

Chapter 901

Bayesian Adjustment using the Posterior Error Approach

Introduction

This procedure implements a general approach for determining sample size that combines both the frequentist and Bayesian framework. The approach was proposed by Lee and Zelen (2000) and summarized in Chow, Shoa, Wang, and Lokhnygina (2018).

The approach may be outlined as follows. Consider planning an experiment to evaluate a noncentrality parameter δ which is often a function of a difference and its associated standard deviation. Instead of basing the sample size on the usual frequentist error rates α and β , Lee and Zelen (2000) suggested that Bayesian posterior error rates could be used. These Bayesian error rates can then be transformed into frequentist error rates which can then be used in the frequentist sample size formulas. This approach assumes that the actual data analysis would follow the frequentist methods.

Error Rate Formulas

The frequentist approach searches for the minimum sample size so the following error rates are achieved

$$\alpha = P(C = +|T = -)$$

$$\beta = P(C = -|T = +)$$

where C is a binary variable (+ or -) that gives the result of the experiment. The positive result is to conclude that $\delta \neq 0$ and the negative result is to conclude that $\delta = 0$. Similarly, T is another binary variable (+ or -) that gives the true state of the hypotheses. That is, positive implies that actually $\delta \neq 0$ and the negative sign implies that $\delta = 0$.

The Bayesian approach defines two posterior error rates

$$\alpha^* = P(T = +|C = -)$$

$$\beta^* = P(T = -|C = +)$$

These are the posterior probabilities of the true state of nature being the opposite of the outcome of the trial.

The Bayesian approach defines another parameter θ which is the prior probability that $T = +$.

$$\theta = P(T = +).$$

Bayesian Adjustment using the Posterior Error Approach

With these definitions, Lee and Zelen (2000) formulate the relationship between these two sets of error rates as

$$1 - \alpha^* = \frac{(1 - \alpha)(1 - \theta)}{[(1 - \alpha)(1 - \theta) + \beta\theta]}$$

$$1 - \beta^* = \frac{(1 - \beta)\theta}{[(1 - \beta)\theta + \alpha(1 - \theta)]}$$

These equations can be inverted to obtain conversion formulas from Bayesian to frequentist as follows

$$\alpha = \frac{(1 - P_2)(\theta + P_1 - 1)}{(1 - \theta)(P_1 + P_2 - 1)}$$

$$\beta = \frac{(1 - P_1)(P_2 - \theta)}{\theta(P_1 + P_2 - 1)}$$

where $P_1 = 1 - \alpha^*$ and $P_2 = 1 - \beta^*$. To avoid negative probabilities, these formulas require $P_1 > 1 - \theta$ and $P_2 > \theta$ or $P_1 < 1 - \theta$ and $P_2 < \theta$.

Hence, one simply specifies α^* , β^* , and θ using the Bayesian approach and translates this to α and β in the frequentist approach. These frequentist error rates can be used to obtain Bayesian results in any **PASS** procedure that calls for these frequentist error rates.

Example 1 – Validation: Finding α and β

Lee and Zelen (2000) on page 98 present a table of values of α and β given various values of α^* , β^* , and θ . A portion of that table is given next (in reverse order).

θ	α^*	β^*	α	β
0.25	0.05	0.05	0.0148	0.1556
0.25	0.10	0.05	0.0118	0.3294
0.25	0.15	0.05	0.0083	0.5250

We will validate this procedure by duplicating these values.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **Frequentist Parameters (α, β)**
 Alternative Hypothesis **Two-Sided**
 θ [Prior Pr(T = +)] **0.25**
 α^* [Posterior Error Pr(T = +|C = -)] **0.05 0.1 0.15**
 β^* [Posterior Error Pr(T = -|C = +)] **0.05**

Bayesian Adjustment using the Posterior Error Approach

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results								
Solve For:		Frequentist Parameters (α, β)						
Alternative Hypothesis:		Two-Sided						
Bayesian Parameters					Frequentist Parameters			
Prior Probability	Posterior Error Probability		Posterior True Probability		Error Probability		Power	
$\Pr(T = +)$ θ	$\Pr(T = + C = -)$ α^*	$\Pr(T = - C = +)$ β^*	$\Pr(T = - C = -)$ $1 - \alpha^*$	$\Pr(T = + C = +)$ $1 - \beta^*$	$\Pr(C = + T = -)$ α	$\Pr(C = - T = +)$ β	$\Pr(C = + T = +)$ $1 - \beta$	
0.25	0.05	0.05	0.95	0.95	0.0148	0.1556	0.8444	
0.25	0.10	0.05	0.90	0.95	0.0118	0.3294	0.6706	
0.25	0.15	0.05	0.85	0.95	0.0083	0.5250	0.4750	
T	True State is $\delta \neq 0$ (+ or -).							
C	Conclusion is $\delta \neq 0$ (+ or -).							
δ	The effect parameter of interest (not specified).							
θ	Prior Probability that $T = +$.							
α^*	Posterior Error Probability that $T = + C = -$.							
β^*	Posterior Error Probability that $T = - C = +$.							
$1 - \alpha^*$	Posterior Correct Probability that $T = - C = -$. P1 in the documentation.							
$1 - \beta^*$	Posterior Correct Probability that $T = + C = +$. P2 in the documentation.							
α	Type I Error. Probability that $C = + T = -$.							
β	Type II Error. Probability that $C = - T = +$.							
$1 - \beta$	Power.							
Summary Statements								
This routine converts Bayesian design parameters to frequentist design parameters. Specifically, the posterior error probabilities α^* (0.05) and β^* (0.05) along with the prior probability θ (0.25) that $T = +$ ($\delta \neq 0$) are converted to the common frequentist error probabilities α (0.0148) and β (0.1556).								
References								
Lee, S.J. and Zelen, M. 2000. 'Clinical Trials and Sample Size Considerations: Another Perspective.' Statistical Science, Vol. 15, No. 2, Pages 95-110.								
Chow, S.C., Shao, J., Wang, H., and Lohknygina, Y. 2018. Sample Size Calculations in Clinical Research, Third Edition. Taylor & Francis/CRC. Boca Raton, Florida.								

This report duplicates the values from Lee and Zelen (2000) exactly, thus validating the procedure.