

## Chapter 285

# Acceptance Sampling for Attributes

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## Introduction

The *Acceptance Sampling for Attributes* procedure is used to determine the number of items to be sampled from a lot to determine whether to accept or reject the lot. The number of items in the sample depend upon a number of parameters, including the lot size, the acceptable quality level (AQL), the desired producer's risk, the limiting quality level (LQL, sometimes called the rejectable quality level or lot tolerance percent/proportion defective), and the desired consumer's risk. This procedure permits the user to enter multiple values of any of these parameters to determine the sensitivity of the sample size to that parameter. When multiple values are entered for a parameter, a sample size curve is also produced. The cutoff value of acceptance, or acceptance number, is also given as part of the output.

In this procedure, the lot size can be assumed to be infinite (or continuous) and use the binomial distribution for calculations, or the lot can have a fixed size, whereupon the calculations are based on the hypergeometric distribution.

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## Technical details

Define  $N$  to be the lot size (possibly infinite),  $n$  as the (unknown) size of the sample to be drawn, and  $c$  to be the acceptance number (the highest number of nonconforming units for which the lot will still be accepted). Let  $X$  denote the number of nonconforming units in the sample. Let  $p_0$  be the AQL, the highest proportion of nonconforming (defective) units for which the lot should still be accepted. Let  $\alpha$  be the producer's risk, the probability of rejecting a lot with a proportion of nonconforming (defective) units that is below the AQL. Let  $p_1$  be the LQL, the proportion of nonconforming (defective) units above which the lot should be routinely rejected. Let  $\beta$  be the probability of accepting a lot with a proportion of nonconforming (defective) units that is above the LQL.

For a given  $N$ ,  $p_0$ ,  $\alpha$ ,  $p_1$ , and  $\beta$ , we desire to obtain an  $n$  and  $c$  such that

$$\Pr\{X \leq c | p_0\} \geq 1 - \alpha$$

and

$$\Pr\{X \leq c | p_1\} \leq \beta$$

If the lot size is finite,  $n$  and  $c$  should satisfy the hypergeometric distribution inequalities

$$H(c; N, M_0, n) \geq 1 - \alpha$$

and

$$H(c; N, M_1, n) \leq \beta$$

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where  $M_0 = [Np_0]$  and  $M_1 = [Np_1]$ . The hypergeometric probability of obtaining exactly  $x$  of  $n$  items with the characteristic of interest is calculated using

$$h(x; N, M, n) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}$$

The cumulative hypergeometric distribution function is

$$H(x; N, M, n) = \sum_{j=0}^x h(j; N, M, n)$$

See Kenett and Zacks (2014) for more details.

If the lot size is infinite (or continuous),  $n$  and  $c$  should satisfy the binomial distribution inequalities

$$\sum_{j=0}^c \frac{n!}{j!(n-j)!} p_0^j (1-p_0)^{n-j} \geq 1 - \alpha$$

and

$$\sum_{j=0}^c \frac{n!}{j!(n-j)!} p_1^j (1-p_1)^{n-j} \leq \beta$$

PASS performs a search to determine an  $n$  and  $c$  value that meet the specified inequality requirements.

## Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

### Design Tab

The Design tab contains the parameters associated with this test such as the proportions, sample sizes, alpha, and power.

#### Solve For

##### Solve For

This option specifies the parameter to be solved for using the other parameters. The parameters that may be selected are *Consumer's Risk*, *Sample Size*, and *Limiting Quality Level*.

#### Lot Size

##### Lot Size (N)

Specify the total number of units in the lot that is subject to quality inspection. If the lot size is infinite or continuous, select Infinite for the Lot Size. In this case, the binomial distribution is used for calculations.

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If the lot size is set to any value other than infinite, the hypergeometric distribution is used in the calculations. You can choose a value from the drop-down, or you can enter a value directly.

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### Sample Size

#### Sample Size (n)

Enter one or more values for the sample size (n). This is the number of elements from the lot that will be randomly sampled and examined. You can enter a single value such as *100* or a series of values such as *50 60 70 80 90 100* or *100 to 150 by 10*. The Sample Size should be less than the Lot Size.

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### Acceptable Quality Level and Producer's Risk

#### Acceptable Quality Level

Sometimes called AQL or P0, this is highest proportion of nonconforming (defective) units for which the lot should still be accepted. That is, lots with proportions of nonconforming (defective) units less than this value should be accepted with high probability. You can enter a single value such as *0.02* or a series of values such as *0.01 0.02 0.03* or *0.01 to 0.05 by 0.01*. The range of permissible values is between 0 and 1 (not inclusive). Typical values are less than 0.1.

#### Producer's Risk (Alpha)

The Producer's Risk, or Alpha, is the probability of rejecting a lot with a proportion of nonconforming (defective) units that is below the acceptable quality level. In short, it is the risk of rejecting a good lot. You can enter a single value such as *0.05* or a series of values such as *0.01 0.05 0.1* or *0.01 to 0.10 by 0.01*. The range of permissible values is between 0 and 1 (not inclusive). Typical values are less than 0.2.

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### Limiting Quality Level and Consumer's Risk

#### Limiting Quality Level

Also called LQL, P1, Rejectable Quality Level, or LTPD (for Lot Tolerance Percent/Proportion Defective), this is the proportion of nonconforming (defective) units above which the lot should be routinely rejected. That is, lots with proportions of nonconforming (defective) units above this value should be rejected with high probability. You can enter a single value such as *0.1* or a series of values such as *0.05 0.1 0.15 0.2* or *0.1 to 0.2 by 0.01*. The range of permissible values is between 0 and 1 (not inclusive). Typical values are less than 0.3.

#### Consumer's Risk (Beta)

The Consumer's Risk, or Beta, is the probability of accepting a lot with a proportion of nonconforming (defective) units that is above the limiting quality level. In short, it is the risk of accepting a bad lot. You can enter a single value such as *0.1* or a series of values such as *0.1 0.15 0.2* or *0.1 to 0.3 by 0.05*. The range of permissible values is between 0 and 1 (not inclusive). Typical values are less than 0.3.

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### Normal Approximation

#### Approximation Cutoff

For typical calculations to determine the needed sample size, the binomial (or hypergeometric) distribution is used. For very large sample sizes, calculations based on the binomial or hypergeometric distributions may be very time-consuming. For large sample sizes the binomial or hypergeometric distribution may be approximated by the Normal distribution. The Approximation Cutoff entered here specifies the sample sizes above which the Normal approximation is used rather than the direct binomial or hypergeometric distribution calculation. Generally, this value should be left at a large number, such as 10,000, so that calculations are made using the binomial or hypergeometric distribution.

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## Example 1 – Finding a Sample Size

Suppose a quality engineer is to receive a lot of 5,000 units. Because it is not feasible to inspect all 5,000 units, the engineer would like to examine an acceptance sample. It is determined that the acceptable quality level is 0.5%, or 0.005. The desired producer's risk (probability of rejecting a good lot) is 0.05. The limiting quality level (or lot tolerance percent defective or rejectable quality level) is 0.07. The desired consumer's risk (probability of accepting a bad lot) is 0.05. The engineer would like to know the number of units to randomly sample and inspect, as well as the acceptance number (the highest number of nonconforming units of the sample for which the lot should still be accepted).

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Acceptance Sampling for Attributes** procedure window by clicking on **Quality Control** and then clicking on **Acceptance Sampling for Attributes**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Lot Size (N) .....	<b>5000</b>
Acceptable Quality Level .....	<b>0.005</b>
Producer's Risk (Alpha) .....	<b>0.05</b>
Limiting Quality Level .....	<b>0.07</b>
Consumer's Risk (Beta) .....	<b>0.05</b>
Approximation Cutoff .....	<b>10000</b>

### Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

#### Numeric Results for Acceptance Sampling for Attributes

Lot Size (N) = 5000

Sample Size (n)	--- # Nonconforming ---		Acceptable Quality Level	- Producer's Risk -		Limiting Quality Level	- Consumer's Risk -	
	Accept Lot If # ≤ c	Reject Lot If # ≥ c + 1		Target	Actual		Target	Actual
66	1	2	0.0050	0.0500	0.0424	0.0700	0.05000	0.04860

#### References

- Chow, S. C., Shao, J., and Wang, H. 2008. Sample Size Calculations in Clinical Research, Second Edition. Chapman & Hall/CRC. Boca Raton, Florida.
- Fliss, J. L., Levin, B., and Paik, M.C. 2003. Statistical Methods for Rates and Proportions. Third Edition. John Wiley & Sons. New York.
- Lachin, John M. 2000. Biostatistical Methods. John Wiley & Sons. New York.
- Machin, D., Campbell, M., Fayers, P., and Pinol, A. 1997. Sample Size Tables for Clinical Studies, 2nd Edition. Blackwell Science. Malden, Mass.
- Ryan, Thomas P. 2013. Sample Size Determination and Power. John Wiley & Sons. Hoboken, New Jersey.
- Zar, Jerrold H. 2010. Biostatistical Analysis, 5th Edition. Prentice-Hall. Englewood Cliffs, New Jersey.
- Kenett, Ron S. and Zacks, Shelemyahu 2014. Modern Industrial Statistics, 2nd Edition. John Wiley & Sons. West Sussex, United Kingdom.

## Acceptance Sampling for Attributes

### Report Definitions

Sample Size ( $n$ ) is the size of the sample drawn from the lot.

$c$ , the acceptance number, is the largest number of nonconforming items from the sample of size  $n$  for which the lot should still be accepted.

$c + 1$ , the rejection number, is the smallest number of nonconforming items from the sample of size  $n$  for which the lot should be rejected.

Acceptable Quality Level, AQL, or PO, is the highest proportion of nonconforming (defective) units for which the lot is still considered acceptable.

Producer's Risk, or alpha, is the probability of rejecting a lot with a proportion of nonconforming (defective) units that is below the acceptable quality level. In short, it is the risk of rejecting a good lot.

The target producer's risk is the specified value entered by the user.

The actual producer's risk is the value of the producer's risk that is generated by the corresponding sample size. It is usually less than the target producer's risk.

Limiting Quality Level, also called LQL, P1, Rejectable Quality Level, or LTPD (for Lot Tolerance Percent/Proportion Defective), this is the proportion of nonconforming (defective) units above which the lot should be routinely rejected.

Consumer's Risk, or beta, is the probability of accepting a lot with a proportion of nonconforming (defective) units that is above the limiting quality level. In short, it is the risk of accepting a bad lot.

The target consumer's risk is the specified value entered by the user.

The actual consumer's risk is the value of the consumer's risk that is generated by the corresponding sample size. It is usually less than the target consumer's risk.

### Summary Statements

A sample size of 66 from a lot of size 5000 with an acceptance cutoff number ( $c$ ) of 1, and based on an acceptable quality level of 0.0050 and a limiting quality level of 0.0700, has a producer's risk of 0.0424 and a consumer's risk of 0.04860. The target producer's risk was 0.0500. The target consumer's risk was 0.05000.

This report shows that the needed sample size is 66. The lot should be accepted if the number of nonconforming units out of the sample of size 66 is 0 or 1. The lot should be rejected if the number of nonconforming units is 2 or greater.

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## Example 2 – Finding Sample Sizes for Multiple Parameter Values

Continuing with Example 1, suppose the quality engineer would like to see how the sample size is affected by varying some of the parameters. The acceptable quality level is to stay at 0.005, but the engineer would like to look at Producer’s Risk values between 0.01 and 0.1, Limiting Quality Level values between 0.05 and 0.1, and Consumer’s Risk values between 0.05 and 0.2.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Acceptance Sampling for Attributes** procedure window by clicking on **Quality Control** and then clicking on **Acceptance Sampling for Attributes**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Lot Size (N) .....	<b>5000</b>
Acceptable Quality Level .....	<b>0.005</b>
Producer’s Risk (Alpha) .....	<b>0.01 to 0.1 by 0.01</b>
Limiting Quality Level .....	<b>0.05 to 0.1 by 0.01</b>
Consumer’s Risk (Beta) .....	<b>0.05 to 0.2 by 0.05</b>
Approximation Cutoff .....	<b>10000</b>

### Output

Click the Calculate button to perform the calculations and generate the following output.

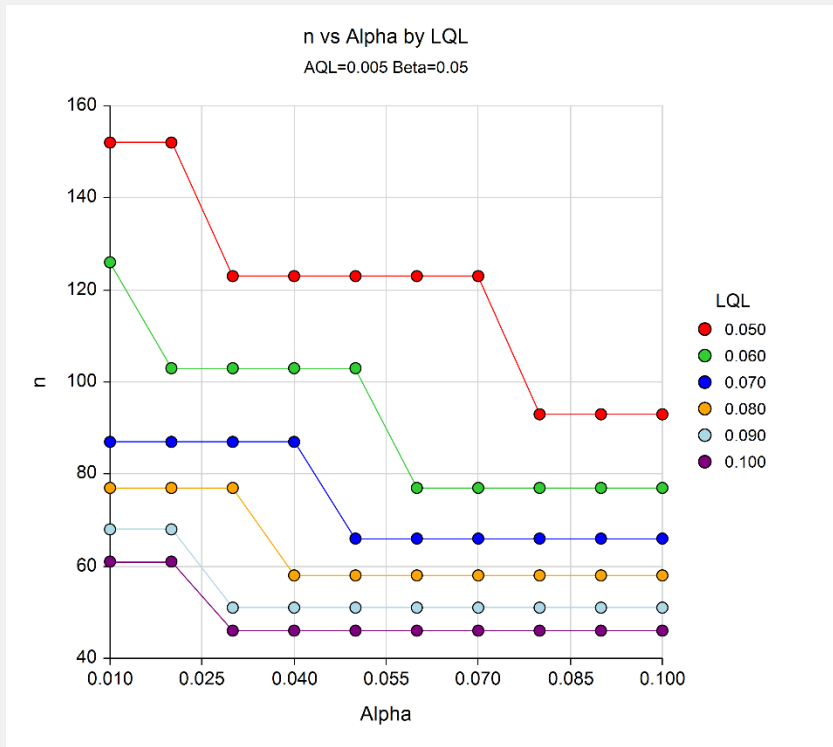
### Numeric Results

Numeric Results for Acceptance Sampling for Attributes									
Lot Size (N) = 5000									
Sample Size (n)	--- # Nonconforming ---		Acceptable Quality Level	- Producer's Risk -		Limiting Quality Level	- Consumer's Risk -		
	Accept Lot If # ≤ c	Reject Lot If # ≥ c + 1		Target	Actual		Target	Actual	
152	3	4	0.0050	0.0100	0.0063	0.0500	0.05000	0.04871	
152	3	4	0.0050	0.0200	0.0063	0.0500	0.05000	0.04871	
123	2	3	0.0050	0.0300	0.0225	0.0500	0.05000	0.04945	
123	2	3	0.0050	0.0400	0.0225	0.0500	0.05000	0.04945	
123	2	3	0.0050	0.0500	0.0225	0.0500	0.05000	0.04945	
123	2	3	0.0050	0.0600	0.0225	0.0500	0.05000	0.04945	
123	2	3	0.0050	0.0700	0.0225	0.0500	0.05000	0.04945	
93	1	2	0.0050	0.0800	0.0779	0.0500	0.05000	0.04853	
93	1	2	0.0050	0.0900	0.0779	0.0500	0.05000	0.04853	
93	1	2	0.0050	0.1000	0.0779	0.0500	0.05000	0.04853	
126	3	4	0.0050	0.0100	0.0032	0.0600	0.05000	0.04968	
103	2	3	0.0050	0.0200	0.0141	0.0600	0.05000	0.04784	
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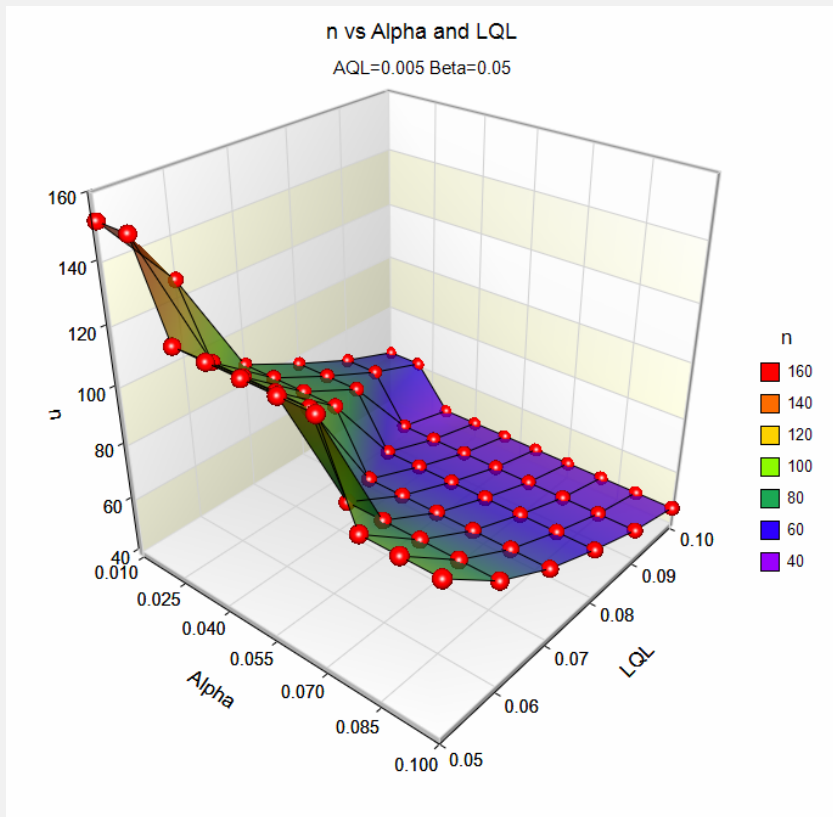
Each line gives a sample size and acceptance number based on the corresponding combination of quality levels and risks.

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Chart Section

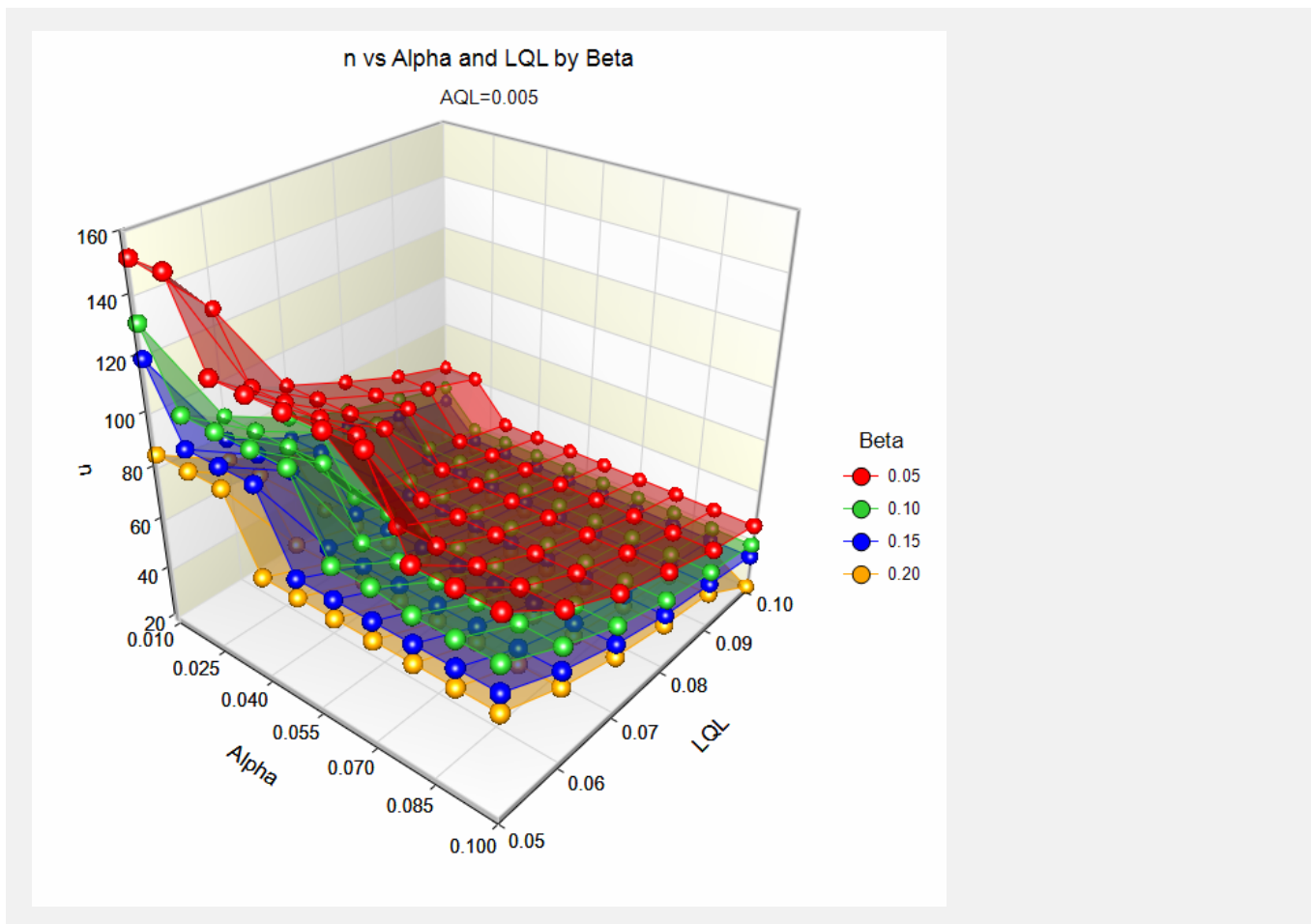


[Several plots between]



[Several plots between]

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These plots show the effect of various combinations of parameters on the sample size needed. The final 3D plot gives a summary of all the combinations in a single plot. This plot can be rotated (if desired) by going to the Plots tab and checking the box 'Live Edit/Rotate' for the X-Y-Z Plots with Groups and then re-running the procedure.



## Example 3 – Validation of Sample Size Calculation using Kenett and Zacks (2013)

Kenett and Zacks (2013), pp. 260-261, give an example in which the lot size is 100, alpha and beta (producer’s risk and consumer’s risk) are both 0.05, the acceptable quality level is 0.01, and the limiting quality level is 0.05. The needed sample size is estimated to be 65, with a *c* of 1.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Acceptance Sampling for Attributes** procedure window by clicking on **Quality Control** and then clicking on **Acceptance Sampling for Attributes**. You may then make the appropriate entries as listed below, or open **Example 3(a-c)** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	Sample Size
Lot Size (N) .....	100
Acceptable Quality Level .....	0.01
Producer’s Risk (Alpha) .....	0.05
Limiting Quality Level.....	0.05
Consumer’s Risk (Beta).....	0.05
Approximation Cutoff .....	10000

### Output

Click the Calculate button to perform the calculations and generate the following output.

#### Numeric Results

Numeric Results for Acceptance Sampling for Attributes									
Lot Size (N) = 100									
Sample Size (n)	--- # Nonconforming ---		Acceptable Quality Level	- Producer's Risk -		Limiting Quality Level	- Consumer's Risk -		
	Accept Lot If # ≤ c	Reject Lot If # ≥ c + 1		Target	Actual		Target	Actual	
65	1	2	0.0100	0.0500	0.0000	0.0500	0.05000	0.04952	

The sample size of 65 and *c* value of 1 matches that of Kenett and Zacks (2013).

In a second example with acceptable quality level of 0.03, and a limiting quality level of 0.05, Kenett and Zacks (2013) estimate a required sample size of 92.

Numeric Results for Acceptance Sampling for Attributes									
Lot Size (N) = 100									
Sample Size (n)	--- # Nonconforming ---		Acceptable Quality Level	- Producer's Risk -		Limiting Quality Level	- Consumer's Risk -		
	Accept Lot If # ≤ c	Reject Lot If # ≥ c + 1		Target	Actual		Target	Actual	
92	3	4	0.0300	0.0500	0.0000	0.0500	0.05000	0.04990	

PASS also gives a sample size estimate of 92.

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In a third example with acceptable quality level of 0.01, and a limiting quality level of 0.2, Kenett and Zacks (2013) estimate a required sample size of 20.

Numeric Results for Acceptance Sampling for Attributes								
Lot Size (N) = 100								
Sample Size (n)	--- # Nonconforming ---		Acceptable Quality Level	- Producer's Risk -		Limiting Quality Level	- Consumer's Risk -	
	Accept Lot If # ≤	Reject Lot If # ≥		Target	Actual		Target	Actual
20	c	c + 1	0.0100	0.0500	0.0000	0.2000	0.050000	0.04985

PASS also estimates a sample size estimate of 20 for this scenario.

## Example 4 – Validation of Sample Size Calculation using Guenther (1977)

Guenther (1977), pp. 7-8, gives an example in which the lot size is infinite (binomial calculations), alpha (producer's risk) is 0.05, beta (consumer's risk) is 0.1, the acceptable quality level is 0.05, and the limiting quality level is 0.2. The needed sample size is estimated to be 38, with an acceptance number ( $c$ ) of 4.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Acceptance Sampling for Attributes** procedure window by clicking on **Quality Control** and then clicking on **Acceptance Sampling for Attributes**. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Lot Size (N) .....	<b>Infinite</b>
Acceptable Quality Level .....	<b>0.05</b>
Producer's Risk (Alpha) .....	<b>0.05</b>
Limiting Quality Level .....	<b>0.2</b>
Consumer's Risk (Beta) .....	<b>0.1</b>
Approximation Cutoff .....	<b>10000</b>

### Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

Numeric Results for Acceptance Sampling for Attributes								
Lot Size (N) = 100								
Sample Size (n)	--- # Nonconforming ---		Acceptable Quality Level	- Producer's Risk -		Limiting Quality Level	- Consumer's Risk -	
	Accept Lot If # ≤	Reject Lot If # ≥		Target	Actual		Target	Actual
38	c	c + 1	0.0500	0.0500	0.0397	0.2000	0.10000	0.09857

The sample size of 38 and  $c$  value of 4 matches that of Guenther (1977).