Chapter 288

Acceptance Sampling for Attributes with Zero Nonconformities

Introduction

The *Acceptance Sampling for Attributes with Zero Nonconformities* procedure is used to determine the number of items to be sampled from a lot to determine whether to accept or reject the lot when the number of nonconformities (defects) is zero.

The number of items in the sample depend upon a number of parameters, including the lot size, the acceptable quality level (AQL), the desired producer's risk, the limiting quality level (LQL, sometimes called the rejectable quality level or lot tolerance percent/proportion defective), the desired consumer's risk, and the number of defective items found in the sample. This procedure gives results for designs with zero nonconformities.

In this procedure, the lot size can be assumed to be infinite (or continuous) and use the binomial distribution for calculations, or the lot can have a fixed size, whereupon the calculations are based on the hypergeometric distribution.

Technical Details

Define N to be the lot size (possibly infinite), n as the (unknown) size of the sample to be drawn, and c to be the acceptance number (the highest number of nonconforming units for which the lot will still be accepted). Let X denote the number of nonconforming units in the sample. Let p_0 be the AQL, the highest proportion of nonconforming (defective) units for which the lot should still be accepted. Let α be the producer's risk, the probability of rejecting a lot with a proportion of nonconforming (defective) units that is below the AQL. Let p_1 be the LQL, the proportion of nonconforming (defective) units above which the lot should be routinely rejected. Let β be the probability of accepting a lot with a proportion of nonconforming (defective) units that is above the LQL.

For a given N, p_0, α, p_1, $c = 0$, and β, we desire to obtain an n such that

$$\Pr\{X \leq c | p_0, c = 0\} \geq 1 - \alpha$$

and

$$\Pr\{X \leq c | p_1, c = 0\} \leq \beta$$

If the lot size is finite, n should satisfy the hypergeometric distribution inequalities

$$H(c; N, M_0, n) \geq 1 - \alpha$$

and

$$H(c; N, M_1, n) \leq \beta$$

where $c = 0$, $M_0 = [Np_0]$ and $M_1 = [Np_1]$. The hypergeometric probability of obtaining exactly x of n items with the characteristic of interest is calculated using

$$h(x; N, M, n) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

The cumulative hypergeometric distribution function is

$$H(x; N, M, n) = \sum_{j=0}^{x} h(j; N, M, n)$$

See Kenett and Zacks (2014) for more details.

If the lot size is infinite (or continuous), n should satisfy the binomial distribution inequalities

$$\sum_{j=0}^{0} \frac{n!}{j!\,(n-j)!} p_0^j (1 - p_0)^{n-j} \geq 1 - \alpha$$

and

$$\sum_{j=0}^{0} \frac{n!}{j!\,(n-j)!} p_1^j (1 - p_1)^{n-j} \leq \beta$$

PASS performs a search to determine an n value that meets the specified inequality requirements.

Example 1 – Comparing Various Sample Sizes

Suppose a quality engineer is to receive lots of between 100 and 300 units. Because it is not feasible to inspect all units in the lot, the engineer would like to examine an acceptance sample. It is determined that the acceptable quality level is 0.1%, or 0.0001, and the limiting quality level is 0.02.

The engineer would like to understand the relationships between producer's risk, consumer's risk, and sample size when the number of defective items allowed is 0. To accomplish this, the sample size will range from 30 to 70.

Note that even though the *Solve For* parameter will be set to *Consumer's Risk (Beta)*, both consumer's risk and producer's risk will be calculated. However, only consumer's risk will be displayed on the plot.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Consumer's Risk (Beta)**
Lot Size (N)..**100 200 300**
Sample Size (n) **30 40 50 60 70**
Acceptable Quality Level (AQL, P0)...............**0.0001**
Limiting Quality Level (LQL, P1) **0.03**
Approximation Cutoff **10000**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results
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Solve For: Consumer's Risk (Beta)
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Lot Size N	Sample Size n	# Nonconforming		Acceptable Quality Level P0	Producer's Risk Alpha	Limiting Quality Level P1	Consumer's Risk Beta
		c Accept Lot If # ≤	c + 1 Reject Lot If # ≥				
100	30	0	1	0.0001	0.30000	0.03	0.33853
100	40	0	1	0.0001	0.40000	0.03	0.21163
100	50	0	1	0.0001	0.50000	0.03	0.12121
100	60	0	1	0.0001	0.60000	0.03	0.06110
100	70	0	1	0.0001	0.70000	0.03	0.02511
200	30	0	1	0.0001	0.15000	0.03	0.37209
200	40	0	1	0.0001	0.20000	0.03	0.25717
200	50	0	1	0.0001	0.25000	0.03	0.17349
200	60	0	1	0.0001	0.30000	0.03	0.11384
200	70	0	1	0.0001	0.35000	0.03	0.07236
300	30	0	1	0.0001	0.10000	0.03	0.38219
300	40	0	1	0.0001	0.13333	0.03	0.27070
300	50	0	1	0.0001	0.16667	0.03	0.18911
300	60	0	1	0.0001	0.20000	0.03	0.13017
300	70	0	1	0.0001	0.23333	0.03	0.08815
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N   Lot Size. The total size of the lot from which the sample drawn.
n   Sample Size. The size of the sample drawn from the lot.
c   Acceptance Number. The largest number of nonconforming items from the sample of size n for which the lot should
           still be accepted.
c + 1   Rejection Number. The smallest number of nonconforming items from the sample of size n for which the lot should
           be rejected.
P0   Acceptable Quality Level (AQL). The highest proportion of nonconforming (defective) units for which the lot is still
           considered acceptable.
Alpha   Producer's Risk. The probability of rejecting a lot with a proportion of nonconforming (defective) units that is below
           the acceptable quality level. In short, it is the risk of rejecting a good lot.
P1   Limiting Quality Level (LQL) (Also called Rejectable Quality Level or LTPD (for Lot Tolerance Percent/Proportion
           Defective)). This is the proportion of nonconforming (defective) units above which the lot should be routinely
           rejected.
Beta   Consumer's Risk. The probability of accepting a lot with a proportion of nonconforming (defective) units that is above
           the limiting quality level. In short, it is the risk of accepting a bad lot.

Summary Statements
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An acceptance sampling design will be used to determine whether to accept or reject a lot of 100 items. The acceptable quality level (P0) is 0.0001. The limiting quality level (P1) is assumed to be 0.03. The lot will be rejected if there are any (one or more) non-conforming units (defectives) in the sample. The determination to accept or reject the lot will be based on a one-sided exact test of a single proportion. With a sample size of 30, the producer's risk (α) is 0.3 and the consumer's risk (β) is 0.33853.
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References
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Guenther, William C. 1977. Sampling inspection in statistical quality control. Griffin's statistical monographs and
    courses, no. 37.
Kenett, R. S. and Zacks, S. 2014. Modern Industrial Statistics, 2nd Edition. John Wiley & Sons. West Sussex,
    United Kingdom.
Schilling, E. G. and Neubauer, D. V. 2017. Acceptance Sampling in Quality Control, Third Edition. CRC Press. New
    York.
Squeglia, Nicholas L. 2017. Zero Acceptance Number Sampling Plans, Fifth Edition. Quality Press. Milwaukee, WI.
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This report shows the values of alpha and beta for various combination of n and c.

Plots Section
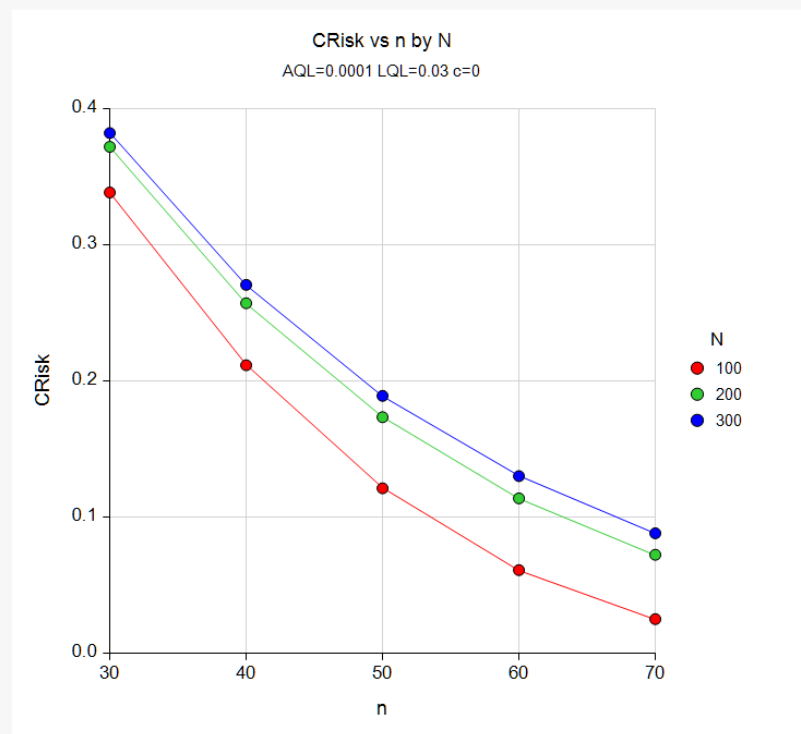
Plots
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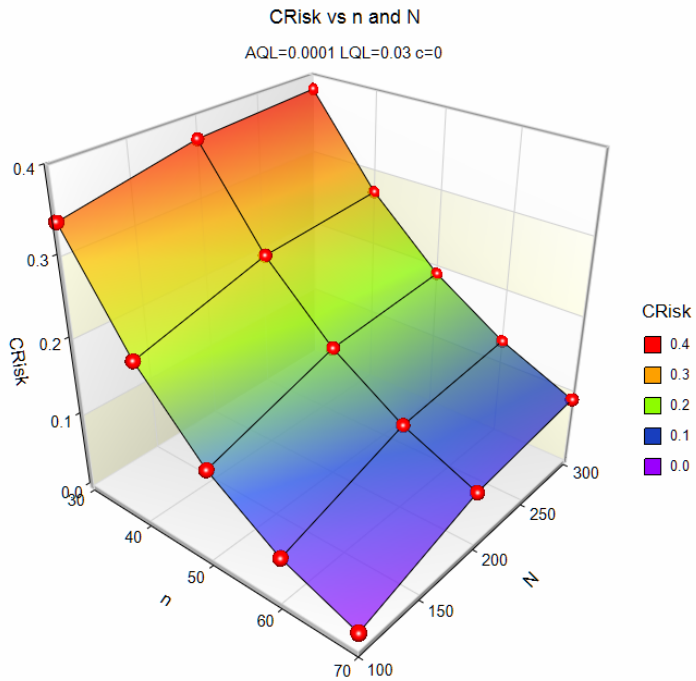
This report allows several parameters to be compared. However, note that the values of the producer's risk as shown in the numeric report are rather high.

Example 2 – Validation of Sample Size Calculation using Schilling and Neubauer (2017)

Schilling and Neubauer (2017), pp. 543, give an example in which the lot size is 5000, *c* is 0, alpha is 0.89 and beta is 0.05, the acceptable quality level is 0.001, and the limiting quality level is 0.005. The required sample size is estimated to be 439.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Lot Size (N)...**5000**
Acceptable Quality Level (AQL, P0)...............**0.001**
Producer's Risk (Alpha) **0.89**
Limiting Quality Level (LQL, P1) **0.005**
Consumer's Risk (Beta) **0.1**
Approximation Cutoff **10000**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results
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Solve For: Sample Size
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Lot Size N	Sample Size n	# Nonconforming		Acceptable Quality Level P0	Producer's Risk (Alpha)		Limiting Quality Level P1	Consumer's Risk (Beta)	
		c Accept Lot If # ≤	c + 1 Reject Lot If # ≥		Target	Actual		Target	Actual
5000	439	0	1	0.001	0.89	0.36851	0.005	0.1	0.09994

The sample size of 439 matches the results of Schilling and Neubauer (2017).