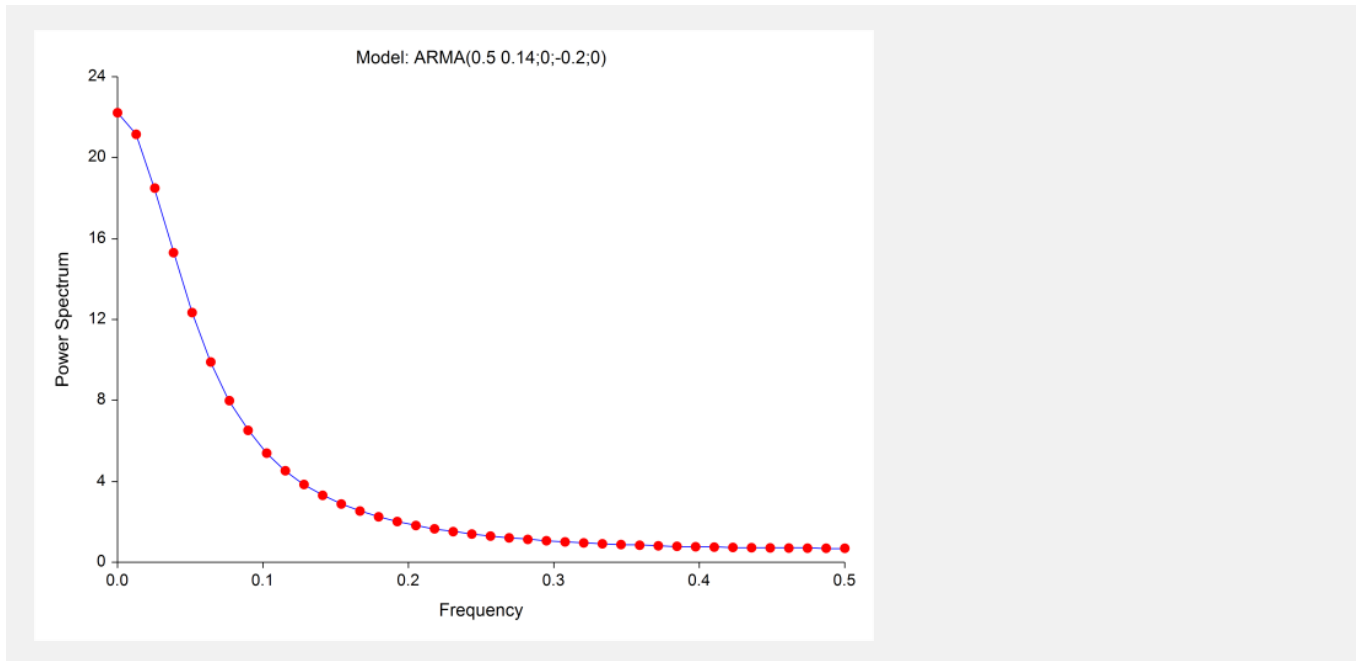
This plot gives the theoretical autocorrelations.

## Partial Autocorrelations Plot



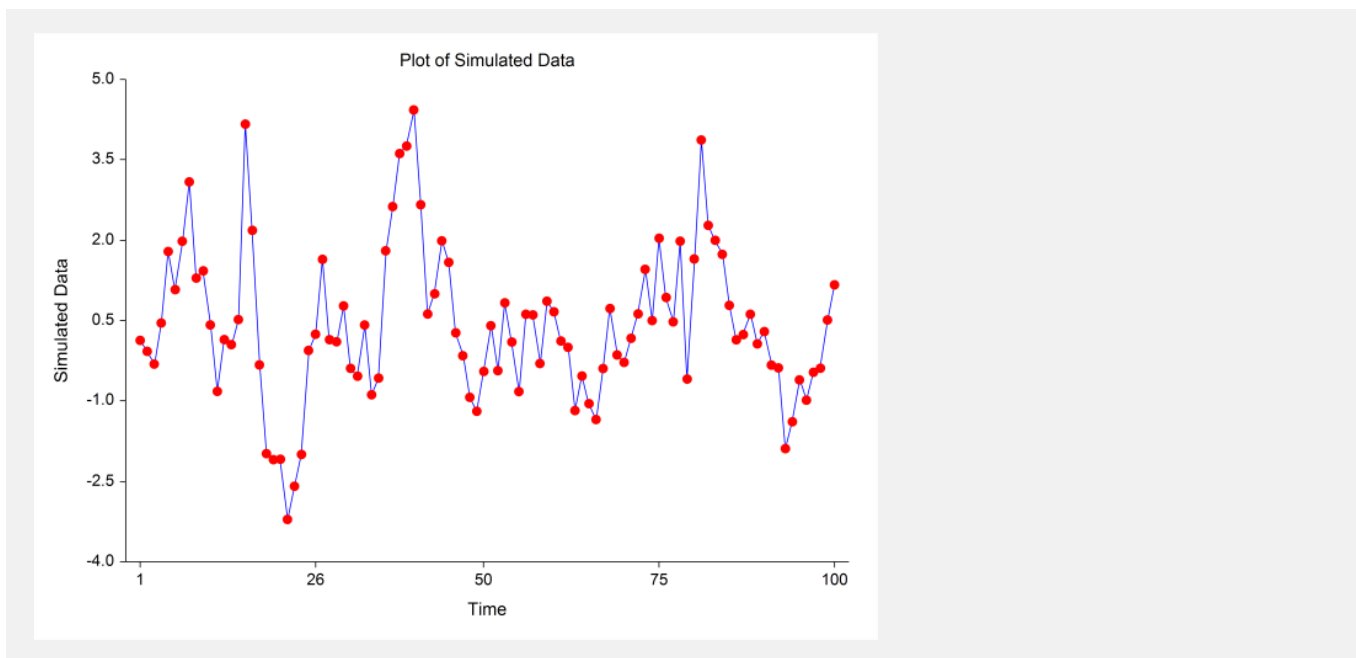
This plot gives the theoretical partial autocorrelations.

## Power Spectrum Plot



This plot shows the power spectrum (see the chapter on spectral analysis).

## Data Plot



This is a plot of a simulated data series from the model.

## Autocorrelation / Power Spectrum Section

Autocorrelations / Power Spectrum Section				
No.	Autocorrelations	Partial Autocorrelations	Frequency	Power Spectrum
1	0.700000	0.700000	0.000000	22.22222
2	0.490000	0.000000	0.012821	21.15512
3	0.343000	0.000000	0.025641	18.49632
4	0.240100	0.000000	0.038462	15.30439
5	0.168070	0.000000	0.051282	12.34195
6	0.117649	0.000000	0.064103	9.89958
7	0.082354	0.000000	0.076923	7.988447
8	0.057648	0.000000	0.089744	6.52031
9	0.040354	0.000000	0.102564	5.394032
10	0.028248	0.000000	0.115385	4.524017
11	0.019773	0.000000	0.128205	3.844779
12	0.013841	0.000000	0.141026	3.308101
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

This section presents the numerical values associated with the plots of the last section.

## Coefficient Analysis Section

Coefficient Analysis Section				
Coefficient Name	Lag	Coefficient Value	Real Root	Imaginary Root
Phi(AR)	0	1.000000	-5.000000	0.000000
Phi(AR)	1	0.500000	1.428571	0.000000
Phi(AR)	2	0.140000		
Theta(MA)	0	1.000000	-5.000000	0.000000
Theta(MA)	1	-0.200000		

Model is stationary and model is invertible.

This report displays an analysis of the coefficients of the model. When the model is written in terms of the backshift operator,  $B$ , it may be thought of as polynomials in  $B$ . Hence in our current example, we have two equations to study, one for the autoregressive operator and one for the moving average operator. These are:

$$(1 - 0.5B - 0.14B^2) = 0$$

and

$$(1 + 0.2B) = 0$$

As we will show, it can be useful to find and compare the roots of these two equations, since knowledge of the roots lets us factor the equations. We see from the report that the roots of the first polynomial are 1.4286 (which is  $10/7$ ) and  $-5$ . We would like to arrange these roots so that the factors may be displayed in a standard form. To do this, we perform the following algebra on each root:

$$B = -5 \qquad B = 1.4286$$

Move the constant to the right side.

$$5 + B = 0 \qquad -1.4286 + B = 0$$

Divide by the constant.

$$1 + .2B = 0 \qquad 1 - .7B = 0$$

### Theoretical ARMA

These are now in the special form that we can easily use them as factors. We note that the polynomial may be factored as

$$(1 - .5B - .14B^2) = (1 + .2B)(1 - .7B)$$

Hence, the model (9.3) may be rewritten as

$$(1 + 0.2B)(1 - 0.7B)X_t = (1 + 0.2B)a_t$$

Notice that the left and right sides of this equation have  $(1 + 0.2B)$  as a common factor. We can cancel this factor out, leaving the simpler model:

$$(1 - 0.7B)X_t = a_t$$

These models are equivalent. Now we can see why the partial autocorrelation plot indicated a single autoregressive parameter even though we had specified two.

A second purpose for studying these coefficients is to look for signals to difference a series. Note that if a root is approximately unity, the factor will be approximately  $(1 - B)$ , the difference operator. Hence, when we find roots on the autoregressive side close to one, we can simplify the model by differencing the series.

One criticism of the Box-Jenkins method is that two well-trained forecasters will most likely arrive at different models. We find that this criticism is not well-founded since often, by factoring the operator polynomials of the two models and studying their roots, we will find that the models are actually quite similar.