

Chapter 722

Assurance for Tests for Two Survival Curves using Cox's Proportional Hazards Model

Introduction

This procedure calculates the assurance of regression coefficient tests from Cox proportional hazards regression. The formulation for testing the significance of a Cox regression coefficient is identical to that of the standard logrank test, so the power and sample size formulas for one analysis also works for the other. This procedure is documented in Chow, Shao, and Wang (2008), which summarizes the work of Schoenfeld (1981, 1983). Refer to the *Tests for Two Survival Curves using Cox's Proportional Hazards Model* procedure for more details. The assurance calculation used in this procedure is based on O'Hagan, Stevens, and Campbell (2005).

The Cox Regression model has the added benefit over the exponential model that it does not assume that the hazard rates are constant across time, but only that they are proportional. That is, that the hazard ratio remains constant throughout the experiment, even if the hazard rates vary from one time point to another.

The assurance calculation in this procedure is based on a user-specified prior distribution of the applicable parameters. This procedure may also be used to determine the needed sample size to obtain a specified assurance.

Assurance

The assurance of a design is the expected value of the power with respect to one or more prior distributions of the design parameters. Assurance is also referred to as *Bayesian assurance*, *expected power*, *average power*, *statistical assurance*, *hybrid classical-Bayesian procedure*, or *probability of success*.

The power of a design is the probability of rejecting the null hypothesis, conditional on a given set of design attributes, such as the test statistic, the significance level, the sample size, and the effect size to be detected. As many of the parameters are typically unknown quantities, the stated power may be very different from the true power if the specified parameter values are inaccurate.

While power is conditional on individual design parameter values, and is highly sensitive to those values, assurance is the average power across a presumed prior distribution of the parameters. Thus, assurance adds a Bayesian element to the frequentist framework, resulting in a hybrid approach to the probability of trial or study success. It should be noted that when it comes time to perform the statistical test on the resulting data, these methods for calculating assurance assume that the traditional (frequentist) tests will be used.

The next section describes some of the ways in which the prior distributions for effect size parameters may be determined.

Elicitation

In order to calculate assurance, a suitable prior distribution for the applicable parameters must be determined. This process is called the *elicitation* of the prior distribution.

The elicitation may be as simple as choosing a distribution that seems plausible for the parameter(s) of interest, or as complex as combining the informed advice of several experts based on experience in the field, available pilot data, or previous studies. The accuracy of the assurance value depends on the accuracy of the elicited prior distribution. The assumption (or hope) is that an informed prior distribution will produce a more accurate estimate of the probability of trial success than a single value estimate. Because clinical trials and other studies are often costly, many institutions now routinely require an elicitation step.

Two reference texts that focus on elicitation are O'Hagan, Buck, Daneshkhah, Eiser, Garthwaite, Jenkinson, Oakley, and Rakow (2006) and Dias, Morton, and Quigley (2018).

Technical Details

We assume that a study is to be made comparing the survival (or healing) of a control group with an experimental group. The control group (group 1) consists of patients that will receive the existing treatment. In cases where no existing treatment exists, group 1 consists of patients that will receive a placebo. Group 2 will receive the new treatment.

We assume that the critical event of interest is death and that two treatments have survival distributions with instantaneous death (hazard) rates, λ_1 and λ_2 . These hazard rates are a subject's probability of death in a short period of time.

Cox's Proportional Hazards Regression

Cox's proportional hazards regression is widely used for survival data. The regression model is

$$h(t|z) = h(t|0) \exp(bz)$$

where

b is the regression coefficient which is equal to $\log[h(t|1)/h(t|0)] = \log(HR)$

z is a binary indicator variable of the treatment group.

t is elapsed time.

$h(t|z)$ is the hazard rate at time t , given covariate z .

HR is the hazard ratio $h(t|1)/h(t|0)$.

The lower, one-sided, statistical hypothesis testing survival equality is a test of whether b is zero. This hypothesis is stated as

$$H_0: b = 0 \quad \text{versus} \quad H_1: b < 0$$

Test Statistic

It can be shown that the test of b based on the partial likelihood method of Cox (1972) coincides with the common logrank test statistic shown next.

Logrank Test

The logrank test statistic is

$$L = \frac{\sum_{k=1}^K \left(I_k - \frac{Y_{1i}}{Y_{1i} + Y_{2i}} \right)}{\left[\sum_{k=1}^K \left(\frac{Y_{1i}Y_{2i}}{(Y_{1i} + Y_{2i})^2} \right) \right]^{-1/2}}$$

where K is the number of deaths, Y_{ij} is the number of subjects at risk just prior to the j^{th} observed event in the i^{th} group, and I_k is a binary variable indicating whether the k^{th} event is from group 1 or not.

The distribution of L is approximately normal with mean $b\sqrt{P_1P_2dN}$ and unit variance, where

P_1 is the proportion of N that is in the control group.

P_2 is the proportion of N that is in the treatment group.

N is the total sample size.

N_1 is the sample size from the control group, $N_1 = N(P_1)$.

N_2 is the sample size from the treatment group, $N_2 = N(P_2)$.

Pev_1 is probability that the event of interest is observed in the control group during the study.

Pev_2 is probability that the event of interest is observed in the treatment group during the study.

d is the overall probability of an event, $d = Pev_1P_1 + Pev_2P_2$.

b is the Cox regression coefficient, $b = \log(HR)$.

Power Calculations

The power of the statistical test of b is given by

$$\Phi(b\sqrt{P_1P_2dN} - z_{1-\alpha})$$

where HR is the actual assumed value of the hazard ratio under the alternative hypothesis.

Assurance Calculation

This assurance computation described here is based on O'Hagan, Stevens, and Campbell (2005).

Let $P_1(R|Pev_1, Pev_2, HR)$ be the power function described above where R is the event that the null hypothesis is rejected conditional on the parameter values. The specification of Pev_1 , Pev_2 , and HR is critical to the power calculation, but the actual values are seldom known. Assurance is defined as the expected power where the expectation is with respect to a joint prior distribution for Pev_1 , Pev_2 , and HR . Hence, assurance is

$$\begin{aligned} Assurance &= E_{Pev_1, Pev_2, HR}(P_1(R|Pev_1, Pev_2, HR)) \\ &= \iiint P_1(R|Pev_1, Pev_2, HR)f(Pev_1, Pev_2, HR)dPev_1dPev_2dHR \end{aligned}$$

where $f(Pev_1, Pev_2, HR)$ is the joint prior distribution of Pev_1 , Pev_2 , and HR .

In **PASS**, the joint prior distribution can be specified as either a discrete approximation to the joint prior distribution, or as individual prior distributions, one for each parameter.

Specifying a Joint Prior Distribution

If the joint prior distribution is to be specified directly, the distribution is specified in **PASS** using a discrete approximation to the function $f(Pev_1, Pev_2, HR)$. This provides flexibility in specifying the joint prior distribution. In the three-parameter case, four columns are entered on the spreadsheet: three for the parameters and a fourth for the probability. Each row gives a value for each parameter and the corresponding parameter-combination probability. The accuracy of the distribution approximation is controlled by the number of points (spreadsheet rows) that are used.

An example of entering a joint prior distribution is included at the end of the chapter.

Specifying Individual Prior Distributions

Ciarleglio, Arendt, and Peduzzi (2016) suggest that more flexibility is available if the joint prior distribution is separated into three independent univariate distributions as follows:

$$f(Pev_1, Pev_2, HR) = f_1(Pev_1)f_2(Pev_2)f_3(HR)$$

where $f_1(Pev_1)$ is the prior distribution of Pev_1 , $f_2(Pev_2)$ is the prior distribution of Pev_2 , and $f_3(HR)$ is the prior distribution of w . This method is also available in **PASS**. The definition of assurance becomes

$$\begin{aligned} Assurance &= E_{Pev_1, Pev_2, HR}(P_1(R|Pev_1, Pev_2, HR)) \\ &= \iiint P_1(R|Pev_1, Pev_2, HR)f_1(Pev_1)f_2(Pev_2)f_3(HR)dPev_1dPev_2dHR \end{aligned}$$

Using this definition, the assurance can be calculated using numerical integration. There are a variety of pre-programmed, univariate prior distributions available in **PASS**.

Fixed Values (No Prior) and Custom Values

For any given parameter, **PASS** also provides the option of entering a single fixed value for the prior distribution, or a series of values and corresponding probabilities (using the spreadsheet), rather than one of the pre-programmed distributions.

Numerical Integration in PASS (and Notes on Computation Speed)

When the prior distribution is specified as independent univariate distributions, **PASS** uses a numerical integration algorithm to compute the assurance value as follows:

The domain of each prior distribution is divided into M intervals. Since many of the available prior distributions are unbounded on one (e.g., Gamma) or both (e.g., Normal) ends, an approximation is made to make the domain finite. This is accomplished by truncating the distribution to a domain between the two quantiles: $q_{0.001}$ and $q_{0.999}$.

The value of M controls the accuracy and speed of the algorithm. If only one parameter is to be given a prior distribution, then a value of M between 50 and 100 usually gives an accurate result in a timely manner. However, if two parameters are given priors, the number of iterations needed increases from M to M^2 . For example, if M is 100, 10000 iterations are needed. Reducing M from 100 to 50 reduces the number of iterations from 10000 to 2500.

The algorithm runtime increases when searching for sample size rather than solving for assurance, as a search algorithm is employed in this case. When solving for sample size, we recommend reducing M to 20 or less while exploring various scenarios, and then increasing M to 50 or more for a final, more accurate, result.

List of Available Univariate Prior Distributions

This section lists the univariate prior distributions that may be used for any of the applicable parameters when the Prior Entry Method is set to Individual.

No Prior

If 'No Prior' is chosen for a parameter, the parameter is assumed to take on a single, fixed value with probability one.

Beta (Shape 1, Shape 2, a, c)

A random variable X that follows the beta distribution is defined on a finite interval $[a, c]$. Two shape parameters (α and β) control the shape of this distribution. Two location parameters a and c give the minimum and maximum of X .

The probability density function of the beta distribution is

$$f(x|\alpha, \beta, a, c) = \frac{\left(\frac{x-a}{c-a}\right)^{\alpha-1} \left(\frac{c-x}{c-a}\right)^{\beta-1}}{(c-a)B(\alpha, \beta)}$$

where $B(\alpha, \beta) = \Gamma(\alpha) \Gamma(\beta) / \Gamma(\alpha + \beta)$ and $\Gamma(z)$ is the gamma function.

The mean of X is

$$\mu_X = \frac{\alpha c + \beta a}{\alpha + \beta}$$

Various distribution shapes are controlled by the values of α and β . These include

Symmetric and Unimodal

$$\alpha = \beta > 1$$

U Shaped

$$\alpha = \beta < 1$$

Bimodal

$$\alpha, \beta < 1$$

Uniform

$$\alpha = \beta = 1$$

Parabolic

$$\alpha = \beta = 2$$

Bell-Shaped

$$\alpha = \beta > 2$$

Gamma (Shape, Scale)

A random variable X that follows the gamma distribution is defined on the interval $(0, \infty)$. A shape parameter, κ , and a scale parameter, θ , control the distribution.

The probability density function of the gamma distribution is

$$f(x|\kappa, \theta) = \frac{x^{\kappa-1} e^{-\frac{x}{\theta}}}{\theta^{\kappa} \Gamma(\kappa)}$$

where $\Gamma(z)$ is the gamma function.

The mean of X is

$$\mu_X = \frac{\kappa}{\theta}$$

A truncated version of the distribution is constructed by dividing the density by $1 - \text{Prob}(Min \leq X \leq Max)$ where Min and Max are two truncation bounds.

Inverse-Gamma (Shape, Scale)

A random variable X that follows the inverse-gamma distribution is defined on the interval $(0, \infty)$. If $Y \sim \text{gamma}$, then $X = 1 / Y \sim \text{inverse-gamma}$. A shape parameter, α , and a scale parameter, β , control the distribution.

The probability density function of the inverse-gamma distribution is

$$f(x|\alpha, \beta) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\frac{\beta}{x}}}{\Gamma(\alpha)}$$

where $\Gamma(z)$ is the gamma function.

The mean of X is

$$\mu_X = \frac{\beta}{\alpha - 1} \text{ for } \alpha > 1$$

A truncated version of the distribution is constructed by dividing the density by $1 - \text{Prob}(Min \leq X \leq Max)$ where Min and Max are two truncation bounds.

Logistic (Location, Scale)

A random variable X that follows the logistic distribution is defined on the interval $(-\infty, \infty)$. A location parameter, μ , and a scale parameter, s , control the distribution.

The probability density function of the logistic distribution is

$$f(x|\mu, s) = \frac{e^{-\frac{x-\mu}{s}}}{s \left(1 + e^{-\frac{x-\mu}{s}}\right)^2}$$

The mean of X is

$$\mu_X = \mu$$

A truncated version of the distribution is constructed by dividing the density by $1 - \text{Prob}(Min \leq X \leq Max)$ where Min and Max are two truncation bounds.

Lognormal (Mean, SD)

A random variable X that follows the lognormal distribution is defined on the interval $(0, \infty)$. A location parameter, $\mu_{\log(X)}$, and a scale parameter, $\sigma_{\log(X)}$, control the distribution. If $Z \sim$ standard normal, then $X = e^{\mu + \sigma Z} \sim$ lognormal. Note that $\mu_{\log(X)} = E(\log(X))$ and $\sigma_{\log(X)} = \text{Standard Deviation}(\log(X))$.

The probability density function of the lognormal distribution is

$$f(x|\mu, \sigma) = \frac{e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2}}{x\sigma\sqrt{2\pi}}$$

The mean of X is

$$\mu_X = e^{\mu + \frac{\sigma^2}{2}}$$

A truncated version of the distribution is constructed by dividing the density by $1 - \text{Prob}(Min \leq X \leq Max)$ where Min and Max are two truncation bounds.

LogT (Mean, SD)

A random variable X that follows the logT distribution is defined on the interval $(0, \infty)$. A location parameter, $\mu_{\log(X)}$, a scale parameter, $\sigma_{\log(X)}$, and a shape parameter, ν , control the distribution. Note that ν is referred to as the *degrees of freedom*.

If $t \sim \text{Student's } t$, then $X = e^{\mu + \sigma t} \sim \text{logT}$.

The probability density function of the logT distribution is

$$f(x|\mu, \sigma, \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{x\Gamma\left(\frac{\nu}{2}\right)\sigma\sqrt{\nu\pi}} \left(1 + \frac{1}{\nu} \left(\frac{\log x - \mu}{\sigma}\right)^2\right)^{\left(\frac{-\nu-1}{2}\right)}$$

The mean of X is not defined.

A truncated version of the distribution is constructed by dividing the density by $1 - \text{Prob}(\text{Min} \leq X \leq \text{Max})$ where Min and Max are two truncation bounds.

Normal (Mean, SD)

A random variable X that follows the normal distribution is defined on the interval $(-\infty, \infty)$. A location parameter, μ , and a scale parameter, σ , control the distribution.

The probability density function of the normal distribution is

$$f(x|\mu, \sigma) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

The mean of X is

$$\mu_X = \mu$$

A truncated version of the distribution is constructed by dividing the density by $1 - \text{Prob}(\text{Min} \leq X \leq \text{Max})$ where Min and Max are two truncation bounds.

T (Mean, SD, DF)

A random variable X that follows Student's t distribution is defined on the interval $(-\infty, \infty)$. A location parameter, μ , a scale parameter, σ , and a shape parameter, ν , control the distribution. Note that ν is referred to as the *degrees of freedom* or *DF*.

The probability density function of the Student's t distribution is

$$f(x|\mu, \sigma, \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sigma\sqrt{\nu\pi}} \left(1 + \frac{1}{\nu} \left(\frac{x-\mu}{\sigma}\right)^2\right)^{\left(\frac{-\nu-1}{2}\right)}$$

The mean of X is μ if $\nu > 1$.

A truncated version of the distribution is constructed by dividing the density by $1 - \text{Prob}(\text{Min} \leq X \leq \text{Max})$ where Min and Max are two truncation bounds.

Triangle (Mode, Min, Max)

Let a = minimum, b = maximum, and c = mode. A random variable X that follows a triangle distribution is defined on the interval (a, b) .

The probability density function of the triangle distribution is

$$f(x|a, b, c) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & \text{for } a \leq x < c \\ \frac{2}{b-a} & \text{for } x = c \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{for } c < x \leq b \end{cases}$$

The mean of X is

$$\frac{a + b + c}{3}$$

Uniform (Min, Max)

Let a = minimum and b = maximum. A random variable X that follows a uniform distribution is defined on the interval $[a, b]$.

The probability density function of the uniform distribution is

$$f(x|a, b) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \end{cases}$$

The mean of X is

$$\frac{a + b}{2}$$

Weibull (Shape, Scale)

A random variable X that follows the Weibull distribution is defined on the interval $(0, \infty)$. A shape parameter, κ , and a scale parameter, λ , control the distribution.

The probability density function of the Weibull distribution is

$$f(x|\kappa, \lambda) = \frac{\kappa}{\lambda} \left(\frac{x}{\lambda}\right)^{\kappa-1} e^{-\left(\frac{x}{\lambda}\right)^\kappa}$$

The mean of X is

$$\mu_X = \kappa \Gamma\left(1 + \frac{1}{\kappa}\right)$$

A truncated version of the distribution is constructed by dividing the density by $1 - \text{Prob}(\text{Min} \leq X \leq \text{Max})$ where Min and Max are two truncation bounds.

Custom (Values and Probabilities in Spreadsheet)

This custom prior distribution is represented by a set of user-specified points and associated probabilities, entered in two columns of the spreadsheet. The points make up the entire set of values that are used for this parameter in the calculation of assurance. The associated probabilities should sum to one. Note that custom values and probabilities can be used to approximate any continuous distribution.

For example, a prior distribution of X might be

X_i	P_i
10	0.2
20	0.2
30	0.3
40	0.2
50	0.1

In this example, the mean of X is

$$\mu_X = \sum_{i=1}^5 X_i P_i$$

Example 1 – Assurance Over a Range of Sample Sizes

A clinical trial is being planned to test the effectiveness of a new treatment compared to the existing treatment using a one-sided test with a significance level of 0.025. The current treatment for this disease achieves 50% survival after two years. The researchers want to compare the assurance across sample sizes when the hazard ratio is 0.75.

To complete their sample size study, the researchers want to run an assurance analysis for a range of group sample sizes from 200 to 800. An elicitation exercise determined that the prior distribution of Pev1 should be normal with mean 0.5 and standard deviation 0.04. The elicitation also concluded that prior distribution of Pev2 should be Normal (0.5, 0.08). Finally, the prior distribution of *HR* should be Normal (0.8, 0.08).

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	Assurance
Prior Entry Method	Individual (Enter a prior distribution for each applicable parameter)
Alternative Hypothesis	One-Sided (H1: HR < 1)
Alpha.....	0.025
Group Allocation	Equal (N1 = N2)
Sample Size Per Group	200 400 600 800
Prior Distribution of Pev1	Normal (Mean, SD)
Mean.....	0.5
SD.....	0.04
Truncation Boundaries.....	None
Prior Distribution of Pev2	Normal (Mean, SD)
Mean.....	0.5
SD.....	0.08
Truncation Boundaries.....	None
Prior Distribution of HR	Normal (Mean, SD)
Mean.....	0.8
SD.....	0.08
Truncation Boundaries.....	None

Options Tab

Number of Computation Points for each.....	50
Prior Distribution	
Maximum N1 in Sample Size Search	5000

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Assurance
 Groups: 1 = Control, 2 = Treatment
 Hazard Ratio (HR): Hazard Rate 2 / Hazard Rate 1
 Hypotheses: H0: HR \geq 1 vs. H1: HR < 1
 Test Statistic: Treatment Regression Coefficient from Cox Regression
 Prior Type: Independent Univariate Distributions

Prior Distributions

Pev1: Normal (Mean = 0.5, SD = 0.04).
 Pev2: Normal (Mean = 0.5, SD = 0.08).
 HR: Normal (Mean = 0.8, SD = 0.08).

Assurance*	Power‡	Required Number of Events			Sample Size			Expected Probability of Observing an Event in Group 1 E(Pev1)	Expected Probability of Observing an Event in Group 2 E(Pev2)	Expected Hazard Ratio E(HR)	Alpha
		E1	E2	E	N1	N2	N				
0.38467	0.35119	100	100	200	200	200	400	0.5	0.5	0.8	0.025
0.58215	0.60699	200	200	400	400	400	800	0.5	0.5	0.8	0.025
0.69380	0.78023	300	300	600	600	600	1200	0.5	0.5	0.8	0.025
0.76233	0.88411	400	400	800	800	800	1600	0.5	0.5	0.8	0.025

* The number of points used for computation of the prior(s) was 50.

‡ Power was calculated using Pev1 = E(Pev1) = 0.5, Pev2 = E(Pev2) = 0.5, and HR = E(HR) = 0.8.

Assurance The expected power where the expectation is with respect to the prior distribution(s).
 Power The power calculated using the means of the prior distributions as the values of the corresponding parameters.
 E1 The required number of events in group 1, the control group.
 E2 The required number of events in group 2, the treatment group.
 E The total number of events required. E = E1 + E2.
 N1 The number of subjects in group 1, the control group.
 N2 The number of subjects in group 2, the treatment group.
 N The total sample size. N = N1 + N2.
 E(Pev1) The expected value over its prior distribution of the probability of observing an event during the study for a subject in group 1.
 E(Pev2) The expected value over its prior distribution of the probability of observing an event during the study for a subject in group 2.
 E(HR) The expected value over its prior distribution of the hazard ratio.
 Alpha The probability of rejecting a true null hypothesis.

Summary Statements

A parallel two-group design will be used to test whether the Group 2 (treatment) hazard rate is less than the Group 1 (control) hazard rate (H0: HR \geq 1 versus H1: HR < 1, HR = Hazard Rate 2 / Hazard Rate 1). The comparison will be made using a one-sided, two-sample logrank test (or an appropriate Cox proportional hazards regression coefficient test), with a Type I error rate (α) of 0.025. It is assumed that the hazard ratio remains constant throughout the study. The prior distribution used for the probability of observing an event in Group 1 is Normal (Mean = 0.5, SD = 0.04). The prior distribution used for the probability of observing an event in Group 2 is Normal (Mean = 0.5, SD = 0.08). The prior distribution used for the hazard ratio (Hazard Rate 2 / Hazard Rate 1) is Normal (Mean = 0.8, SD = 0.08). With sample sizes of 200 for Group 1 (control) and 200 for Group 2 (treatment), the assurance (average power) is 0.38467.

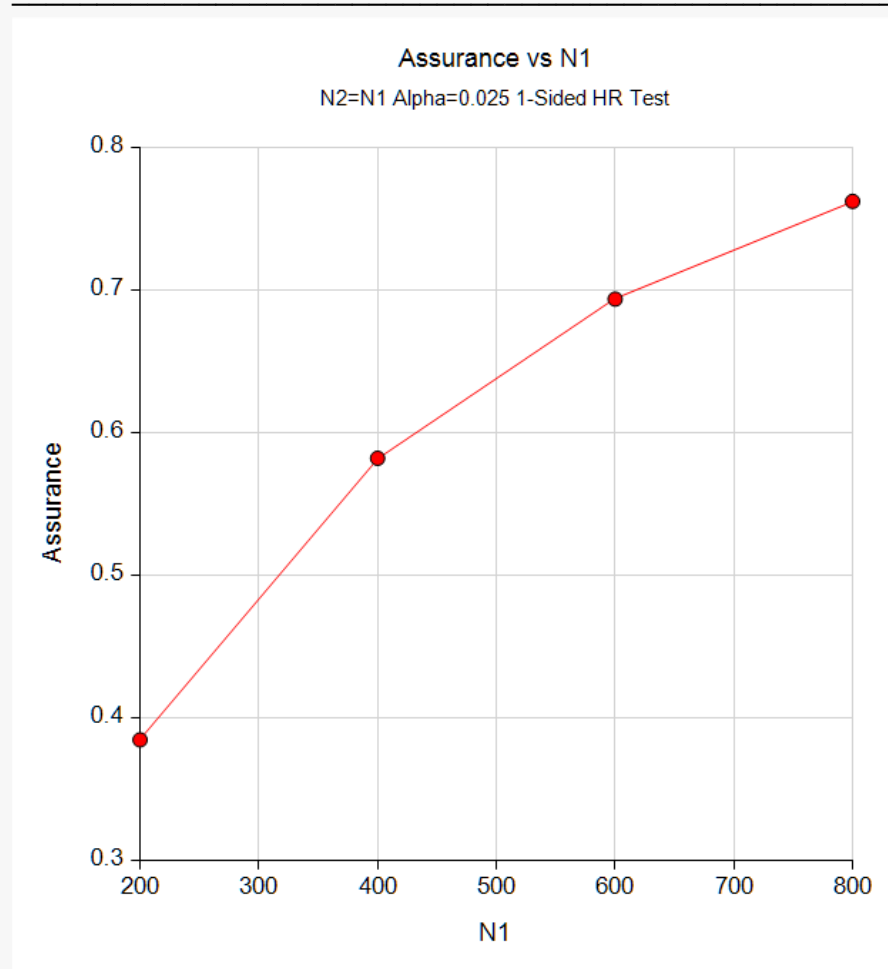
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These reports show the assurance values obtained by each sample size.

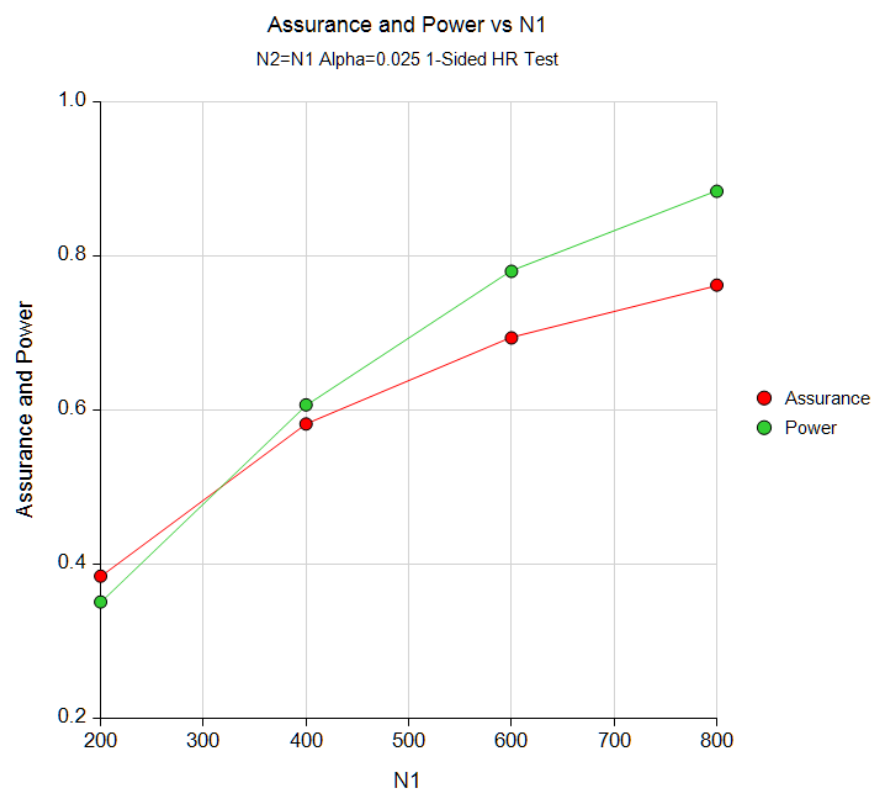
Plots Section

Plots

This plot shows the relationship between the assurance and sample size. Note the diminishing impact on assurance of each increase in the number of subjects.

Comparison Plots Section

Comparison Plots



This plot compares the assurance and power across values of sample size. Note that assurance does not increase as fast as power.

Example 2 – Validation using Hand Computation

We could not find a validation example in the literature for this procedure, so we have developed a validation example of our own.

Suppose a one-sided test based on a Cox regression will be used in which $N1 = N2 = 650$ and the significance level is 0.025.

The prior distribution of $Pev1$ will be approximated by the following table.

<u>Pev1</u>	<u>Prob</u>
0.46	0.2
0.5	0.6
0.54	0.2

The prior distribution of the $Pev2$ will be approximated by the following table.

<u>Pev2</u>	<u>Prob</u>
0.44	0.2
0.5	0.6
0.56	0.2

The prior distribution of HR will be approximated by the following table.

<u>HR</u>	<u>Prob</u>
0.65	0.3
0.8	0.4
0.95	0.3

The *Tests for Two Survival Curves using Cox's Proportional Hazards Model* procedure is used to compute the power for each of the 27 combinations of $Pev1$, $Pev2$, and HR . The results of these calculations are shown next.

Numeric Results

Solve For: [Power](#)

Groups: 1 = Control, 2 = Treatment

Hypotheses: $H0: HR \geq 1$ vs. $H_a: HR < 1$

Power	Sample Size			Percent Group 1 %N1	Number of Events			Hazard Ratio HR1	Probability of an Event		Alpha
	N	N1	N2		E	E1	E2		Pev1	Pev2	
0.99942	1300	650	650	50	585	299	286	0.65	0.46	0.44	0.025
0.99969	1300	650	650	50	624	299	325	0.65	0.46	0.50	0.025
0.99983	1300	650	650	50	663	299	364	0.65	0.46	0.56	0.025
0.99962	1300	650	650	50	611	325	286	0.65	0.50	0.44	0.025
0.99979	1300	650	650	50	650	325	325	0.65	0.50	0.50	0.025
0.99989	1300	650	650	50	689	325	364	0.65	0.50	0.56	0.025
0.99975	1300	650	650	50	637	351	286	0.65	0.54	0.44	0.025
0.99986	1300	650	650	50	676	351	325	0.65	0.54	0.50	0.025
0.99993	1300	650	650	50	715	351	364	0.65	0.54	0.56	0.025
0.76992	1300	650	650	50	585	299	286	0.80	0.46	0.44	0.025
0.79591	1300	650	650	50	624	299	325	0.80	0.46	0.50	0.025
0.81935	1300	650	650	50	663	299	364	0.80	0.46	0.56	0.025
0.78754	1300	650	650	50	611	325	286	0.80	0.50	0.44	0.025
0.81181	1300	650	650	50	650	325	325	0.80	0.50	0.50	0.025
0.83364	1300	650	650	50	689	325	364	0.80	0.50	0.56	0.025
0.80400	1300	650	650	50	637	351	286	0.80	0.54	0.44	0.025
0.82662	1300	650	650	50	676	351	325	0.80	0.54	0.50	0.025
0.84694	1300	650	650	50	715	351	364	0.80	0.54	0.56	0.025
0.09018	1300	650	650	50	585	299	286	0.95	0.46	0.44	0.025
0.09353	1300	650	650	50	624	299	325	0.95	0.46	0.50	0.025
0.09687	1300	650	650	50	663	299	364	0.95	0.46	0.56	0.025

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0.09242	1300	650	650	50	611	325	286	0.95	0.50	0.44	0.025
0.09576	1300	650	650	50	650	325	325	0.95	0.50	0.50	0.025
0.09909	1300	650	650	50	689	325	364	0.95	0.50	0.56	0.025
0.09465	1300	650	650	50	637	351	286	0.95	0.54	0.44	0.025
0.09798	1300	650	650	50	676	351	325	0.95	0.54	0.50	0.025
0.10130	1300	650	650	50	715	351	364	0.95	0.54	0.56	0.025

The assurance calculation is made by summing the quantities $\left[(power_{i,j,k}) (p(Pev1_i)) (p(Pev2_j)) (p(HR_k)) \right]$ as follows

$$Assurance = (0.99942 \times 0.2 \times 0.2 \times 0.3) + (0.99969 \times 0.2 \times 0.6 \times 0.3) + \cdots + (0.10130 \times 0.2 \times 0.2 \times 0.3) = 0.6531.$$

To run this example, the spreadsheet will need to be loaded with the following six columns in which the first two are for Pev1, the second two are for Pev2, and the third two are for HR.

C1	C2	C3	C4	C5	C6
0.46	0.2	0.44	0.2	0.65	0.3
0.5	0.6	0.5	0.6	0.8	0.4
0.54	0.2	0.56	0.2	0.95	0.3

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	Assurance
Prior Entry Method	Individual (Enter a prior distribution for each applicable parameter)
Alternative Hypothesis	One-Sided (H1: HR < 1)
Alpha.....	0.025
Group Allocation	Equal (N1 = N2)
Sample Size Per Group	650
Prior Distribution of Pev1	Custom (Values and Probabilities in Spreadsheet)
Column of Values	C1
Column of Pr(Values).....	C2
Prior Distribution of Pev2	Custom (Values and Probabilities in Spreadsheet)
Column of Values	C3
Column of Pr(Values).....	C4
Prior Distribution of HR	Custom (Values and Probabilities in Spreadsheet)
Column of Values	C5
Column of Pr(Values).....	C6

Assurance for Tests for Two Survival Curves using Cox's Proportional Hazards Model

Options Tab

Number of Computation Points for each50

Prior Distribution

Maximum N1 in Sample Size Search5000

Input Spreadsheet Data

Row	C1	C2	C3	C4	C5	C6
1	0.46	0.2	0.44	0.2	0.65	0.3
2	0.50	0.6	0.50	0.6	0.80	0.4
3	0.54	0.2	0.56	0.2	0.95	0.3

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Assurance
 Groups: 1 = Control, 2 = Treatment
 Hazard Ratio (HR): Hazard Rate 2 / Hazard Rate 1
 Hypotheses: H0: HR \geq 1 vs. H1: HR < 1
 Test Statistic: Treatment Regression Coefficient from Cox Regression
 Prior Type: Independent Univariate Distributions

Prior Distributions

Pev1: Point List (Values = C1, Probs = C2).

C1: 0.46 0.5 0.54

C2: 0.2 0.6 0.2

Pev2: Point List (Values = C3, Probs = C4).

C3: 0.44 0.5 0.56

C4: 0.2 0.6 0.2

HR: Point List (Values = C5, Probs = C6).

C5: 0.65 0.8 0.95

C6: 0.3 0.4 0.3

Assurance	Power‡	Required Number of Events			Sample Size			Expected Probability of Observing an Event in Group 1 E(Pev1)	Expected Probability of Observing an Event in Group 2 E(Pev2)	Expected Hazard Ratio E(HR)	Alpha
		E1	E2	E	N1	N2	N				
0.6531	0.81181	325	325	650	650	650	1300	0.5	0.5	0.8	0.025

‡ Power was calculated using Pev1 = E(Pev1) = 0.5, Pev2 = E(Pev2) = 0.5, and HR = E(HR) = 0.8.

PASS has also calculated the assurance as 0.6531 which validates the procedure.

Example 3 – Finding the Sample Size Needed to Achieve a Specified Assurance

Continuing with Example 1, the researchers want to investigate the sample sizes necessary to achieve assurances of 0.4, 0.6, and 0.8.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	Sample Size
Prior Entry Method	Individual (Enter a prior distribution for each applicable parameter)
Alternative Hypothesis	One-Sided ($H_1: HR < 1$)
Assurance	0.4 0.6 0.8
Alpha	0.025
Group Allocation	Equal ($N_1 = N_2$)
Prior Distribution of Pev1	Normal (Mean, SD)
Mean	0.5
SD	0.04
Truncation Boundaries	None
Prior Distribution of Pev2	Normal (Mean, SD)
Mean	0.5
SD	0.08
Truncation Boundaries	None
Prior Distribution of HR	Normal (Mean, SD)
Mean	0.8
SD	0.08
Truncation Boundaries	None

Options Tab

Number of Computation Points for each	50
Prior Distribution	
Maximum N_1 in Sample Size Search	5000

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: [Sample Size](#)
 Groups: 1 = Control, 2 = Treatment
 Hazard Ratio (HR): Hazard Rate 2 / Hazard Rate 1
 Hypotheses: H0: HR \geq 1 vs. H1: HR < 1
 Test Statistic: Treatment Regression Coefficient from Cox Regression
 Prior Type: Independent Univariate Distributions

Prior Distributions

Pev1: Normal (Mean = 0.5, SD = 0.04).
 Pev2: Normal (Mean = 0.5, SD = 0.08).
 HR: Normal (Mean = 0.8, SD = 0.08).

Assurance*			Required Number of Events			Sample Size			Expected Probability of Observing an Event in Group 1 E(Pev1)	Expected Probability of Observing an Event in Group 2 E(Pev2)	Expected Hazard Ratio E(HR)	Alpha
Actual	Target	Power‡	E1	E2	E	N1	N2	N				
0.40022	0.4	0.36864	106	106	212	212	212	424	0.5	0.5	0.8	0.025
0.60027	0.6	0.63414	213	213	426	426	426	852	0.5	0.5	0.8	0.025
0.80000	0.8	0.93303	481	480	961	961	961	1922	0.5	0.5	0.8	0.025

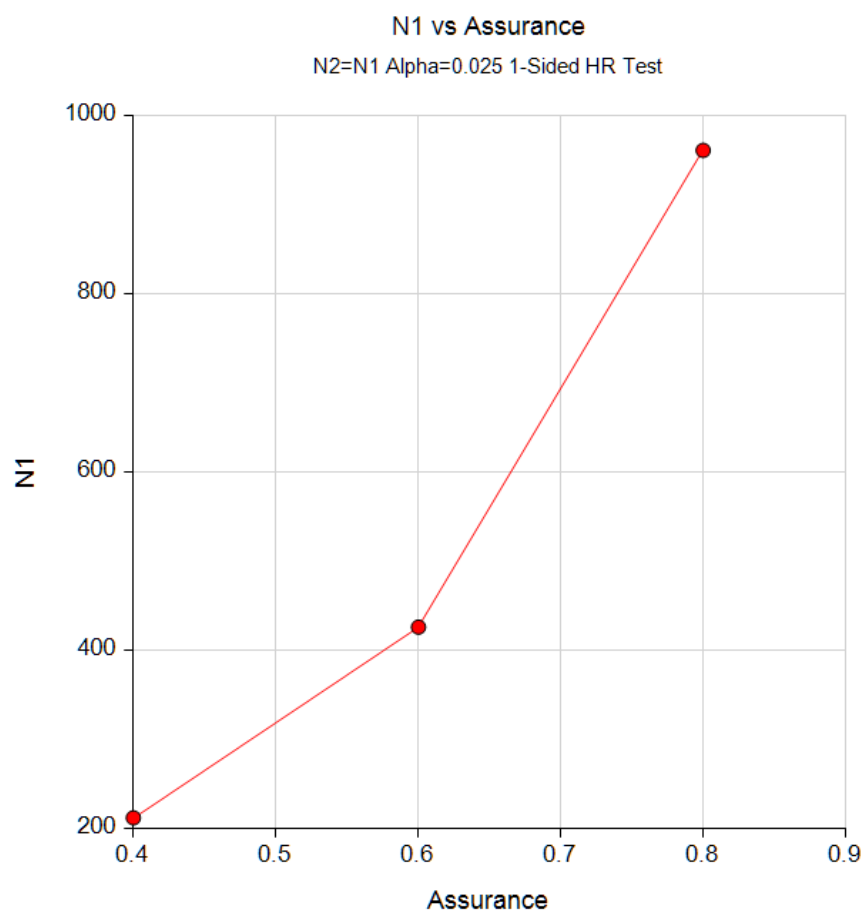
* The number of points used for computation of the prior(s) was 50.

‡ Power was calculated using Pev1 = E(Pev1) = 0.5, Pev2 = E(Pev2) = 0.5, and HR = E(HR) = 0.8.

This report shows the assurance values obtained by each sample size.

Plots Section

Plots



This plot shows the relationship between the assurance and sample size. Note the large increase in required sample size that occurs between an assurance of 0.6 and 0.8.

Example 4 – Joint Prior Distribution

The following example shows the complexity required to specify a joint distribution for three or more parameters.

Suppose a one-sided test will be used in which $N1 = N2 = (200\ 400\ 600\ 800)$ and the significance level is 0.025. Further suppose that the joint prior distribution of the Pev1, Pev2, and HR is approximated by the following table. In a real study, the values in this table would be provided by an elicitation study.

Note that the program will rescale the probabilities so they sum to one.

<u>Pev1</u>	<u>Pev2</u>	<u>HR</u>	<u>Prob</u>
0.60	0.60	0.9	0.2
0.65	0.65	0.9	0.4
0.70	0.70	0.9	0.2
0.63	0.63	0.9	0.4
0.68	0.68	0.9	0.6
0.73	0.73	0.9	0.4
0.66	0.66	0.9	0.2
0.71	0.71	0.9	0.4
0.76	0.76	0.9	0.2
0.60	0.60	0.7	0.1
0.65	0.65	0.7	0.2
0.70	0.70	0.7	0.1
0.63	0.63	0.7	0.2
0.68	0.68	0.7	0.3
0.73	0.73	0.7	0.2
0.66	0.66	0.7	0.1
0.71	0.71	0.7	0.3
0.76	0.76	0.7	0.1

To run this example, the spreadsheet will need to be loaded with the following four columns.

<u>C1</u>	<u>C2</u>	<u>C3</u>	<u>C4</u>
0.60	0.60	0.9	0.2
0.65	0.65	0.9	0.4
0.70	0.70	0.9	0.2
0.63	0.63	0.9	0.4
0.68	0.68	0.9	0.6
0.73	0.73	0.9	0.4
0.66	0.66	0.9	0.2
0.71	0.71	0.9	0.4
0.76	0.76	0.9	0.2
0.60	0.60	0.7	0.1
0.65	0.65	0.7	0.2
0.70	0.70	0.7	0.1
0.63	0.63	0.7	0.2
0.68	0.68	0.7	0.3

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0.73	0.73	0.7	0.2
0.66	0.66	0.7	0.1
0.71	0.71	0.7	0.3
0.76	0.76	0.7	0.1

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **Assurance**
 Prior Entry Method **Combined (Enter parameter values and probabilities on spreadsheet)**
 Alternative Hypothesis **One-Sided (H1: HR < 1)**
 Alpha **0.025**
 Group Allocation **Equal (N1 = N2)**
 Sample Size Per Group **200 400 600 800**
 Column of Pev1 Values **C1**
 Column of Pev2 Values **C2**
 Column of HR Values **C3**
 Column of Pr(Values) **C4**

Options Tab

Number of Computation Points for each **50**
 Prior Distribution
 Maximum N1 in Sample Size Search **5000**

Input Spreadsheet Data

Row	C1	C2	C3	C4
1	0.60	0.60	0.9	0.2
2	0.65	0.65	0.9	0.4
3	0.70	0.70	0.9	0.2
4	0.63	0.63	0.9	0.4
5	0.68	0.68	0.9	0.6
6	0.73	0.73	0.9	0.4
7	0.66	0.66	0.9	0.2
8	0.71	0.71	0.9	0.4
9	0.76	0.76	0.9	0.2
10	0.60	0.60	0.7	0.1
11	0.65	0.65	0.7	0.2
12	0.70	0.70	0.7	0.1
13	0.63	0.63	0.7	0.2
14	0.68	0.68	0.7	0.3
15	0.73	0.73	0.7	0.2
16	0.66	0.66	0.7	0.1
17	0.71	0.71	0.7	0.3
18	0.76	0.76	0.7	0.1

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: [Assurance](#)
 Groups: 1 = Control, 2 = Treatment
 Hazard Ratio (HR): Hazard Rate 2 / Hazard Rate 1
 Hypotheses: H0: HR \geq 1 vs. H1: HR < 1
 Test Statistic: Treatment Regression Coefficient from Cox Regression
 Prior Type: Joint Multivariate Distribution

Prior Distribution

Point Lists

Pev1: C1: 0.6 0.65 0.7 0.63 0.68 0.73 0.66 0.71 0.76 0.6 0.65 0.7 0.63 0.68 0.73 0.66 0.71 0.76

Pev2: C2: 0.6 0.65 0.7 0.63 0.68 0.73 0.66 0.71 0.76 0.6 0.65 0.7 0.63 0.68 0.73 0.66 0.71 0.76

HR: C3: 0.9 0.9 0.9 0.9 0.9 0.9 0.9 0.9 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.7

Prob: C4: 0.2 0.4 0.2 0.4 0.6 0.4 0.2 0.4 0.2 0.1 0.2 0.1 0.2 0.3 0.2 0.1 0.3 0.1

Assurance	Power‡	Required Number of Events			Sample Size			Expected Probability of Observing an Event in Group 1 E(Pev1)	Expected Probability of Observing an Event in Group 2 E(Pev2)	Expected Hazard Ratio E(HR)	Alpha
		E1	E2	E	N1	N2	N				
0.38067	0.33468	137	136	273	200	200	400	0.68065	0.68065	0.83043	0.025
0.49430	0.58236	273	272	545	400	400	800	0.68065	0.68065	0.83043	0.025
0.55907	0.75652	409	408	817	600	600	1200	0.68065	0.68065	0.83043	0.025
0.61639	0.86561	545	545	1090	800	800	1600	0.68065	0.68065	0.83043	0.025

‡ Power was calculated using Pev1 = E(Pev1) = 0.68065, Pev2 = E(Pev2) = 0.68065, and HR = E(HR) = 0.83043.

This report shows the assurance values obtained by each sample size.

Example 5 – Joint Prior Validation

The problem given in Example 2 will be used to validate the joint prior distribution method. This will be done by running the independent-prior scenario used in that example through the joint-prior method and checking that the assurance values match.

In Example 2, the prior distributions of the parameters are

<u>Pev1</u>	<u>Prob</u>
0.46	0.2
0.5	0.6
0.54	0.2

<u>Pev2</u>	<u>Prob</u>
0.44	0.2
0.5	0.6
0.56	0.2

<u>HR</u>	<u>Prob</u>
0.65	0.3
0.8	0.4
0.95	0.3

The joint prior distribution can be found by multiplying the three independent probabilities in each row. This results in the following discrete probability distribution.

<u>Pev1</u>	<u>Pev2</u>	<u>HR</u>	<u>Prob</u>
0.46	0.44	0.65	0.012
0.46	0.44	0.8	0.016
0.46	0.44	0.95	0.012
0.46	0.5	0.65	0.036
0.46	0.5	0.8	0.048
0.46	0.5	0.95	0.036
0.46	0.56	0.65	0.012
0.46	0.56	0.8	0.016
0.46	0.56	0.95	0.012
0.5	0.44	0.65	0.036
0.5	0.44	0.8	0.048
0.5	0.44	0.95	0.036
0.5	0.5	0.65	0.108
0.5	0.5	0.8	0.144
0.5	0.5	0.95	0.108
0.5	0.56	0.65	0.036
0.5	0.56	0.8	0.048
0.5	0.56	0.95	0.036
0.54	0.44	0.65	0.012
0.54	0.44	0.8	0.016
0.54	0.44	0.95	0.012
0.54	0.5	0.65	0.036
0.54	0.5	0.8	0.048

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0.54	0.5	0.95	0.036
0.54	0.56	0.65	0.012
0.54	0.56	0.8	0.016
0.54	0.56	0.95	0.012

To run this example, the spreadsheet is loaded with the following four columns.

<u>C1</u>	<u>C2</u>	<u>C3</u>	<u>C4</u>
0.46	0.44	0.65	0.012
0.46	0.44	0.8	0.016
0.46	0.44	0.95	0.012
0.46	0.5	0.65	0.036
0.46	0.5	0.8	0.048
0.46	0.5	0.95	0.036
0.46	0.56	0.65	0.012
0.46	0.56	0.8	0.016
0.46	0.56	0.95	0.012
0.5	0.44	0.65	0.036
0.5	0.44	0.8	0.048
0.5	0.44	0.95	0.036
0.5	0.5	0.65	0.108
0.5	0.5	0.8	0.144
0.5	0.5	0.95	0.108
0.5	0.56	0.65	0.036
0.5	0.56	0.8	0.048
0.5	0.56	0.95	0.036
0.54	0.44	0.65	0.012
0.54	0.44	0.8	0.016
0.54	0.44	0.95	0.012
0.54	0.5	0.65	0.036
0.54	0.5	0.8	0.048
0.54	0.5	0.95	0.036
0.54	0.56	0.65	0.012
0.54	0.56	0.8	0.016
0.54	0.56	0.95	0.012

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **Assurance**
 Prior Entry Method **Combined (Enter parameter values and probabilities on spreadsheet)**
 Alternative Hypothesis **One-Sided ($H_1: HR < 1$)**
 Alpha **0.025**
 Group Allocation **Equal ($N_1 = N_2$)**
 Sample Size Per Group **650**
 Column of Pev1 Values **C1**
 Column of Pev2 Values **C2**
 Column of HR Values **C3**
 Column of Pr(Values) **C4**

Options Tab

Number of Computation Points for each **50**
 Prior Distribution
 Maximum N_1 in Sample Size Search **5000**

Input Spreadsheet Data

Row	C1	C2	C3	C4
1	0.46	0.44	0.65	0.012
2	0.46	0.44	0.80	0.016
3	0.46	0.44	0.95	0.012
4	0.46	0.50	0.65	0.036
5	0.46	0.50	0.80	0.048
6	0.46	0.50	0.95	0.036
7	0.46	0.56	0.65	0.012
8	0.46	0.56	0.80	0.016
9	0.46	0.56	0.95	0.012
10	0.50	0.44	0.65	0.036
11	0.50	0.44	0.80	0.048
12	0.50	0.44	0.95	0.036
13	0.50	0.50	0.65	0.108
14	0.50	0.50	0.80	0.144
15	0.50	0.50	0.95	0.108
16	0.50	0.56	0.65	0.036
17	0.50	0.56	0.80	0.048
18	0.50	0.56	0.95	0.036
19	0.54	0.44	0.65	0.012
20	0.54	0.44	0.80	0.016
21	0.54	0.44	0.95	0.012
22	0.54	0.50	0.65	0.036
23	0.54	0.50	0.80	0.048

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24	0.54	0.50	0.95	0.036
25	0.54	0.56	0.65	0.012
26	0.54	0.56	0.80	0.016
27	0.54	0.56	0.95	0.012

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: [Assurance](#)
 Groups: 1 = Control, 2 = Treatment
 Hazard Ratio (HR): Hazard Rate 2 / Hazard Rate 1
 Hypotheses: H0: HR \geq 1 vs. H1: HR < 1
 Test Statistic: Treatment Regression Coefficient from Cox Regression
 Prior Type: Joint Multivariate Distribution

Prior Distribution

Point Lists

Pev1: C1: 0.46 0.46 0.46 0.46 0.46 0.46 0.46 0.46 0.46 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.54 0.54 0.54 0.54 0.54 0.54 0.54 0.54 0.54
 Pev2: C2: 0.44 0.44 0.44 0.5 0.5 0.5 0.56 0.56 0.56 0.44 0.44 0.44 0.5 0.5 0.5 0.56 0.56 0.56 0.44 0.44 0.44 0.5 0.5 0.5 0.56 0.56 0.56
 HR: C3: 0.65 0.8 0.95 0.65 0.8 0.95 0.65 0.8 0.95 0.65 0.8 0.95 0.65 0.8 0.95 0.65 0.8 0.95 0.65 0.8 0.95 0.65 0.8 0.95 0.65 0.8 0.95
 Prob: C4: 0.012 0.016 0.012 0.036 0.048 0.036 0.012 0.016 0.012 0.036 0.048 0.036 0.108 0.144 0.108 0.036 0.048 0.036 0.012 0.016
 0.012 0.036 0.048 0.036 0.012 0.016 0.012

Assurance	Power‡	Required Number of Events			Sample Size			Expected Probability of Observing an Event in Group 1 E(Pev1)	Expected Probability of Observing an Event in Group 2 E(Pev2)	Expected Hazard Ratio E(HR)	Alpha
		E1	E2	E	N1	N2	N				
0.6531	0.81181	325	325	650	650	650	1300	0.5	0.5	0.8	0.025

‡ Power was calculated using Pev1 = E(Pev1) = 0.5, Pev2 = E(Pev2) = 0.5, and HR = E(HR) = 0.8.

PASS has also calculated the assurance as 0.6531 which matches Example 2 and thus validates the procedure.