

Chapter 250

Chi-Square Tests

Introduction

The *Chi-square test* is often used to test whether sets of frequencies or proportions follow certain patterns. The two most common instances are tests of goodness of fit using multinomial tables and tests of independence in contingency tables.

The *Chi-square goodness of fit* test is used to test whether the distribution of a set of data follows a particular pattern. For example, the goodness-of-fit Chi-square may be used to test whether a set of values follow the normal distribution or whether the proportions of Democrats, Republicans, and other parties are equal to a certain set of values, say 0.4, 0.4, and 0.2.

The *Chi-square test for independence* in a contingency table is the most common Chi-square test. Here individuals (people, animals, or things) are classified by two (nominal or ordinal) classification variables into a two-way, contingency table. This table contains the counts of the number of individuals in each combination of the row categories and column categories. The Chi-square test determines if there is dependence (association) between the two classification variables. Hence, many surveys are analyzed with Chi-square tests.

The following table is an example of data arranged in a two-way contingency table. The rows of the table represent the stated political party of a respondent. The columns represent the respondent's answer to a question about whether they favor a certain proposition. The body of the table represents the number of individuals that fall into each cell (category). Note that the opinions of 311 individuals are recorded in this table.

(Count)	<u>Favor Proposition A</u>	
	<u>Yes</u>	<u>No</u>
<u>Political Party</u>		
Democrats	86	21
Republican	54	59
Others	34	57

The table below presents the row percentages for each category.

(Row Percentage)	<u>Favor Proposition A</u>	
	<u>Yes</u>	<u>No</u>
<u>Political Party</u>		
Democrats	80.4	19.6
Republican	47.8	52.2
Others	37.4	62.6

The Chi-square statistic tests whether the percentage of *Yes* responses remains constant across the three political parties. Notice that 80% of the Democrats said *Yes*, while only 37% of those in the Other category chose *Yes*. The Chi-square value for the above table is 41.71, which is statistically significant. Obviously, there is quite a shift in response pattern on this item across political parties.

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Effect Size

We begin by defining what we will call the *effect size*. For each cell of a table containing m cells, suppose there are two proportions considered: one specified by a null hypothesis and the other specified by an alternative hypothesis. Often, the proportions specified by the alternative hypothesis are those occurring in the data. Define p_{0i} to be the proportion in cell i under the null hypothesis and p_{1i} to be the proportion in cell i under the alternative hypothesis. The effect size, w , is calculated using the formula

$$w = \sqrt{\sum_{i=1}^m \frac{(p_{1i} - p_{0i})^2}{p_{0i}}}$$

The formula for computing the Chi-square value, χ^2 , is

$$\begin{aligned} \chi^2 &= \sum_{i=1}^m \frac{(O_i - E_i)^2}{E_i} \\ &= N \sum_{i=1}^m \frac{(p_{0i} - p_{1i})^2}{p_{0i}}, \end{aligned}$$

where N is the total count in all the cells. Hence, the relationship between w and χ^2 is

$$\chi^2 = Nw^2$$

Note that when you are dealing with a contingency table, the cell index, i , is often replaced by two indices, one representing columns and the other representing rows.

The effect size, w , was used by Cohen (1988) because it does not depend on the sample size. He sets a small value of w at 0.1, a medium value at 0.3, and a large value at 0.5. Although these are rather arbitrary settings, they are useful for planning purposes.

Chi-Square Effect Size Estimator

PASS provides a special module to aid in finding an appropriate value for w called the Chi-Square Effect Size Estimator. This module may be loaded by pressing the *CS* button near the *W* (Effect Size) box or from the menus by selecting *PASS* and then *Other*.

You will find that as values are typed into the body of the table, the value of the effect size (shown at the bottom in the box labeled Effect Size - *W*) is also changed. Using this utility program, you can quickly determine the impact of table configurations on the value of w .

For example, suppose the cell proportions under the null and alternative hypotheses are as follows:

Cell	1	2	3	4	
p_{0i}	0.25	0.25	0.25	0.25	(Null: Equal distribution across the four cells.)
p_{1i}	0.40	0.20	0.20	0.20	(Alternative: cell 1 has twice the probability as the rest.)

To calculate w , first create the differences (ignoring the signs since the differences will be squared

Diff	0.15	0.05	0.05	0.05
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Next, square the differences

Diff^2	0.0225	0.0025	0.0025	0.0025
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Divide by the null

X² 0.09 0.01 0.01 0.01

When these are summed, the result is 0.12. Taking the square of 0.12 gives the value of w as 0.3464.

As an experiment, load the Chi-Square Effect Size Estimator and enter these values on the Multinomial Test window. Enter the p_{oi} values in the column labeled Data Values and the p_{li} values in the column marked Hypothesized Proportions. Check that the value of w is 0.3464. Next, change the Data Values to 4,2,2,2 and the Hypothesized Proportions to 1,1,1,1. Check that the value of w is still the same.

Calculating the Power

The power is calculated as follows:

1. Find x_α such that $1 - \chi^2(x_\alpha | df) = \alpha$, where $\chi^2(x_\alpha | df)$ is the area to the left of x under a Chi-square distribution with df degrees of freedom.
2. Power = $1 - \chi'^2_{df,\lambda}$, where $\chi'^2_{k,\lambda}$ is the left-tail area of the noncentral Chi-square distribution with k degrees of freedom and non-centrality parameter λ . Note that $\lambda = Nw^2$.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters. The parameters that may be selected are W (*Effect Size*), DF (*Degrees of Freedom*), $Sample Size$, $Alpha$, and $Power$. Under most situations, you will select either $Power$ or $Sample Size$.

Select $Sample Size$ when you want to calculate the sample size needed to achieve a given power and alpha level.

Select $Power$ when you want to calculate the power of an experiment that has already been run.

Test

DF (Degrees of Freedom)

This options specifies the degrees of freedom of the Chi-square test. For a test of independence in a contingency table, the degrees of freedom is $(R-1)(C-1)$ where R is the number of rows and C is the number of columns. For example, for a 3-by-4 table, $DF = (3-1)(4-1) = 6$.

In a goodness of fit test, the degrees of freedom is the number of cells minus one. You may have to further adjust it for every distributional parameter that is estimated from the data. For example, suppose a Chi-square goodness-of-fit will be used to test the adequacy of the normality assumption on a set of 300 observations. Two parameters, the mean and variance, are estimated from the data. Suppose the data are categorized into six categories. $DF = 6 - 2 - 1 = 3$.

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Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected. In this procedure, a type-II error occurs when you fail to reject the null hypothesis of equal row proportions when in fact they are different.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected. For this procedure, a type-I error occurs when you reject the null hypothesis of equal row (or column) proportions when in fact they are equal.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

Sample Size

N (Sample Size)

This option specifies the number of individuals whose responses are recorded in the table. This number should be greater than or equal to the number of cells in the table.

Effect Size

W (Effect Size)

This is the value of w , the effect size. If you have Chi-square values that you want to analyze, use the following formula to transform them to w 's:

$$w = \sqrt{\frac{\chi^2}{N}}$$

Remember that a small value of w is 0.1, a medium value is 0.3, and a large value is 0.5.

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Example 1 – Finding the Power for an Existing Contingency Table

This example will compute the power of the Chi-square test of independence of the data in the contingency table that was discussed at the beginning of this chapter. If you would like to follow along, load the Chi-Square Effect Size Estimator window, select the Contingency Table tab, enter 86, 54, 34 in the first column and 21, 59, 57 in the second column. The results are Chi-square = 41.708829, $DF = 2$, $N = 311$, and $W = 0.366213$.

We will compute the power when alpha = 0.01, 0.05, and 0.10. For evaluation purposes, we will compute the power when $N = 20, 50, 100$, and 200 as well as at 311.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Chi-Square Tests** procedure window by expanding **Proportions**, then clicking on **Contingency Table (Chi-Square)**, and then clicking on **Chi-Square Tests**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Power
DF (Degrees of Freedom)	2
Alpha.....	0.01 0.05 0.10
N (Sample Size).....	20 50 100 200 311
W (Effect Size).....	0.366213

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results						
Power	N	W	Chi-Square	DF	Alpha	Beta
0.12127	20	0.3662	2.6822	2	0.01000	0.87873
0.29104	20	0.3662	2.6822	2	0.05000	0.70896
0.41007	20	0.3662	2.6822	2	0.10000	0.58993
0.39621	50	0.3662	6.7056	2	0.01000	0.60379
0.63538	50	0.3662	6.7056	2	0.05000	0.36462
0.74622	50	0.3662	6.7056	2	0.10000	0.25378
0.78214	100	0.3662	13.4112	2	0.01000	0.21786
0.91678	100	0.3662	13.4112	2	0.05000	0.08322
0.95512	100	0.3662	13.4112	2	0.10000	0.04488
0.98840	200	0.3662	26.8224	2	0.01000	0.01160
0.99795	200	0.3662	26.8224	2	0.05000	0.00205
0.99927	200	0.3662	26.8224	2	0.10000	0.00073
0.99980	311	0.3662	41.7088	2	0.01000	0.00020
0.99998	311	0.3662	41.7088	2	0.05000	0.00002
1.00000	311	0.3662	41.7088	2	0.10000	0.00000

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Report Definitions

Power is the probability of rejecting a false null hypothesis. It should be close to one.
 N is the size of the sample drawn from the population. To conserve resources, it should be small.
 W is the effect size--a measure of the magnitude of the Chi-Square that is to be detected.
 DF is the degrees of freedom of the Chi-Square distribution.
 Alpha is the probability of rejecting a true null hypothesis.
 Beta is the probability of accepting a false null hypothesis.

Summary Statements

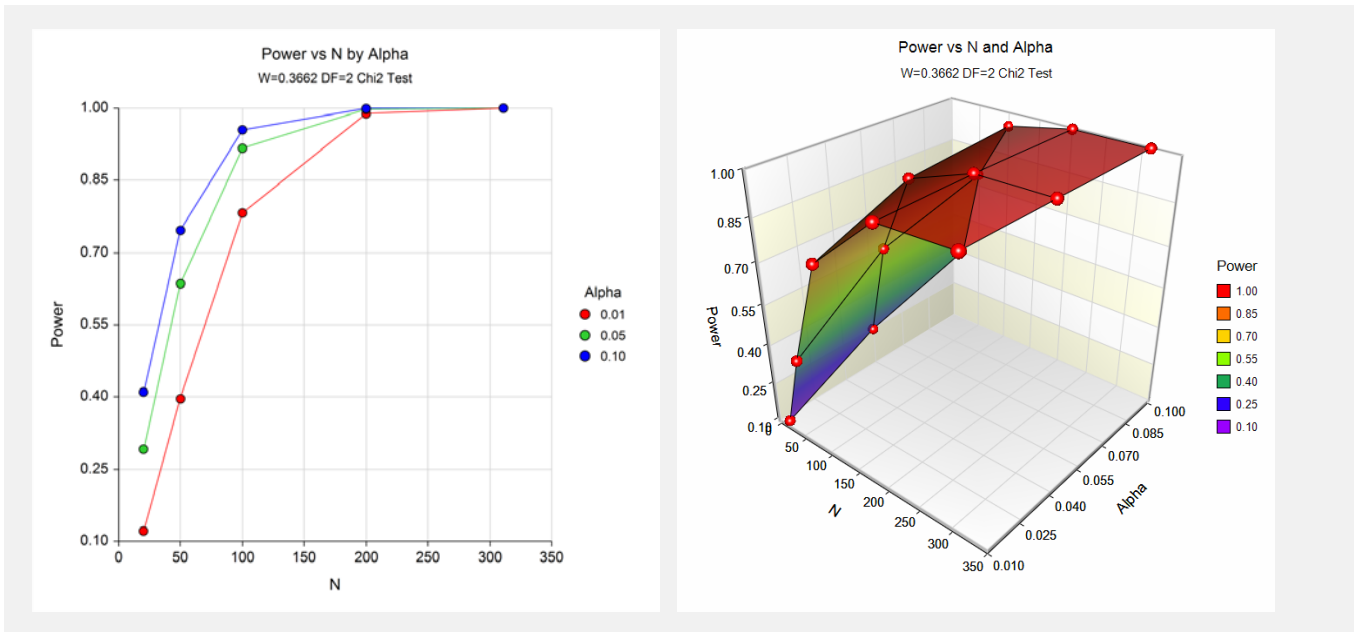
A sample size of 20 achieves 12% power to detect an effect size (W) of 0.3662 using a 2 degrees of freedom Chi-Square Test with a significance level (alpha) of 0.01000.

This report shows the values of each of the parameters, one scenario per row. The definitions of each column are given in the Report Definitions section.

Note that in this particular example, a reasonable power of about 0.80 is reached for all values of alpha once the sample size is greater than 100.

The values from this table are plotted in the chart below.

Plots Section



These plots show the relationship between sample size, power, and alpha.

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Example 2 – Finding the Sample Size

A survey is being planned that will contain several questions with three possible answers: agree, neutral, disagree. The researchers are planning to analyze the questionnaires using Chi-square tests of independence in two-way contingency tables. How many respondents are needed to detect small ($w = 0.1$), medium ($w = 0.3$), or large ($w = 0.5$) effects if all hypothesis testing will be done at the 0.05 significance level?

Since the researchers are planning for 3-by-3 tables, $DF = (3 - 1)(3 - 1) = 4$.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Chi-Square Tests** procedure window by expanding **Proportions**, then clicking on **Contingency Table (Chi-Square)**, and then clicking on **Chi-Square Tests**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
DF (Degrees of Freedom)	4
Power.....	0.80 0.90
Alpha.....	0.05
W (Effect Size).....	0.1 0.3 0.5

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results						
Power	N	W	Chi-Square	DF	Alpha	Beta
0.80018	1194	0.1000	11.9400	4	0.05000	0.19982
0.90010	1541	0.1000	15.4100	4	0.05000	0.09990
0.80130	133	0.3000	11.9700	4	0.05000	0.19870
0.90157	172	0.3000	15.4800	4	0.05000	0.09843
0.80243	48	0.5000	12.0000	4	0.05000	0.19757
0.90198	62	0.5000	15.5000	4	0.05000	0.09802

This report shows that for 80% power, 1194 (about 1200) respondents are needed to detect small effects, 133 respondents are needed to detect medium effects, and 48 respondents are needed to detect large effects.

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Example 3 – Validation using Cohen

Cohen (1988) page 251 presents an example in which $W = 0.30$ and 0.40 , $N = 140$, $\alpha = 0.01$, and $DF = 2$. He gives the power as 0.75 for $W = 0.3$ and 0.97 for $W = 0.4$.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Chi-Square Tests** procedure window by expanding **Proportions**, then clicking on **Contingency Table (Chi-Square)**, and then clicking on **Chi-Square Tests**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Power
DF (Degrees of Freedom)	2
Alpha.....	0.01
N (Sample Size).....	140
W (Effect Size).....	0.3 0.4

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results						
Power	N	W	Chi-Square	DF	Alpha	Beta
0.74841	140	0.3000	12.6000	2	0.01000	0.25159
0.96641	140	0.4000	22.4000	2	0.01000	0.03359

PASS matches Cohen's power values of 0.75 and 0.97 .

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Example 4 – Finding the Sample Size for a Normality Goodness-of-Fit Test

A researcher is planning a study to determine if the distribution of scores on a certain test is normal. He plans to divide the test scores from his sample into five intervals of equal probability under the normal distribution using the sample mean and sample variance. After experimenting with the Chi-Square Effect Size Estimator, he decides that he must be able to detect a departure from normality of $w = 0.20$. He sets his significance level at 0.10 so that he will be lenient in his rejection of normality. He decides to focus on a power of 0.80. How large of a sample size will the researcher need?

The value of $DF = 5 - 2 - 1 = 2$, since there are five intervals and two parameters, mean and variance, are used.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Chi-Square Tests** procedure window by expanding **Proportions**, then clicking on **Contingency Table (Chi-Square)**, and then clicking on **Chi-Square Tests**. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
DF (Degrees of Freedom)	2
Power.....	0.80
Alpha.....	0.10
W (Effect Size).....	0.20

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results						
Power	N	W	Chi-Square	DF	Alpha	Beta
0.80046	193	0.2000	7.7200	2	0.10000	0.19954

This report shows that for 80% power, 193 observations are needed.