

## Chapter 262

# Bridging Study using the Equivalence Test of Two Groups (Binary Outcome)

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## Introduction

This procedure calculates the power and sample size required for bridging studies that use an equivalence test to compare the efficacy of a treatment in two regions. The response data is binary. Schuirmann's (1987) two one-sided tests (TOST) approach is used to test the equivalence of a bridging study and the original study. Only a brief introduction to the subject will be given here. For a comprehensive discussion, refer to Liu, Hsueh, and Chen (2002).

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## Bridging Studies

Once a pharmaceutical product has been approved for use in one or more regions (countries) through a set of clinical trials, it is often desirable to register the product in a new region that was not included in the original study. When the cost and time needed to complete an additional set of clinical trials in the new region is prohibitive, a *bridging methodology* may be used to obtain the approval. The bridging analysis compares the results of a smaller and shorter *bridging study* in the new region with the data obtained in the original study.

The bridging analysis makes use of a two-group design in which the effectiveness in the new region is compared to the effectiveness in the original region using a TOST equivalence test. The effectiveness in each region is measured by the difference between the proportions of a treatment group and a control group. The equivalence test shows that the differences between the two regions do not differ by more than a small amount, called the margin of equivalence.

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## Test Statistics

This section summarizes the results found in Liu, Hsueh, and Chen (2002), page 974. Note that in the following presentation, since the response is binary, the mean response is also the proportion of responses in which the outcome is positive.

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## Original Study

Let  $Y_{ijk}$  be the binary response (0 or 1) of subject  $k$  on receiving treatment  $j$  in original study  $i$ . It is assumed that  $i = 1, \dots, I$ . Also,  $j = T$  (treatment),  $C$  (control) and  $k = 1, \dots, N_{ij}$ . Hence  $Y_{ijk}$  includes the response data from each of the original trials. Assume that the  $Y_{ijk}$  are independently distributed with means  $\mu_{ij}$  and variance  $\sigma_{ij}^2$ . Further assume that  $\mu_{ij}$  has a normal distribution with mean  $\mu_{0j}$  and variance  $\gamma_{0j}^2$ . Hence, the  $Y_{ijk}$ 's are independently distributed with mean  $\mu_{0j}$  and variance  $\omega_{ij}^2 = \sigma_{ij}^2 + \gamma_{0j}^2$ .

## Bridging Study using the Equivalence Test of Two Groups (Binary Outcome)

Let  $Y_{ij}$  be the sample means. The MLE of  $\mu_{0j}$  is

$$t_{0j} = \frac{\sum Y_{ij}/(w_{ij}^2/N_{ij})}{\sum 1/(w_{ij}^2/N_{ij})}, i = 1, \dots, I; j = T, C$$

where

$$w_{ij}^2 = \sum \frac{(Y_{ijk} - t_{0j})^2}{N_{ij}}$$

is the MLE of  $\omega_{ij}^2$ . The MLE's  $t_{0j}$  and  $\omega_{ij}^2$  are solved for iteratively.

## Bridging Study

Let  $Y_{Bjk}$  be the binary response of subject  $k$  on receiving treatment  $j$  in the bridging study conducted in the new region. It is assumed that  $j = T, C$  and  $k = 1, \dots, N_{Bj}$ . As before, the  $Y_{Bjk}$ 's are independently distributed with mean  $\mu_{Bj}$  and variance  $\omega_{Bj}^2$ .

The MLE of  $\mu_{Bj}$  is the sample mean  $Y_{Bj}$ . Let  $t_{Bj} = Y_{Bj}, j = T, C$ .

## Equivalence Test

The MLEs  $t_{0j}$  and  $t_{Bj}$  are independently normally distributed with asymptotic variances estimated by

$$s_{0j}^2 = \frac{1}{\sum 1/(w_{ij}^2/N_{ij})}$$

and

$$s_{Bj}^2 = \sum \frac{(Y_{Bjk} - t_{Bj})^2}{N_{Bj}^2}$$

Let  $E_L = -E$  and  $E_U = E$  be the lower and upper equivalence limits for the mean differences, assuming  $E > 0$ . Usually,  $E$  is set using  $E = f(t_{0T} - t_{0C})$  where  $f$  is between 0 and 1.

The TOST equivalence hypotheses are

$$H_0: \theta \leq -E \text{ or } \theta \geq E \text{ vs } H_1: -E < \theta < E$$

where

$$\theta = (\mu_{BT} - \mu_{BC}) - (\mu_{0T} - \mu_{0C})$$

is the difference in treatment effects between the two regions.

The test statistic

$$t = (t_{BT} - t_{BC}) - (t_{0T} - t_{0C})$$

is an asymptotically unbiased estimate for  $\theta$ .

The variance of  $t$  is given by

$$s^2 = s_{BT}^2 + s_{BC}^2 + s_{0T}^2 + s_{0C}^2.$$

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The two test statistics for the equivalence test are

$$T_L = \frac{(t + E)}{s} \text{ and } T_U = \frac{(t - E)}{s}$$

The null hypothesis is rejected, and equivalence is concluded at significance level  $\alpha$  if and only if  $T_L > z_\alpha$  and  $T_U < -z_\alpha$ , where  $z_\alpha$  is the  $\alpha^{th}$  upper percentile of the standard normal distribution.

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## Power Calculation

Based on the above results, Liu *et al.* (2002) estimate the sample size required to meet the power, significance level, and effect size requirement as

$$N_B \geq \frac{A_1}{A_2 - A_3}$$

where

$$A_1 = \frac{\sigma_{BT}^2}{g_{BT}} + \frac{\sigma_{BC}^2}{E^2(1 - g_{BT})}$$

$$A_2 = \frac{1}{(z_\alpha + z_{\beta/2})^2}$$

$$A_3 = s_{OT}^2 + s_{OC}^2$$

$$g_{BT} = \frac{N_{BT}}{N_B}$$

where  $\beta = 1 - \text{Power}$ ,  $\sigma_{BT}^2$  is often estimated by  $s_{OT}^2$ ,  $\sigma_{BC}^2$  is often estimated by  $s_{OC}^2$ , and the actual difference between the two study differences is zero.

Note that since the variance of a Bernoulli random variable is  $p(1 - p)$ , all four variance terms may be estimated from the corresponding proportions. For example,  $\sigma_{BT}^2 = P_{BT}(1 - P_{BT})$ .

The power is obtained by rearranging this formula.

## Example 1 – Finding Sample Size

A certain drug has been cleared for use in North America using parallel-group, treatment versus control clinical trials. The primary endpoint was binary. These trials resulted in the following summary statistics:

$$N_{OT} = 973 \quad \hat{\mu}_{OT} = 0.732$$

$$N_{OC} = 948 \quad \hat{\mu}_{OC} = 0.508$$

Researchers in a region not included in the original study would like to register the new drug for use in that region. To do so, they are planning a bridging study with a significance level of 0.05 and a power of 0.8. They will set  $P_{BT} = P_{OT}$  and  $P_{BC} = P_{OC}$ . They want to calculate the necessary sample size when  $f$  is 0.3, 0.4, or 0.5. They are planning a balanced study.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Sample Size</b>
N <sub>OT</sub> (Sample Size of Group OT).....	<b>973</b>
N <sub>OC</sub> (Sample Size of Group OC) .....	<b>948</b>
P <sub>OT</sub> (Group OT Proportion) .....	<b>0.732</b>
P <sub>OC</sub> (Group OC Proportion) .....	<b>0.508</b>
Power.....	<b>0.8</b>
Alpha.....	<b>0.05</b>
Group Allocation .....	<b>Equal (N<sub>BT</sub> = N<sub>BC</sub>)</b>
Equivalence Limit Input Type.....	<b>Enter f, the proportion E is of  P<sub>OT</sub>-P<sub>OC</sub> </b>
f (Proportion E is of  P <sub>OT</sub> -P <sub>OC</sub>  ).....	<b>0.4 0.5 0.6</b>
P <sub>BT</sub> (Group BT Proportion) .....	<b>0.732</b>
P <sub>BC</sub> (Group BC Proportion) .....	<b>0.508</b>

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Reports

#### Numeric Results

Solve For: [Sample Size](#)  
 Definition:  $\theta = (P_{BT} - P_{BC}) - (P_{OT} - P_{OC})$   
 Hypotheses:  $H_0: \theta \leq -E$  or  $\theta \geq E$  vs.  $H_1: -E < \theta < E$   
 H1 Assumption:  $\theta = 0$

Bridging Study									Original Study					
Power	N <sub>BT</sub>	N <sub>BC</sub>	N <sub>B</sub>	Equivalence		Proportions			Alpha	N <sub>OT</sub>	N <sub>OC</sub>	Proportions		
				Prop f	Value E	Trt P <sub>BT</sub>	Ctrl P <sub>BC</sub>	Diff Do				Trt P <sub>OT</sub>	Ctrl P <sub>OC</sub>	
0.80006	945	945	1890	0.4	0.0896	0.732	0.508	0.05	973	948	0.224	0.732	0.508	
0.80052	447	447	894	0.5	0.1120	0.732	0.508	0.05	973	948	0.224	0.732	0.508	
0.80094	272	272	544	0.6	0.1344	0.732	0.508	0.05	973	948	0.224	0.732	0.508	

- Power The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
- N<sub>BT</sub> The number of bridging study subjects assigned to the treatment group.
- N<sub>BC</sub> The number of bridging study subjects assigned to the control group.
- N<sub>B</sub> The total sample size of the bridging study.
- f The proportion of  $|P_{OT} - P_{OC}|$  used as the magnitude of the equivalence limits.  $E = f \times |P_{OT} - P_{OC}|$ .
- E The magnitude of the lower and upper equivalence limits: EL and EU.  $EL = -|E|$  and  $EU = |E|$ .
- P<sub>BT</sub> The response proportion of subjects assigned to the treatment group in the bridging study.
- P<sub>BC</sub> The response proportion of subjects assigned to the control group in the bridging study.
- Alpha The probability of rejecting a true null hypothesis.
- N<sub>OT</sub> The number of subjects assigned to the treatment group in the original study.
- N<sub>OC</sub> The number of subjects assigned to the control group in the original study.
- Do The difference between the group proportions ( $\tau - c$ ) in the original study.
- P<sub>OT</sub> The response proportion of subjects assigned to the treatment group in the original study.
- P<sub>OC</sub> The response proportion of subjects assigned to the control group in the original study.

#### Summary Statements

The bridging study sample sizes of 945 in the treatment group and 945 in the control group achieve 80% power using a TOST equivalence test of the difference between the two group proportions. The significance level (alpha) of the equivalence test is 0.05. The lower equivalence limit is -0.0896. The upper equivalence limit is 0.0896. The bridging-study treatment-group proportion is 0.732. The bridging-study control-group proportion is 0.508. The summary statistics of the original study are as follows. The treatment group sample size was 973. The control group sample size was 948. The difference between the group proportions was 0.224. The treatment group proportion was 0.732. The control group proportion was 0.508.

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## Dropout-Inflated Sample Size

Dropout Rate	Sample Size			Dropout-Inflated Enrollment Sample Size			Expected Number of Dropouts		
	N <sub>BT</sub>	N <sub>BC</sub>	N <sub>B</sub>	N <sub>BT'</sub>	N <sub>BC'</sub>	N <sub>B'</sub>	D <sub>T</sub>	D <sub>C</sub>	D
20%	945	945	1890	1182	1182	2364	237	237	474
20%	447	447	894	559	559	1118	112	112	224
20%	272	272	544	340	340	680	68	68	136

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N <sub>BT</sub> , N <sub>BC</sub> , and N <sub>B</sub>	The evaluable sample sizes at which power is computed (as entered by the user). If N <sub>BT</sub> and N <sub>BC</sub> subjects are evaluated out of the N <sub>BT'</sub> and N <sub>BC'</sub> subjects that are enrolled in the study, the design will achieve the stated power.
N <sub>BT'</sub> , N <sub>BC'</sub> , and N <sub>B'</sub>	The number of subjects that should be enrolled in the study in order to obtain N <sub>BT</sub> , N <sub>BC</sub> , and N <sub>B</sub> evaluable subjects, based on the assumed dropout rate. N <sub>BT'</sub> and N <sub>BC'</sub> are calculated by inflating N <sub>BT</sub> and N <sub>BC</sub> using the formulas $N_{BT'} = N_{BT} / (1 - DR)$ and $N_{BC'} = N_{BC} / (1 - DR)$ , with N <sub>BT'</sub> and N <sub>BC'</sub> always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lohknygina, Y. (2018) pages 32-33.)
D <sub>T</sub> , D <sub>C</sub> , and D	The expected number of dropouts. $D_T = N_{BT'} - N_{BT}$ , $D_C = N_{BC'} - N_{BC}$ , and $D = D_T + D_C$ .

## Dropout Summary Statements

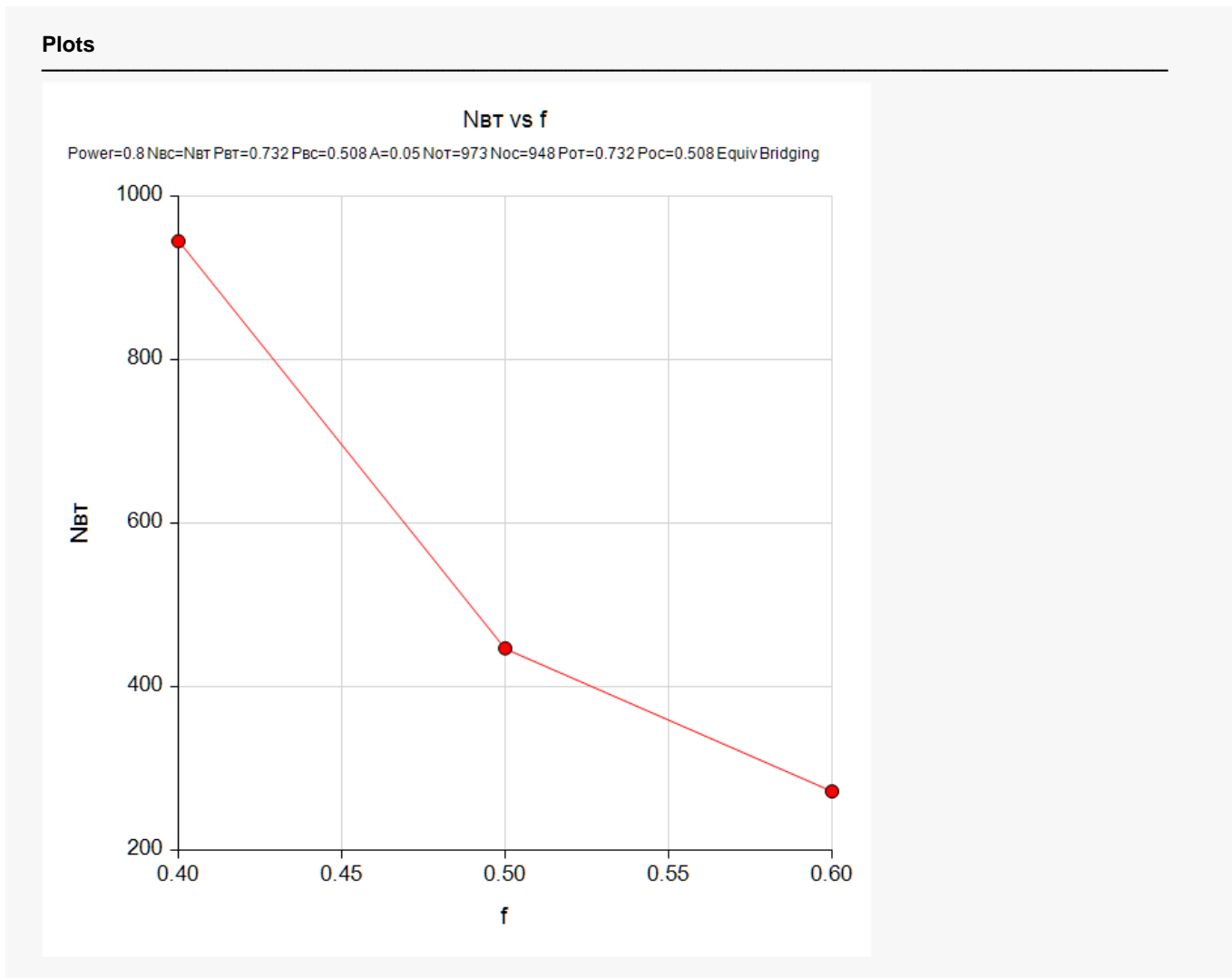
Anticipating a 20% dropout rate, 1182 subjects should be enrolled in Group 1, and 1182 in Group 2, to obtain final group sample sizes of 945 and 945, respectively.

## References

Liu, J.P., Hsueh, H., Chen, J.J. 2002. 'Sample Size Requirements for Evaluation of Bridging Evidence.' Biometrical Journal, Volume 44 (8), Pages 969-981.

This report shows the sample size for the indicated parameter configurations.

## Plots Section



This plot shows the power versus the sample size. Note that we had to reduce the font size of the subtitle so that it would fit in the space allotted.

## Example 2 – Validation using a Previously Validated Procedure

We could not find a validation example in the literature, so we will validate this procedure using the *Bridging Study using the Equivalence Test of Two Groups (Continuous Outcomes)* procedure that has been validated.

The following example will be used for the validation. The summary statistics of the original study are

$$N_{OT} = 1000 \quad P_{OT} = 0.8$$

$$N_{OC} = 1000 \quad P_{OC} = 0.5$$

Note that  $D_0 = 0.8 - 0.5 = 0.3$ ,  $s_{OT} = \sqrt{0.8(0.2)} = 0.4$  and  $s_{OC} = \sqrt{0.5(0.5)} = 0.5$ . In the bridging study, set  $f = 0.4$ ,  $alpha = 0.05$ , and  $power = 0.8$ .

These values translate to the following in the Continuous Outcomes procedure.

$$N_{OT} = 1000 \quad D_0 = 0.3 \quad s_{OT} = 0.4$$

$$N_{OC} = 1000 \quad s_{OC} = 0.5$$

Running these values through that procedure results in the following sample sizes for the bridging study:

$$N_{BT} = N_{BC} = 323.$$

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Sample Size</b>
N <sub>OT</sub> (Sample Size of Group OT).....	<b>1000</b>
N <sub>OC</sub> (Sample Size of Group OC) .....	<b>1000</b>
P <sub>OT</sub> (Group OT Proportion) .....	<b>0.8</b>
P <sub>OC</sub> (Group OC Proportion) .....	<b>0.5</b>
Power.....	<b>0.8</b>
Alpha.....	<b>0.05</b>
Group Allocation .....	<b>Equal (N<sub>BT</sub> = N<sub>BC</sub>)</b>
Equivalence Limit Input Type.....	<b>Enter f, the proportion E is of  P<sub>OT</sub>-P<sub>OC</sub> </b>
f (Proportion E is of  P <sub>OT</sub> -P <sub>OC</sub>  ).....	<b>0.4</b>
P <sub>BT</sub> (Group BT Proportion) .....	<b>0.8</b>
P <sub>BC</sub> (Group BC Proportion) .....	<b>0.5</b>



## Bridging Study using the Equivalence Test of Two Groups (Binary Outcome)

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

Solve For: [Sample Size](#)  
 Definition:  $\theta = (P_{BT} - P_{BC}) - (P_{OT} - P_{OC})$   
 Hypotheses:  $H_0: \theta \leq -E \text{ or } \theta \geq E$  vs.  $H_1: -E < \theta < E$   
 H1 Assumption:  $\theta = 0$

Bridging Study									Original Study					
Power	N <sub>BT</sub>	N <sub>BC</sub>	N <sub>B</sub>	Equivalence		Proportions			Alpha	N <sub>OT</sub>	N <sub>OC</sub>	Proportions		
				Prop f	Value E	Trt P <sub>BT</sub>	Ctrl P <sub>BC</sub>	Diff D <sub>O</sub>				Trt P <sub>OT</sub>	Ctrl P <sub>OC</sub>	
0.80065	323	323	646	0.4	0.12	0.8	0.5	0.05	1000	1000	0.3	0.8	0.5	

This procedure has also calculated a sample size of 323 per group, so the procedure is validated.