

## Chapter 255

# Cochran-Armitage Test for Trend in Proportions

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## Introduction

This module computes power and sample size for the Cochran-Armitage test for a linear trend in proportions based on the results in Nam (1987). Asymptotic and exact power calculations for the uncorrected and continuity-corrected tests are available. The results assume that the proportions follow a linear trend on the logistic scale, with  $X$  being the covariate (or dose) variable, and that random samples are drawn from  $k$  separate populations.

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## Technical Details

Suppose we have  $k$  independent binomial variates  $y_i$ , with response probabilities  $p_i$  based on samples of size  $n_i$  at covariate (or dose) levels  $x_i$ , for  $i = 1, 2, \dots, k$ , where  $x_1 < x_2 < \dots < x_k$ . Define the following:

$$N = \sum_{i=1}^k n_i$$

$$\bar{p} = \frac{1}{N} \sum_{i=1}^k y_i$$

$$\bar{q} = 1 - \bar{p}$$

$$\bar{x} = \frac{1}{N} \sum_{i=1}^k n_i x_i$$

If we assume that the probability of response follows a linear trend on the logistic scale, then

$$p_i = \frac{\exp(\alpha + \beta x_i)}{1 + \exp(\alpha + \beta x_i)}.$$

## Hypothesis Tests

The Cochran-Armitage test can be used to test the following hypotheses:

$$\text{One-Sided (Increasing Trend)} \quad H_0: p_1 = p_2 = \dots = p_k \quad \text{vs.} \quad H_1: p_1 < p_2 < \dots < p_k$$

$$\text{One-Sided (Decreasing Trend)} \quad H_0: p_1 = p_2 = \dots = p_k \quad \text{vs.} \quad H_1: p_1 > p_2 > \dots > p_k$$

$$\text{Two-Sided} \quad H_0: p_1 = p_2 = \dots = p_k \quad \text{vs.} \quad H_1: p_1 < p_2 < \dots < p_k \\ \text{or } p_1 > p_2 > \dots > p_k$$

## One-Sided Test of Increasing Linear Trend in Proportions

### Continuity-Corrected Test

Nam (1987) presents the following continuity-corrected asymptotic test statistic for detecting an increasing linear trend in proportions

$$z_{c.c.} = \frac{\sum_{i=1}^k y_i(x_i - \bar{x}) - \frac{\Delta}{2}}{\sqrt{\bar{p}\bar{q}[\sum_{i=1}^k n_i(x_i - \bar{x})^2]}}$$

The factor  $\Delta/2$  is the continuity correction. If the covariates  $x_i$  are equally spaced, then

$$\Delta = x_{i+1} - x_i \text{ for all } i < k$$

or the interval between adjacent covariates. **PASS** computes  $\Delta$  for unequally spaced covariates as

$$\Delta = \frac{1}{k-1} \sum_{i=1}^{k-1} (x_{i+1} - x_i).$$

For the case of unequally spaced covariates, Nam (1987) has this to say, "For unequally spaced doses, no constant correction is adequate for all outcomes." Therefore, we caution against the use of the continuity-corrected test statistic in the case of unequally spaced covariates.

The test rejects  $H_0$  if  $z_{c.c.} \geq z_{1-\alpha}$ , where  $z_{1-\alpha}$  is the value that leaves  $1 - \alpha$  in the upper tail of the standard normal distribution.

### Uncorrected Test

The uncorrected test statistic is equivalent to the corrected test statistic except that  $\Delta = 0$ ,

$$z = \frac{\sum_{i=1}^k y_i(x_i - \bar{x})}{\sqrt{\bar{p}\bar{q}[\sum_{i=1}^k n_i(x_i - \bar{x})^2]}}$$

The test rejects  $H_0$  if  $z \geq z_{1-\alpha}$ , where  $z_{1-\alpha}$  is the value that leaves  $1 - \alpha$  in the upper tail of the standard normal distribution.

## Cochran-Armitage Test for Trend in Proportions

## One-Sided Test of Decreasing Linear Trend in Proportions

## Continuity-Corrected Test

Nam (1987) presents a continuity-corrected asymptotic test statistic for detecting an increasing linear trend in proportions. The continuity-corrected test statistic for a decreasing trend is the same as that for an increasing trend, except that  $\Delta/2$  is added in the numerator instead of subtracted

$$z_{c.c.} = \frac{\sum_{i=1}^k y_i(x_i - \bar{x}) + \frac{\Delta}{2}}{\sqrt{\bar{p}\bar{q}[\sum_{i=1}^k n_i(x_i - \bar{x})^2]}}$$

The factor  $\Delta$  is defined the same as in the case of a test for increasing trend, and the caution about the use of the continuity-corrected test statistic in the case of unequally spaced covariates also applies here.

The test rejects  $H_0$  if  $z_{c.c.} \leq z_{\alpha}$ , where  $z_{\alpha}$  is the value that leaves  $\alpha$  in the lower tail of the standard normal distribution.

## Uncorrected Test

The uncorrected test statistic is equivalent to the corrected test statistic except that  $\Delta = 0$ ,

$$z = \frac{\sum_{i=1}^k y_i(x_i - \bar{x})}{\sqrt{\bar{p}\bar{q}[\sum_{i=1}^k n_i(x_i - \bar{x})^2]}}$$

The test rejects  $H_0$  if  $z \leq z_{\alpha}$ , where  $z_{\alpha}$  is the value that leaves  $\alpha$  in the lower tail of the standard normal distribution.

## Two-Sided Test for Linear Trend in Proportions

## Continuity-Corrected Test

Nam (1987) presents a continuity-corrected asymptotic test statistic for detecting an increasing linear trend in proportions. A two-sided test statistic utilizes a combination of the upper- and lower-tailed test statistics.

$$z_{c.c.U} = \frac{\sum_{i=1}^k y_i(x_i - \bar{x}) - \frac{\Delta}{2}}{\sqrt{\bar{p}\bar{q}[\sum_{i=1}^k n_i(x_i - \bar{x})^2]}} \quad \text{and} \quad z_{c.c.L} = \frac{\sum_{i=1}^k y_i(x_i - \bar{x}) + \frac{\Delta}{2}}{\sqrt{\bar{p}\bar{q}[\sum_{i=1}^k n_i(x_i - \bar{x})^2]}}$$

The factor  $\Delta$  is defined the same as in the case of a test for increasing trend, and the caution about the use of the continuity-corrected test statistic in the case of unequally spaced covariates also applies here.

The test rejects  $H_0$  if  $z_{c.c.U} \geq z_{1-\alpha/2}$  or if  $z_{c.c.L} \leq z_{\alpha/2}$ .

## Cochran-Armitage Test for Trend in Proportions

## Uncorrected Test

The uncorrected test statistic is the same as the corrected test statistic except that  $\Delta = 0$ , which reduces the upper- and lower-tailed statistics to a single test statistic

$$z = \frac{\sum_{i=1}^k y_i (x_i - \bar{x})}{\sqrt{\bar{p}\bar{q}[\sum_{i=1}^k n_i (x_i - \bar{x})^2]}}$$

The test rejects  $H_0$  if  $|z| \geq z_{1-\alpha/2}$ .

## Exact Power Calculations

The power for the previous test statistics that are based on the normal approximation can be computed exactly using the binomial distribution. The following steps are taken to compute exact power.

## One-Sided Test of Increasing Linear Trend in Proportions

1. Find the critical value using the standard normal distribution. The critical value,  $z_{critical}$ , is that value of  $z$  that leaves exactly the target value of alpha in the upper tail of the normal distribution. For example, for an upper-tailed test (increasing trend) with a target alpha of 0.05, the critical value is 1.645.
2. Compute the value of the test statistic,  $z_t$ , for every  $\mathbf{y}$ , where  $\mathbf{y} = (y_1, y_2, \dots, y_k)$ . Note that  $y_1$  ranges from 0 to  $n_1$ ,  $y_2$  ranges from 0 to  $n_2$ , and so on. The test statistic  $z_t$  can be either the corrected or uncorrected test statistic.
3. If  $z_t \geq z_{critical}$ , the combination is in the rejection region. Call all  $\mathbf{y}$  that lead to a rejection the set  $A$ .
4. Compute the power for given values of  $\mathbf{p} = (p_1, p_2, \dots, p_k)$  as

$$1 - \beta = \sum_A \left\{ \prod_{i=1}^k \binom{n_i}{y_i} p_i^{y_i} (1 - p_i)^{n_i - y_i} \right\}$$

When the values of  $n_i$  are large (say over 50) or  $k$  is large (say over 5), these formulas may take a little time to evaluate. In this case, a large sample approximation may be used.

## One-Sided Test of Decreasing Linear Trend in Proportions

1. Find the critical value using the standard normal distribution. The critical value,  $z_{critical}$ , is that value of  $z$  that leaves exactly the target value of alpha in the upper tail of the normal distribution. For example, for a lower-tailed test (decreasing trend) with a target alpha of 0.05, the critical value is 1.645.
2. Compute the value of the test statistic,  $z_t$ , for every  $\mathbf{y}$ , where  $\mathbf{y} = (y_1, y_2, \dots, y_k)$ . Note that  $y_1$  ranges from 0 to  $n_1$ ,  $y_2$  ranges from 0 to  $n_2$ , and so on. The test statistic  $z_t$  can be either the corrected or uncorrected test statistic.
3. If  $z_t \leq -z_{critical}$ , the combination is in the rejection region. Call all  $\mathbf{y}$  that lead to a rejection the set  $A$ .

## Cochran-Armitage Test for Trend in Proportions

4. Compute the power for given values of  $\mathbf{p} = (p_1, p_2, \dots, p_k)$  as

$$1 - \beta = \sum_A \left\{ \prod_{i=1}^k \left[ \binom{n_i}{y_i} p_i^{y_i} (1 - p_i)^{n_i - y_i} \right] \right\}$$

When the values of  $n_i$  are large (say over 50) or  $k$  is large (say over 5), these formulas may take a little time to evaluate. In this case, a large sample approximation may be used.

## Two-Sided Test of Linear Trend in Proportions

1. Find the critical value using the standard normal distribution. The critical value,  $z_{critical}$ , is that value of  $z$  that leaves exactly  $\alpha/2$  in the upper tail of the normal distribution. For example, for a two-sided test with a target alpha of 0.05, the critical value is 1.96.
2. Compute the value of the test statistics,  $z_U$  and  $z_L$ , for every  $\mathbf{y}$ , where  $\mathbf{y} = (y_1, y_2, \dots, y_k)$ . Note that  $y_1$  ranges from 0 to  $n_1$ ,  $y_2$  ranges from 0 to  $n_2$ , and so on. The test statistics  $z_U$  and  $z_L$  can be either the corrected or uncorrected test statistics. In the case of the uncorrected test,  $z_U = z_L$ .
3. If  $z_U \geq z_{critical}$  or  $z_L \leq -z_{critical}$ , the combination is in the rejection region. Call all  $\mathbf{y}$  that lead to a rejection the set  $A$ .
4. Compute the power for given values of  $\mathbf{p} = (p_1, p_2, \dots, p_k)$  as

$$1 - \beta = \sum_A \left\{ \prod_{i=1}^k \left[ \binom{n_i}{y_i} p_i^{y_i} (1 - p_i)^{n_i - y_i} \right] \right\}$$

When the values of  $n_i$  are large (say over 50) or  $k$  is large (say over 5), these formulas may take a little time to evaluate. In this case, a large sample approximation may be used.

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## Approximate Power Calculation

The power for the Cochran-Armitage test can be computed quickly using the normal approximation to the binomial distribution. The following steps are taken to compute approximate power.

### One-Sided Test of Increasing Linear Trend in Proportions

1. Find the critical value using the standard normal distribution. The critical value,  $z_{critical}$ , is that value of  $z$  that leaves exactly the target value of alpha in the upper tail of the normal distribution. For example, for an upper-tailed test with a target alpha of 0.05, the critical value is 1.645.
2. For a one-sided test of the alternative hypothesis that  $p_i$  is a monotone increasing function of  $x_i$ , compute the power for given values of  $\mathbf{p} = (p_1, p_2, \dots, p_k)$  as

$$\begin{aligned} 1 - \beta &= \Pr(z \geq z_{critical} | H_1) \\ &= 1 - \Phi(u_U) \end{aligned}$$

## Cochran-Armitage Test for Trend in Proportions

where  $\Phi()$  is the cumulative normal distribution and

$$u_U = \frac{-\left[\sum_{i=1}^k n_i p_i (x_i - \bar{x}) - \frac{\Delta}{2}\right] + z_{critical} \sqrt{p(1-p) \sum_{i=1}^k n_i (x_i - \bar{x})^2}}{\sqrt{\sum_{i=1}^k n_i p_i (1-p_i) (x_i - \bar{x})^2}},$$

with

$$p = \frac{1}{N} \sum_{i=1}^k n_i p_i.$$

The power for the uncorrected test is computed with  $\Delta = 0$ .

## One-Sided Test of Decreasing Linear Trend in Proportions

1. Find the critical value using the standard normal distribution. The critical value,  $z_{critical}$ , is that value of  $z$  that leaves exactly the target value of alpha in the upper tail of the normal distribution. For example, for a lower-tailed test with a target alpha of 0.05, the critical value is 1.645.
2. For a one-sided test of the alternative hypothesis that  $p_i$  is a monotone decreasing function of  $x_i$ , compute the power for given values of  $\mathbf{p} = (p_1, p_2, \dots, p_k)$  as

$$\begin{aligned} 1 - \beta &= \Pr(z \leq -z_{critical} | H_1) \\ &= \Phi(u_L) \end{aligned}$$

where  $\Phi()$  is the cumulative normal distribution and

$$u_L = \frac{-\left[\sum_{i=1}^k n_i p_i (x_i - \bar{x}) + \frac{\Delta}{2}\right] - z_{critical} \sqrt{p(1-p) \sum_{i=1}^k n_i (x_i - \bar{x})^2}}{\sqrt{\sum_{i=1}^k n_i p_i (1-p_i) (x_i - \bar{x})^2}},$$

with

$$p = \frac{1}{N} \sum_{i=1}^k n_i p_i.$$

The power for the uncorrected test is computed with  $\Delta = 0$ .

## Cochran-Armitage Test for Trend in Proportions

**Two-Sided Test of Linear Trend in Proportions**

1. Find the critical value using the standard normal distribution. The critical value,  $z_{critical}$ , is that value of  $z$  that leaves exactly  $\alpha/2$  in the upper tail of the normal distribution. For example, for a two-tailed test with a target alpha of 0.05, the critical value is 1.96.
2. For a two-sided test of the alternative hypothesis that  $p_i$  is a monotone decreasing or increasing function of  $x_i$ , compute the power for given values of  $\mathbf{p} = (p_1, p_2, \dots, p_k)$  as

$$\begin{aligned} 1 - \beta &= \Pr(z_U \geq z_{critical} | H_1) + \Pr(z_L \leq -z_{critical} | H_1) \\ &= 1 - \Phi(u_U) + \Phi(u_L) \end{aligned}$$

where  $\Phi()$  is the cumulative normal distribution and  $u_U$  and  $u_L$  are as defined previously. The power for the uncorrected test is computed with  $\Delta = 0$ .

## Example 1 – Finding the Power

An experiment is being designed to determine if there exists a dose-response relationship for a particular drug. Researchers will administer the drug at three dose levels: control (no drug), low, and high. The low dose is exactly half of the high dose, so the dosage structure is equally spaced. They expect to find response proportions of 0.05, 0.15, and 0.25 corresponding to the three doses, control, low, and high, respectively. A two-sided test with an alpha level of 0.05 will be used along with the continuity-corrected Cochran-Armitage test. They wish to compute the power for conducting the study with equal-sized groups ranging from 30 to 70 subjects in size.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Power</b>
Test Type.....	<b>Z test with continuity correction</b>
Alternative Hypothesis (H1) .....	<b>Two-Sided</b>
Alpha.....	<b>0.05</b>
n (Sample Size Multiplier).....	<b>30 to 70 by 5</b>
k (Number of Groups).....	<b>3</b>
Group Sample Size Pattern .....	<b>Equal</b>
P (Proportions).....	<b>0.05 0.15 0.25</b>
Equally Spaced X Values.....	<b>Checked</b>
Group Sample Size Max for Exact Power.....	<b>20</b>
Sample Size Product Max for Exact Power .....	<b>1000000</b>



## Cochran-Armitage Test for Trend in Proportions

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Reports

#### Numeric Results

Solve For: Power  
 Number of Groups (k): 3  
 Test Type: Two-sided Z test with continuity correction  
 Correction Factor: 0.5  
 Hypotheses: H0:  $P_1 = P_2 = \dots = P_k$  vs. H1: Increasing or Decreasing Trend  
 X Values: Equally Spaced

#### Sample Size

Power	Group Average n	Total N	Alpha	Proportions
0.51187	30	90	0.05	0.05, 0.15, 0.25
0.58893	35	105	0.05	0.05, 0.15, 0.25
0.65710	40	120	0.05	0.05, 0.15, 0.25
0.71640	45	135	0.05	0.05, 0.15, 0.25
0.76724	50	150	0.05	0.05, 0.15, 0.25
0.81029	55	165	0.05	0.05, 0.15, 0.25
0.84635	60	180	0.05	0.05, 0.15, 0.25
0.87629	65	195	0.05	0.05, 0.15, 0.25
0.90093	70	210	0.05	0.05, 0.15, 0.25

Power The probability of rejecting a false null hypothesis when the alternative hypothesis is true.  
 n The average group sample size.  
 N The total sample size of all groups combined.  
 Alpha The probability of rejecting a true null hypothesis.  
 Proportions Lists the set of proportions used. The number of proportions is equal to k.

#### Summary Statements

A 3-group design (with the groups at equally spaced X values) will be used to test whether there is an increasing or decreasing linear trend in proportions (H0:  $P_1 = P_2 = \dots = P_k$  versus H1:  $P_1 < P_2 < \dots < P_k$  or  $P_1 > P_2 > \dots > P_k$ ). The hypotheses will be evaluated using a two-sided Cochran-Armitage Z-test with continuity correction, with a Type I error rate ( $\alpha$ ) of 0.05. To detect a proportion sequence of 0.05, 0.15, and 0.25, with group sample sizes of 30, 30, and 30 (totaling 90 subjects), the power is 0.51187.

#### References

Nam, J. 1987. 'A Simple Approximation for Calculating Sample Sizes for Detecting Linear Trend in Proportions'. Biometrics 43, 701-705.

This report shows the numeric results of this power study.

## Cochran-Armitage Test for Trend in Proportions

## Detailed Results Report

## Details when Power = 0.51187 and Alpha = 0.05

Group	Sample Size Ni	Percent Ni of Total N	Proportion Pi
1	30	33.33	0.05
2	30	33.33	0.15
3	30	33.33	0.25
ALL	90	100.00	

## Details when Power = 0.58893 and Alpha = 0.05

Group	Sample Size Ni	Percent Ni of Total N	Proportion Pi
1	35	33.33	0.05
2	35	33.33	0.15
3	35	33.33	0.25
ALL	105	100.00	

(More Reports Follow)

This report shows the details of each row of the previous report.

**Group**

The number of the group shown on this line. The last line, labeled *ALL*, gives the total sample size for the scenario.

**Ni**

This is the sample size of each group. This column is especially useful when the sample sizes are unequal.

**Percent Ni of Total Ni**

This is the percentage of the total sample that is allocated to each group.

**Pi**

This is the value of the hypothesized proportion.

## Cochran-Armitage Test for Trend in Proportions

## Dropout-Inflated Sample Size Report

## Dropout-Inflated Sample Size

Average Group Sample Size n	Group	Dropout Rate	Sample Size Ni	Dropout-Inflated Enrollment Sample Size Ni'	Expected Number of Dropouts Di
30	1 - 3	20%	30	38	8
	Total		90	114	24
35	1 - 3	20%	35	44	9
	Total		105	132	27
40	1 - 3	20%	40	50	10
	Total		120	150	30
45	1 - 3	20%	45	57	12
	Total		135	171	36
50	1 - 3	20%	50	63	13
	Total		150	189	39
55	1 - 3	20%	55	69	14
	Total		165	207	42
60	1 - 3	20%	60	75	15
	Total		180	225	45
65	1 - 3	20%	65	82	17
	Total		195	246	51
70	1 - 3	20%	70	88	18
	Total		210	264	54

n	The average group sample size.
Group	Lists the group numbers.
Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
Ni	The evaluable sample size for each group at which power is computed (as entered by the user). If Ni subjects are evaluated out of the Ni' subjects that are enrolled in the study, the design will achieve the stated power.
Ni'	The number of subjects that should be enrolled in each group in order to obtain Ni evaluable subjects, based on the assumed dropout rate. Ni' is calculated by inflating Ni using the formula $Ni' = Ni / (1 - DR)$ , with Ni' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lohknygina, Y. (2018) pages 32-33.)
Di	The expected number of dropouts in each group. $Di = Ni' - Ni$ .

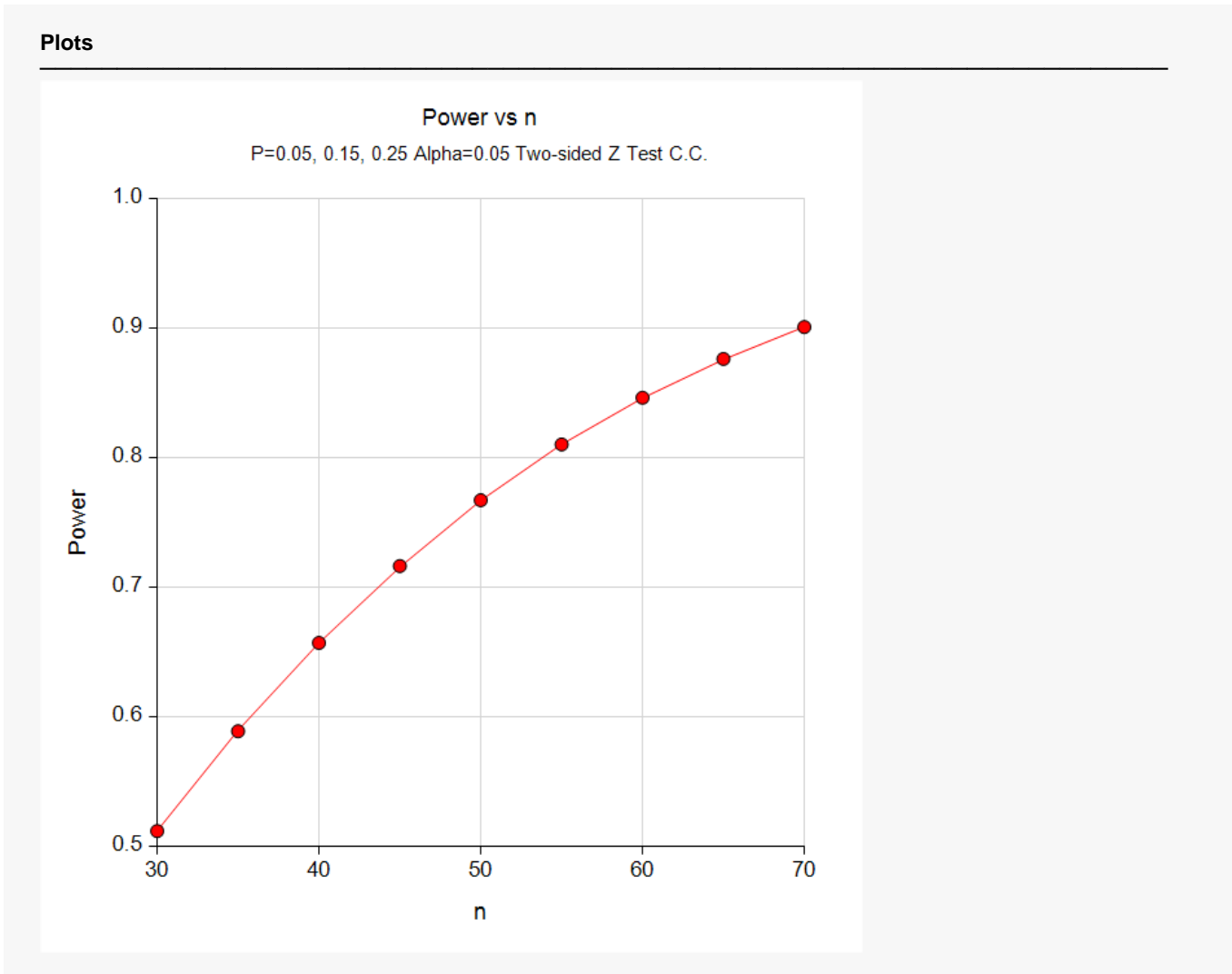
## Dropout Summary Statements

Anticipating a 20% dropout rate, group sizes of 38, 38, and 38 subjects should be enrolled to obtain final group sample sizes of 30, 30, and 30 subjects.

This report shows the sample sizes adjusted for dropout. In this example, dropout is assumed to be 20%. You can change the dropout rate on the Reports tab.

## Cochran-Armitage Test for Trend in Proportions

## Plots Section



This plot gives a visual presentation to the results in the Numeric Report. We can quickly see the impact on the power of increasing the sample size.

When you create one of these plots, it is important to use trial and error to find an appropriate range for the horizontal variable so that you have results with both low and high power.

Cochran-Armitage Test for Trend in Proportions

### Exact Power Calculation

You can calculate the exact power for this scenario by setting the maximum group sample size for exact power calculations to **70**. You can do this yourself or load the completed template **Example1b** from the Template tab on the procedure window.

**Numeric Results**

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Solve For: **Power**  
 Number of Groups (k): 3  
 Test Type: Two-sided Z test with continuity correction  
 Correction Factor: 0.5  
 Hypotheses: H0: P1 = P2 = ... = Pk vs. H1: Increasing or Decreasing Trend  
 X Values: Equally Spaced

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**Sample Size**

<b>Power</b>	<b>Group Average n</b>	<b>Total N</b>	<b>Alpha</b>	<b>Proportions</b>
0.51173*	30	90	0.05	0.05, 0.15, 0.25
0.60387*	35	105	0.05	0.05, 0.15, 0.25
0.67534*	40	120	0.05	0.05, 0.15, 0.25
0.74067*	45	135	0.05	0.05, 0.15, 0.25
0.78352*	50	150	0.05	0.05, 0.15, 0.25
0.83170*	55	165	0.05	0.05, 0.15, 0.25
0.86462*	60	180	0.05	0.05, 0.15, 0.25
0.89489*	65	195	0.05	0.05, 0.15, 0.25
0.91511*	70	210	0.05	0.05, 0.15, 0.25

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\* Values in this row are based on exact power calculations. Exact power was calculated for scenarios in which the largest group sample size is less than or equal to 70 and the product of all group samples sizes is less than or equal to 1000000.

This report indicates that all power values were calculated exactly based on the binomial distribution. The approximate power values calculated earlier are very close to these values.

## Example 2 – Finding the Sample Size

Continuing with the last example, we will determine how large the sample size would need to be to have the power at least 0.95 with an alpha level of 0.05.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For .....	<b>Sample Size</b>
Test Type.....	<b>Z test with continuity correction</b>
Alternative Hypothesis (H1) .....	<b>Two-Sided</b>
Power.....	<b>0.95</b>
Alpha.....	<b>0.05</b>
k (Number of Groups) .....	<b>3</b>
Group Sample Size Pattern .....	<b>Equal</b>
P (Proportions).....	<b>0.05 0.15 0.25</b>
Equally Spaced X Values.....	<b>Checked</b>
Group Sample Size Max for Exact Power.....	<b>20</b>
Sample Size Product Max for Exact Power .....	<b>1000000</b>

### Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results				
Solve For:	Sample Size			
Number of Groups (k):	3			
Test Type:	Two-sided Z test with continuity correction			
Correction Factor:	0.5			
Hypotheses:	H0: P1 = P2 = ... = Pk vs. H1: Increasing or Decreasing Trend			
X Values:	Equally Spaced			
Sample Size				
Power	Group Average n	Total N	Alpha	Proportions
0.95054	85	255	0.05	0.05, 0.15, 0.25

The required sample size is 85 per group or 255 subjects.

## Example 3 – Calculating Power with Unequal Group Sample Sizes

Continuing with the last example, consider the impact of allowing the group sample sizes to be unequal. Suppose we have twice as many control subjects receiving no drug as subjects at the low and high dose levels. What is the power for group sample sizes of 120, 60, and 60?

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Pay particular attention to how the sample size parameters were changed. The sample size multiplier,  $n$ , was set to 1 so that it is essentially ignored. The Group Sample Size Pattern contains the three sample sizes.

Design Tab	
Solve For .....	<b>Power</b>
Test Type.....	<b>Z test with continuity correction</b>
Alternative Hypothesis (H1) .....	<b>Two-Sided</b>
Alpha.....	<b>0.05</b>
n (Sample Size Multiplier).....	<b>1</b>
k (Number of Groups).....	<b>3</b>
Group Sample Size Pattern .....	<b>120 60 60</b>
P (Proportions).....	<b>0.05 0.15 0.25</b>
Equally Spaced X Values.....	<b>Checked</b>
Group Sample Size Max for Exact Power.....	<b>20</b>
Sample Size Product Max for Exact Power .....	<b>1000000</b>

## Cochran-Armitage Test for Trend in Proportions

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

---

Solve For: [Power](#)  
 Number of Groups (k): 3  
 Test Type: Two-sided Z test with continuity correction  
 Correction Factor: 0.5  
 Hypotheses:  $H_0: P_1 = P_2 = \dots = P_k$  vs.  $H_1: \text{Increasing or Decreasing Trend}$   
 X Values: Equally Spaced

---

Sample Size				
Power	Group Average n	Total N	Alpha	Proportions
0.95196	80	240	0.05	0.05, 0.15, 0.25

---

### Details when Power = 0.95196 and Alpha = 0.05

Group	Sample Size Ni	Percent Ni of Total N	Proportion Pi
1	120	50	0.05
2	60	25	0.15
3	60	25	0.25
ALL	240	100	

---

Group sample sizes of 120, 60, and 60 yield just over 95% power. The total sample size of 240 for 95% power for this scenario is actually less than the total of 255 from Example 2, where equal group sample sizes were used.



## Example 4 – Calculating Power with Unequally Spaced X Values

Continuing with Example 1, consider the impact of using unequally spaced dose levels: 0, 2, and 5. Because the doses are not equally spaced, we will use the uncorrected z test for power calculations.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For .....	<b>Power</b>
Test Type.....	<b>Z test</b>
Alternative Hypothesis (H1) .....	<b>Two-Sided</b>
Alpha.....	<b>0.05</b>
n (Sample Size Multiplier).....	<b>30 to 70 by 5</b>
k (Number of Groups) .....	<b>3</b>
Group Sample Size Pattern .....	<b>Equal</b>
P (Proportions).....	<b>0.05 0.15 0.25</b>
Equally Spaced X Values.....	<b>Unchecked</b>
X Values .....	<b>0 2 5</b>
Group Sample Size Max for Exact Power.....	<b>20</b>
Sample Size Product Max for Exact Power .....	<b>1000000</b>

## Cochran-Armitage Test for Trend in Proportions

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

---

Solve For: **Power**  
 Number of Groups (k): 3  
 Test Type: Two-sided Z test  
 Hypotheses: H0:  $P_1 = P_2 = \dots = P_k$  vs. H1: Increasing or Decreasing Trend  
 X Values: 0, 2, 5

---

Sample Size				
Power	Group Average n	Total N	Alpha	Proportions
0.57754	30	90	0.05	0.05, 0.15, 0.25
0.64383	35	105	0.05	0.05, 0.15, 0.25
0.70190	40	120	0.05	0.05, 0.15, 0.25
0.75214	45	135	0.05	0.05, 0.15, 0.25
0.79514	50	150	0.05	0.05, 0.15, 0.25
0.83161	55	165	0.05	0.05, 0.15, 0.25
0.86229	60	180	0.05	0.05, 0.15, 0.25
0.88790	65	195	0.05	0.05, 0.15, 0.25
0.90915	70	210	0.05	0.05, 0.15, 0.25

---

The power values are quite different from those calculated with the continuity-corrected z test when the dose-spacing is equal. Of course, the covariate spacing you use will likely depend on more factors than the achievable power.

## Example 5 – Validation of Sample Size Calculations with Approximate Power using Nam

Nam (1987) page 703 presents a table of calculated sample sizes with three equally spaced doses and equal group sample sizes using the one-sided continuity-corrected z test for an increasing trend in proportions. Sample size is calculated for various proportion sets, alpha levels of 0.05 and 0.025, and power values of 0.5, 0.7, and 0.9. The table based on approximate power calculations is given below.

Alternative Proportion			Specified Nominal Power					
			0.50		0.70		0.90	
$p_0$	$p_1$	$p_2$	$\alpha = 0.025$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.05$
0.05	0.10	0.15	79	58	120	94	197	162
0.10	0.15	0.20	108	79	167	129	276	226
0.20	0.25	0.30	154	111	241	186	402	329
0.30	0.35	0.40	185	133	290	223	486	398
0.05	0.15	0.25	29	22	44	34	70	58
0.10	0.20	0.30	36	26	54	42	87	72
0.20	0.30	0.40	45	33	69	54	113	93
0.30	0.40	0.50	51	37	78	61	129	106
0.05	0.25	0.45	11	9	16	13	25	21
0.10	0.30	0.50	12	9	18	14	28	23
0.20	0.40	0.60	14	10	20	16	32	26
0.30	0.50	0.70	14	11	21	17	33	28

This example will replicate these results.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For .....	<b>Sample Size</b>
Test Type.....	<b>Z test with continuity correction</b>
Alternative Hypothesis (H1) .....	<b>One-Sided (Increasing Trend)</b>
Power.....	<b>0.5 0.7 0.9</b>
Alpha.....	<b>0.025 0.05</b>
k (Number of Groups) .....	<b>3</b>
Group Sample Size Pattern .....	<b>Equal</b>
P (Proportions).....	<b>=C1-C12</b>
Equally Spaced X Values.....	<b>Checked</b>

Cochran-Armitage Test for Trend in Proportions

Group Sample Size Max for Exact Power..... **0**  
 Sample Size Product Max for Exact Power ..... **1000000**

**Input Spreadsheet Data**

Row	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
1	0.05	0.10	0.20	0.30	0.05	0.1	0.2	0.3	0.05	0.1	0.2	0.3
2	0.10	0.15	0.25	0.35	0.15	0.2	0.3	0.4	0.25	0.3	0.4	0.5
3	0.15	0.20	0.30	0.40	0.25	0.3	0.4	0.5	0.45	0.5	0.6	0.7

**Output**

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results**

Solve For: [Sample Size](#)  
 Number of Groups (k): 3  
 Test Type: One-sided Z test with continuity correction  
 Correction Factor: 0.5  
 Hypotheses: H0: P1 = P2 = ... = Pk vs. H1: P1 < P2 < ... < Pk  
 X Values: Equally Spaced

**Sample Size**

Power	Group Average n	Total N	Alpha	Proportions
0.50098	79	237	0.025	0.05, 0.1, 0.15
0.70301	121	363	0.025	0.05, 0.1, 0.15
0.90012	197	591	0.025	0.05, 0.1, 0.15
0.50493	59	177	0.050	0.05, 0.1, 0.15
0.70061	94	282	0.050	0.05, 0.1, 0.15
0.90150	163	489	0.050	0.05, 0.1, 0.15
0.50110	108	324	0.025	0.1, 0.15, 0.2
0.70115	167	501	0.025	0.1, 0.15, 0.2
0.90025	276	828	0.025	0.1, 0.15, 0.2
0.50156	79	237	0.050	0.1, 0.15, 0.2
0.70244	130	390	0.050	0.1, 0.15, 0.2
0.90073	227	681	0.050	0.1, 0.15, 0.2
0.50029	154	462	0.025	0.2, 0.25, 0.3
0.70057	241	723	0.025	0.2, 0.25, 0.3
0.90008	402	1206	0.025	0.2, 0.25, 0.3
0.50249	112	336	0.050	0.2, 0.25, 0.3
0.70052	186	558	0.050	0.2, 0.25, 0.3
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

The sample sizes calculated by **PASS** match those of Nam (1987). In many cases, **PASS** reports a sample size that is one greater than that reported Nam (1987). This difference is due to rounding. Nam (1987) rounds some power values up when they are actually slightly lower than the nominal value. **PASS** does not round power values up when computing the sample size. All sample sizes result in at least the nominal power.

## Example 6 – Validation of Exact Power Calculations using Nam

Nam (1987) page 703 presents a table of calculated sample sizes with three equally spaced doses and equal group sample sizes using the one-sided continuity-corrected z test for an increasing trend in proportions. Sample size is calculated for various proportion sets, alpha levels of 0.05 and 0.025, and power values of 0.5, 0.7, and 0.9. The table of calculated sample sizes is given in Example 5. Nam (1987) further calculates the exact power for scenarios in which the resulting sample size is less than or equal to 50. The results are given below.

Alternative Proportion			Specified Nominal Power					
			0.50		0.70		0.90	
$p_0$	$p_1$	$p_2$	$\alpha = 0.025$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.05$
0.05	0.15	0.25	29 (0.51)	22 (0.51)	44 (0.73)	34 (0.71)		
0.10	0.20	0.30	36 (0.52)	26 (0.50)		42 (0.71)		
0.20	0.30	0.40	45 (0.50)	33 (0.51)				
0.30	0.40	0.50		37 (0.49)				
0.05	0.25	0.45	11 (0.50)	9 (0.57)	16 (0.71)	13 (0.71)	25 (0.92)	21 (0.91)
0.10	0.30	0.50	12 (0.50)	9 (0.54)	18 (0.72)	14 (0.71)	28 (0.91)	23 (0.91)
0.20	0.40	0.60	<b>14 (0.53)</b>	10 (0.47)	20 (0.71)	16 (0.69)	32 (0.90)	26 (0.89)
0.30	0.50	0.70	<b>14 (0.53)</b>	11 (0.50)	21 (0.69)	17 (0.69)	33 (0.90)	28 (0.91)

This example will replicate the results in bold type.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 6** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ..... **Power**  
 Test Type..... **Z test with continuity correction**  
 Alternative Hypothesis (H1) ..... **One-Sided (Increasing Trend)**  
 Alpha..... **0.025**  
 n (Sample Size Multiplier)..... **14**  
 k (Number of Groups)..... **3**  
 Group Sample Size Pattern ..... **Equal**  
 P (Proportions)..... **=C11-C12**  
 Equally Spaced X Values..... **Checked**

## Cochran-Armitage Test for Trend in Proportions

Group Sample Size Max for Exact Power.....**50**  
 Sample Size Product Max for Exact Power ... **1000000**

## Input Spreadsheet Data

Row	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
1	0.05	0.10	0.20	0.30	0.05	0.1	0.2	0.3	0.05	0.1	0.2	0.3
2	0.10	0.15	0.25	0.35	0.15	0.2	0.3	0.4	0.25	0.3	0.4	0.5
3	0.15	0.20	0.30	0.40	0.25	0.3	0.4	0.5	0.45	0.5	0.6	0.7

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

Solve For: [Power](#)  
 Number of Groups (k): 3  
 Test Type: One-sided Z test with continuity correction  
 Correction Factor: 0.5  
 Hypotheses:  $H_0: P_1 = P_2 = \dots = P_k$  vs.  $H_1: P_1 < P_2 < \dots < P_k$   
 X Values: Equally Spaced

### Sample Size

Power	Group Average n	Total N	Alpha	Proportions
0.53000*	14	42	0.025	0.2, 0.4, 0.6
0.52761*	14	42	0.025	0.3, 0.5, 0.7

\* Values in this row are based on exact power calculations. Exact power was calculated for scenarios in which the largest group sample size is less than or equal to 50 and the product of all group samples sizes is less than or equal to 1000000.

The exact power values calculated by **PASS** match those calculated in Nam (1987) exactly if you round to two decimal places. Group sample sizes of 14 results in power of 0.53 for both scenarios. If you replicate the other scenarios in the table, you will find that the **PASS** results for exact power match Nam (1987) after rounding to two decimal places.