Chapter 585

Tolerance Intervals

Introduction

This procedure calculates one-, and two-, sided tolerance intervals based on either a distribution-free (nonparametric) method or a method based on a normality assumption (parametric). A two-sided *tolerance interval* consists of two limits between which a given proportion β of the population falls with a given confidence level $1-\alpha$. A one-sided tolerance interval is similar but consists of a single upper or lower limit.

Technical Details

Let $X_1, X_2, \cdots, X_n$ be a random sample for a population with distribution function $F(X)$. A $(\beta, 1-\alpha)$ two-sided β-content tolerance interval (T_L, T_U) is defined by

$$\Pr[F(T_U) - F(T_L) \geq \beta] \geq 1-\alpha$$

A $(\beta, 1-\alpha)$ lower, one-sided β-content tolerance bound T_L is defined by

$$\Pr[1 - F(T_L) \geq \beta] \geq 1-\alpha$$

A $(\beta, 1-\alpha)$ upper, one-sided β-content tolerance bound T_U is defined by

$$\Pr[F(T_U) \geq \beta] \geq 1-\alpha$$

Note that a one-sided tolerance limit is the same as the one-sided confidence limit of the quantile of *F*.

Distribution-Free Tolerance Intervals

The definition of two-sided distribution-free tolerance intervals is found in many places. We use the formulation given by Bury (1999). The only distributional assumption made about *F* is that it is a continuous, non-decreasing, probability distribution. That is, these intervals should not be used with discrete data. Given this, the tolerance limits are

$$T_L = X_{(r)}, \quad T_U = X_{(s)}$$

where *r* and *s* are two order indices. The values of *r* and *s* are determined using the formula

$$\sum_{i=0}^{n-2c} \binom{n}{i} \beta^i (1-\beta)^{n-i} \geq 1-\alpha$$

where

$$r = c$$

$$s = n - c + 1$$

The value of *c* is found as the largest value for which the above inequality is true.

A lower, one-sided tolerance bound is $X_{(r)}$ where *r* is the largest value for which the following inequality is true.

$$\sum_{i=0}^{n-r} \binom{n}{i} \beta^i (1-\beta)^{n-i} \geq 1 - \alpha$$

An upper, one-sided tolerance bound is $X_{(s)}$ where *s* is the largest value for which the following inequality is true.

$$\sum_{i=0}^{s-1} \binom{n}{i} \beta^i (1-\beta)^{n-i} \geq 1 - \alpha$$

Normal-Distribution Tolerance Interval

The limits discussed in this section are based on the assumption that *F* is the normal distribution.

Two-Sided Tolerance Interval Limits

In this case, the two-sided tolerance interval is defined by the interval

$$T_L = \bar{x} - ks, \quad T_U = \bar{x} + ks$$

The construction reduces to the determination of the constant *k*. Howe (1969) provides the following approximation which is 'nearly' exact for all values of *n* greater than one

$$k = uvw$$

where

$$u = z_{\frac{1+\beta}{2}} \sqrt{1 + \frac{1}{n}}$$

$$v = \sqrt{\frac{n-1}{\chi^2_{n-1,\alpha}}}$$

$$w = \sqrt{1 + \frac{n - 3 - \chi^2_{n-1,\alpha}}{2(n+1)^2}}$$

Note that originally, Howe (1969) used *n* – 2 in the above definition of *w*. But Guenther (1977) gives the corrected version using *n* – 3 shown above.

One-Sided Tolerance Bound

A one-sided tolerance bound ('bound' is used instead of 'limit' in the one-sided case) is given by

$$T_U = \bar{x} + ks$$

Here *k* is selected so that

$$Pr\left(t'_{n-1,\delta} = k\sqrt{n}\right) = 1 - \alpha$$

where $t'_{f,\delta}$ represents a noncentral *t* distribution with *f* degrees of freedom and noncentrality

$$\delta = z_\beta \sqrt{n}\,.$$

Data Structure

The data are contained in a single column.

Example 1 – Generating Tolerance Intervals

This section presents a detailed example of how to generate tolerance intervals for the *Height* variable in the Height dataset.

Setup

To run this example, complete the following steps:

1 Open the Height example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Height** and click **OK**.

2 Specify the Tolerance Intervals procedure options

- Find and open the **Tolerance Intervals** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Variables..**Height**
Population Percentages for Tolerances **50 75 80 90 95 99**

Reports Tab

Values Decimal Places....................................**3**
Means Decimal Places **3**
Probabilities Decimal Places...........................**2**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Descriptive Statistics

Descriptive Statistics for Height

Count	Mean	Standard Deviation	Standard Error	Minimum	Maximum	Range
20	62.1	8.4411	1.8875	51	79	28

This report was defined and discussed in the Descriptive Statistics procedure chapter. We refer you to the Summary Section of that chapter for details.

Two-Sided Tolerance Intervals

Two-Sided Tolerance Intervals for Height

Percent of Population Between Limits	95% Tolerance Interval Limits Parametric Lower	Parametric Upper	Nonparametric Lower	Nonparametric Upper
50	54.0738	70.1262	52	73
75	48.4113	75.7887	51	79
80	46.8500	77.3500		
90	42.5268	81.6732		
95	38.7771	85.4229		
99	31.4486	92.7514		

Notes:
The parametric (normal-based) limits assume that the data follow the normal distribution.
The nonparametric (distribution-free) limits make no special distributional assumption.

This section gives the parametric and nonparametric two-sided tolerance intervals.

Percent of Population Between Limits

This is the percentage of population values that are contained in the tolerance interval.

Parametric Tolerance Interval Limits

These are the values of the limits of a tolerance interval based on the assumption that the population is normally distributed.

Nonparametric Tolerance Interval Limits

These are the values of the limits of a distribution-free tolerance interval. These intervals make no distributional assumption.

Lower One-Sided Tolerance Bounds

Lower One-Sided Tolerance Bounds for Height

Percent of Population Greater Than Bound	Lower 95% Tolerance Bound Parametric	Nonparametric
50	60.2643	56
75	52.2538	52
80	50.5242	51
90	45.8425	
95	41.8750	
99	34.2852	

Notes:
The parametric (normal-based) limit assumes that the data follow the normal distribution.
The nonparametric (distribution-free) limit makes no special distributional assumption.

This section gives the parametric and nonparametric one-sided tolerance bounds.

Percent of Population Greater Than Bound

This is the percentage of population values that are above the tolerance bound.

Lower Parametric Tolerance Bound

This is the lower parametric (normal distribution) tolerance bound.

Lower Nonparametric Tolerance Bound

This is the lower nonparametric (distribution-free) tolerance bound. Note that some values are missing because of the small sample size in this example.

Upper One-Sided Tolerance Bounds

Upper One-Sided Tolerance Bounds for Height

Percent of Population Less Than Bound	Upper 95% Tolerance Bound	
	Parametric	Nonparametric
50	63.9357	67
75	71.9462	76
80	73.6758	79
90	78.3575	
95	82.3250	
99	89.9148	

Notes:
The parametric (normal-based) limit assumes that the data follow the normal distribution.
The nonparametric (distribution-free) limit makes no special distributional assumption.

This section gives the parametric and nonparametric one-sided tolerance bounds.

Percent of Population Less Than Bound

This is the percentage of population values that are less than the tolerance bound.

Upper Parametric Tolerance Bound

This is the upper parametric (normal distribution) tolerance bound.

Upper Nonparametric Tolerance Bound

This is the upper nonparametric (distribution-free) tolerance bound.

Normality Tests

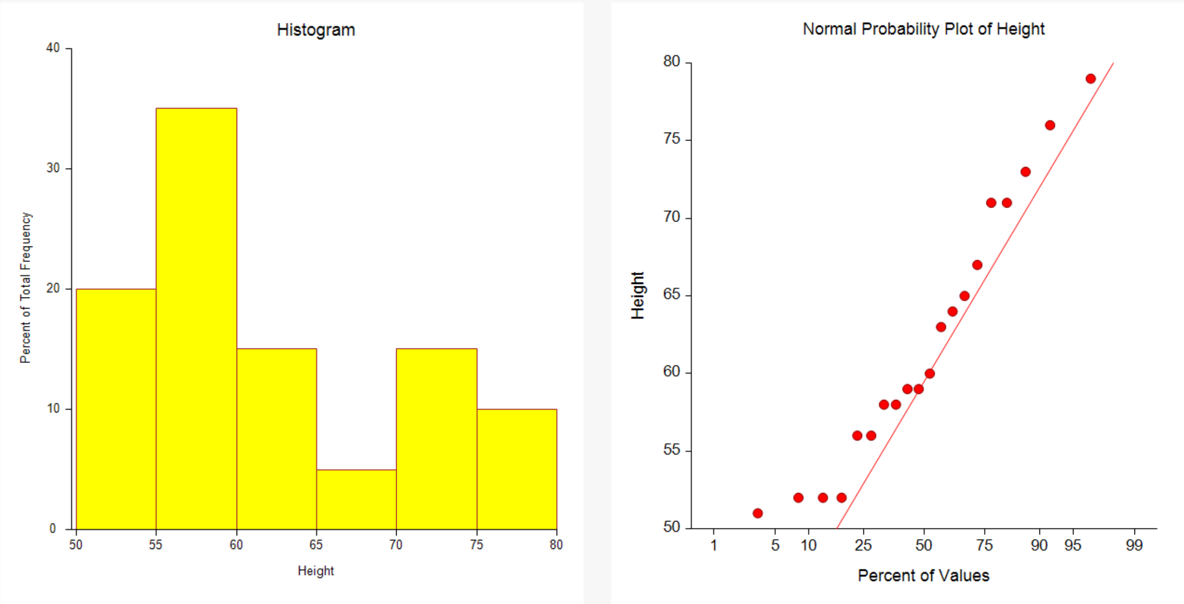
Normality Tests for Height

			Critical Values		
Test Name	**Test Statistic**	**P-Value**	**α = 0.1**	**α = 0.05**	**Decision at α = 0.05**
Shapiro-Wilk W	0.9374	0.2137			Can't reject normality
Anderson-Darling	0.4434	0.2863			Can't reject normality
Kolmogorov-Smirnov	0.1482		0.176	0.192	Can't reject normality
D'Agostino Skewness	1.0367	0.2999	1.645	1.960	Can't reject normality
D'Agostino Kurtosis	-0.7855	0.4322	1.645	1.960	Can't reject normality
D'Agostino Omnibus	1.6918	0.4292	4.605	5.991	Can't reject normality

This report was defined and discussed in the Descriptive Statistics procedure chapter. We refer you to the Normality Tests section of that chapter for details.

Plots

Plots for Height



The plots section displays a histogram and a probability plot to allow you to assess the accuracy of the normality assumption.