

Chapter 401

Conditional Power and Sample Size Reestimation of Tests for Two Means in a 2x2 Cross-Over Design

Introduction

In sequential designs, one or more intermediate analyses of the emerging data are conducted to evaluate whether the experiment should be continued. This may be done to conserve resources or to allow a data monitoring board to evaluate safety and efficacy when subjects are entered in a staggered fashion over a long period of time. *Conditional power* (a frequentist concept) is the probability that the final result will be significant, given the data obtained up to the time of the interim look. *Predictive power* (a Bayesian concept) is the result of averaging the conditional power over the posterior distribution of effect size. Both of these methods fall under the heading of *stochastic curtailment* techniques. Further reading about the theory of these methods can be found in Jennison and Turnbull (2000), Chow and Chang (2007), Chang (2008), Proschan et. al (2006), and Dmitrienko et. al (2005).

This program module computes conditional and predicted power for the case when a *t*-test, computed from data obtained from a 2x2 cross-over design, is used to test whether two population means are different. It also provides *sample size reestimation* to achieve a specified conditional power value.

Technical Details

The 2x2 cross-over design may be described as follows. Randomly assign the subjects to one of two sequence groups so that there are n_1 subjects in sequence one and n_2 subjects in sequence two. In order to achieve design balance, the sample sizes n_1 and n_2 are assumed to be equal so that $n_1 = n_2 = N/2$.

Sequence one is given treatment A followed by treatment B. Sequence two is given treatment B followed by treatment A.

The data may be analyzed by a *t*-test. See one of the cross-over chapters for further details.

Conditional Power

The power of an experiment indicates whether a study is likely to result in useful results, given the sample size. Low power means that the study is *futile*: little chance of statistical significance even though the alternative hypothesis is true. A study that is futile should not be started. However, futility may be determined only after the study has started. When this happens, the study is *curtailed*.

The futility of a study that is underway can be determined by calculating its *conditional power*: the probability of statistical significance at the completion of the study given the data obtained so far.

It is important to note that conditional power at the beginning of the study before any data are collected is equal to the unconditional power. So, conditional power will be high even if early results are negative. Hence, conditional power will seldom result in study curtailment very early in the study.

From Jennison and Turnbull (2000) pages 205 to 208, the general upper one-sided conditional power at stage k for rejecting a null hypothesis about a parameter θ at the end of the study, given the observed test statistic, Z_k , is computed as

$$P_{uk}(\theta) = \Phi\left(\frac{Z_k\sqrt{I_k} - z_{1-\alpha}\sqrt{I_K} + \theta(I_K - I_k)}{\sqrt{I_K - I_k}}\right),$$

the general lower one-sided conditional power at stage k is computed as

$$P_{lk}(\theta) = \Phi\left(\frac{-Z_k\sqrt{I_k} - z_{1-\alpha}\sqrt{I_K} - \theta(I_K - I_k)}{\sqrt{I_K - I_k}}\right),$$

and the general two-sided conditional power at stage k is computed as

$$P_k(\theta) = \Phi\left(\frac{Z_k\sqrt{I_k} - z_{1-\alpha/2}\sqrt{I_K} + \theta(I_K - I_k)}{\sqrt{I_K - I_k}}\right) + \Phi\left(\frac{-Z_k\sqrt{I_k} - z_{1-\alpha/2}\sqrt{I_K} - \theta(I_K - I_k)}{\sqrt{I_K - I_k}}\right),$$

where

θ = the parameter being tested by the hypothesis

k = an interim stage at which the conditional power is computed ($k = 1, \dots, K - 1$)

K = the stage at which the study is terminated and the final test computed

Z_k = the test statistic calculated from the observed data that has been collected up to stage k

I_k = the information level at stage k

I_K = the information level at the end of the study

$z_{1-\alpha}$ = the standard normal value for the test with a type I error rate of α .

Conditional Power and Sample Size Reestimation of Tests for Two Means in a 2x2 Cross-Over Design

For a 2x2 cross-over design t -test of two means with null hypothesis $H_0: \delta = \delta_0$, where δ_0 is the hypothesized mean difference under the null hypothesis, and if δ_1 is the actual mean difference under the alternative hypothesis, these components are computed in Chang (2008) page 71 as

$$\theta = \delta_1 - \delta_0 \quad (\text{the expected difference of mean differences under the alternative hypothesis})$$

$$Z_k = \left(\frac{\bar{d}_{Y1k} + \bar{d}_{Y2k}}{2} - \delta_0 \right) \sqrt{\hat{I}_k} \quad (\text{the } t\text{-statistic computed from the observed data})$$

$$I_k = \frac{n_k}{\sigma_d^2} \quad (\text{the interim information level})$$

$$I_K = \frac{N}{\sigma_d^2} \quad (\text{the final information level})$$

where

\bar{d}_{Yik} the sample mean difference at stage k for those subjects receiving sequence i

\hat{I}_k is the estimated information from the sample at stage k

n_k is the sample size at stage k

N is the total sample size

σ_d^2 is the variance of the subject differences

The value of σ_d^2 may computed directly from the differences or from the original data using

$\sigma_d^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$. Here, σ_1^2 is the variance of the first variable from which the difference is computed and σ_2^2 is the variance of the second variable. The correlation coefficient ρ is correlation between the first and second variables.

Computing conditional power requires you to set σ_d^2 , δ_0 , and δ_1 . Their values can come from the values used during the planning of the study, from similar studies, or from estimates made from the data that has emerged.

Futility Index

The *futility index* is $1 - P_k(\theta)|H_1$. The study may be stopped if this index is above 0.8 or 0.9 (that is, if conditional power falls below 0.2 or 0.1).

Predictive Power

Predictive power (a Bayesian concept) is the result of averaging the conditional power over the posterior distribution of effect size. From Jennison and Turnbull (2000) pages 210 to 213, the general upper one-sided predictive power at stage k is given by

$$P_{uk} = \Phi \left(\frac{Z_k \sqrt{I_K} - z_{1-\alpha} \sqrt{I_k}}{\sqrt{I_K - I_k}} \right).$$

the general lower one-sided predictive power at stage k is given by

$$P_{lk} = \Phi \left(\frac{-Z_k \sqrt{I_K} - z_{1-\alpha} \sqrt{I_k}}{\sqrt{I_K - I_k}} \right),$$

the general two-sided predictive power at stage k is given by

$$P_k = \Phi \left(\frac{|Z_k| \sqrt{I_K} - z_{1-\alpha/2} \sqrt{I_k}}{\sqrt{I_K - I_k}} \right) + \Phi \left(\frac{-|Z_k| \sqrt{I_K} - z_{1-\alpha/2} \sqrt{I_k}}{\sqrt{I_K - I_k}} \right),$$

with all terms defined as in the equations for conditional power.

Sample Size Reestimation

As Chang (2014) points out, after an interim analysis, it is often desirable to recalculate the target sample size using updated values for various nuisance parameters such as the variance. This process is known as *sample size reestimation*.

One method of calculating an adjusted sample size estimate is to search for the sample size that results in a predetermined value of conditional power. PASS conducts a binary search using the conditional power as the criterion. The result is called the *target sample size*.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters. The parameters that may be selected are *Conditional Power* or *Sample Size Reestimation*.

Note that the value selected here always appears as the vertical axis on the charts.

The program is set up to calculate conditional power directly. To find appropriate values of sample size, a binary search is made using an iterative procedure until an appropriate value is found.

Test

Alternative Hypothesis

Specify the alternative hypothesis of the test. Since the null hypothesis is the opposite, specifying the alternative is all that is needed.

When you choose a one-sided test option, you must be sure that the value(s) of δ_0 and δ_1 match your choice. For example, if you select $H1: \delta_1 < \delta_0$, the value δ_1 should be less than δ_0 .

Conditional Power and Alpha

Conditional Power

Enter one or more values for the conditional power that is to be attained by the sample size search.

Conditional power is the probability of rejecting a false null hypothesis given the data that has been obtained so far. It is a function of the effect size and the data that have been collected.

The valid range is between 0 to 1. Typically, values over 0.70 are considered high enough to continue a study, while values below 0.10 considered grounds for curtailing the study.

You can enter a single value such as 0.80 or a series of values such as 0.80 0.85 0.90 0.95 or 0.7 to 0.97 by 0.05.

Alpha

This option specifies one or more values for the probability of a type-I error at the end of the study. A type-I error occurs when a true null hypothesis is rejected.

Values must be between zero and one. Historically, the value of 0.05 has been used for two-sided tests and 0.025 for one-sided tests.

You may enter a range of values such as 0.01 0.05 0.10 or 0.01 to 0.10 by 0.01.

Sample Size

N (Target Sample Size)

This option specifies one or more values of the target sample size, the total number of subjects planned for the study. To achieve balance in the design, it is assumed that half of the subjects are given the first sequence of treatment A then treatment B while the other half are given treatment B then treatment A.

This value must be an integer greater than one. Note that you may enter a list of values using the syntax 50,100,150,200,250 or 50 to 250 by 50.

nk (Sample Size at Look k)

Enter the sample size obtained through look k. If this value is greater than N, the value of N is increased to this amount. To achieve balance in the design, it is assumed that half of the subjects are given the first sequence of treatment A then treatment B while the other half are given treatment B then treatment A.

Effect Size – Mean Difference

δ_0 (Null Difference)

Enter a value for δ_0 , the mean difference assumed by the null hypothesis. Only the difference, $\delta_1 - \delta_0$, is needed to compute the conditional power. Usually, δ_0 and δ_1 are the values upon which the original sample size of the study was based. This value is often set to zero.

You may enter a range of values such as 0 0.5 1 or 0 to 3 by 0.5.

Conditional Power and Sample Size Reestimation of Tests for Two Means in a 2x2 Cross-Over Design **$\delta 0$ (Actual Difference to Detect)**

Enter a value for $\delta 1$, the mean difference assumed by the alternative hypothesis at which conditional power is calculated. Only the difference, $\delta 1 - \delta 0$, is needed to compute the conditional power. Usually, $\delta 0$ and $\delta 1$ are the values upon which the original sample size of the study was based.

You may enter a range of values such as *0 0.5 1* or *0 to 3 by 0.5*.

Effect Size – Standard Deviation **σd (Std Dev of Differences)**

Enter a value for σ_d , the standard deviation of the pair differences of subjects within the same sequence. It is assumed that the standard deviations of the two sequences are the same.

Often, cross-over designs are analyzed using repeated measures ANOVA. If this is your case, then $\sigma d = \text{SQRT}(\text{WMSE}/2)$, where WMSE is the within mean square error of the ANOVA table.

The value entered is controversial. Some think you should enter the value used in planning the study. Others think you should use the value estimated from the data obtained so far. Still others think you should use a confidence limit for σd created from the current sample.

You can enter a range of values such as *1 2 3* or *1 to 5 by 1*.

Press the σ button to the right to load the Standard Deviation Estimator window.

Effect Size – Current Test Statistic **Z_k (Current Test Statistic)**

Enter the value of the t -statistic calculated from the data obtained through stage k . This value may be positive or negative. Typically, the t -statistic ranges between -5 and 5.

Example 1 – Computing Conditional Power

Suppose a 2x2 cross-over study has been planned to detect a mean change of 1 at an alpha of 0.025 using a one-sided *t*-test. The total sample size is 52. The standard deviation is expected to be about 1.8. An interim analysis is run after half the data have been collected. This analysis yields a *t*-test value of 2.12.

The data monitoring board would also like to have the conditional power calculated for mean changes of 0.2, 0.4, 0.6, and 0.8.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may make the appropriate entries as listed below or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	Conditional Power
Alternative Hypothesis	H1: $\delta > \delta_0$ (One-Sided)
Alpha.....	0.025
N (Target Sample Size)	52
nk (Sample Size at Look k).....	26
δ_0 (Null Difference).....	0
δ_1 (Actual Difference to Detect)	0.2 0.4 0.6 0.8 1
σ_d (Std Dev of Differences)	1.8
Zk (Current Test Statistic).....	2.12

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for 2x2 Cross-Over T-Test

Hypotheses: $H_0: \delta \leq \delta_0$ vs. $H_1: \delta > \delta_0$

Cond. Power	Pred. Power	Target Sample Size N	Current Sample Size nk	Null Diff δ_0	Actual Diff δ_1	Diff of Diffs $\delta_1 - \delta_0$	Std Dev σ_d	Test Stat Zk	Alpha	Futility
0.46603	0.85040	52	26	0.00	0.20	0.20	1.80	2.120	0.025	0.53397
0.68485	0.85040	52	26	0.00	0.40	0.40	1.80	2.120	0.025	0.31515
0.85265	0.85040	52	26	0.00	0.60	0.60	1.80	2.120	0.025	0.14735
0.94678	0.85040	52	26	0.00	0.80	0.80	1.80	2.120	0.025	0.05322
0.98541	0.85040	52	26	0.00	1.00	1.00	1.80	2.120	0.025	0.01459

References

Jennison, C., and Turnbull, B.W. 2000. Group Sequential Methods with Applications to Clinical Trials. Chapman & Hall/CRC. New York.

Proschan, M., Lan, K.K.G., Wittes, J.T. 2006. Statistical Monitoring of Clinical Trials. Springer. New York.

Chang, Mark. 2008. Classical and Adaptive Clinical Trial Designs. John Wiley & Sons. Hoboken, New Jersey.

Chang, Mark. 2014. Adaptive Design Theory and Implementation Using SAS and R. CRC Press. New York.

Conditional Power and Sample Size Reestimation of Tests for Two Means in a 2x2 Cross-Over Design

Report Definitions

Conditional Power is the probability of rejecting a false null hypothesis at the end of the study given the data that have emerged so far.

Predictive Power is the result of averaging the conditional power over the posterior distribution of the effect size.

N is the target sample size from both sequences combined.

n_k is the total sample size obtained through stage k from both sequences combined.

δ_0 is the mean difference assuming the null hypothesis.

δ_1 is the actual difference to detect under the alternative hypothesis at which conditional power is calculated.

$\delta_1 - \delta_0$ is the difference of the mean differences. This is the minimum value that is to be detected by the completed study.

σ_d is the standard deviation of the pair differences.

Z_k is the value of the test statistic from the observed data at stage k .

Alpha is the probability of rejecting a true null hypothesis.

Futility is one minus the conditional power. A value greater than 0.9 or 0.8 indicates the study should be stopped because there is little chance of achieving statistical significance.

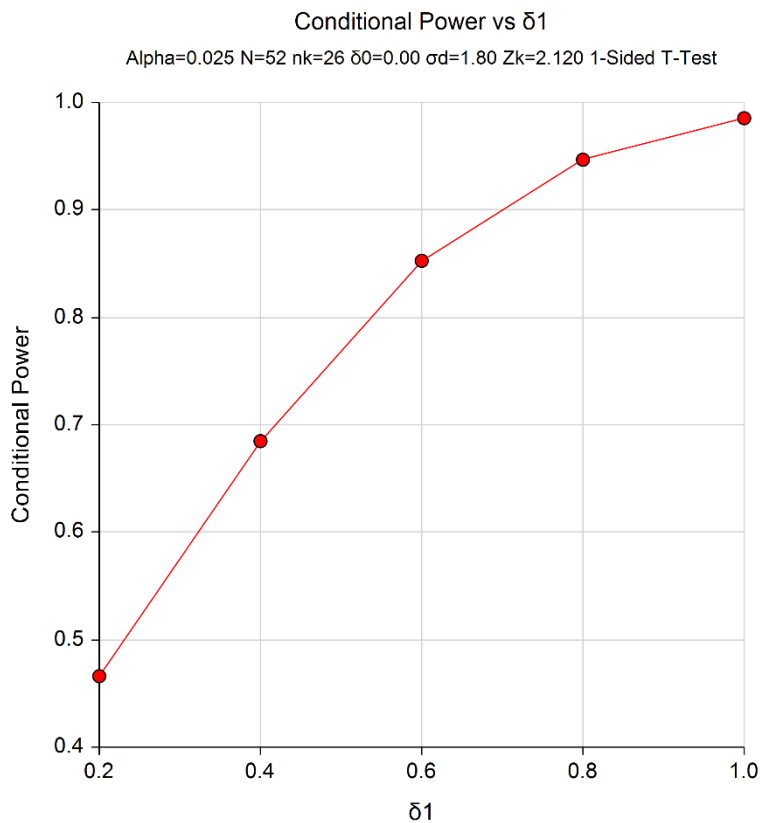
Summary Statements

The first 26 of the planned 52 subjects in a 2x2 cross-over design achieve 46.603% conditional power to detect a difference of $\delta_1 - \delta_0 = 0.20$ between the alternative mean difference $\delta_1 = 0.20$ and the null mean difference $\delta_0 = 0.00$ using a one-sided t-test with a standard deviation of differences of 1.80 and a significance level of 0.025.

The t-value of the data that have emerged so far is 2.120. The futility index is 0.53397.

This report shows the values of each of the parameters, one scenario per row. The definitions of each column are given in the Report Definitions section.

Plots Section



This plot shows the relationship between conditional power and δ_1 .

Example 2 – Validation

We could not find an example of a conditional power calculation for a 2x2 cross-over t -test in the literature. Since the calculations are relatively simple, we will validate the calculation of the third scenario ($\delta_1 = 0.6$) of Example 1 by hand.

In this case

$$\begin{aligned} I_k &= n_k / \sigma_d^2 & I_K &= N / \sigma_d^2 \\ &= \frac{26}{1.8^2} & &= \frac{52}{1.8^2} \\ &= 8.0246914 & &= 16.0493827 \end{aligned}$$

$$\begin{aligned} P_{uk}(\theta) &= \Phi\left(\frac{Z_k \sqrt{I_k} - z_{1-\alpha} \sqrt{I_K} + (\delta_1 - \delta_0)(I_K - I_k)}{\sqrt{I_K - I_k}}\right) \\ &= \Phi\left(\frac{2.12\sqrt{8.0246914} - (1.959964)\sqrt{16.0493827} + (0.6 - 0)(16.0493827 - 8.0246914)}{\sqrt{16.0493827 - 8.0246914}}\right) \\ &= \Phi\left(\frac{2.9683814}{2.8327886}\right) \\ &= \Phi(1.0478655) \\ &= 0.8526497 \end{aligned}$$

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may make the appropriate entries as listed below or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Conditional Power
Alternative Hypothesis	H1: $\delta_1 > \delta_0$ (One-Sided)
Alpha	0.025
N (Target Sample Size)	52
nk (Sample Size at Look k)	26
δ_0 (Null Difference)	0
δ_1 (Actual Difference to Detect)	0.6
σ_d (Std Dev of Differences)	1.8
Zk (Current Test Statistic)	2.12

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for 2x2 Cross-Over T-Test

Hypotheses: $H_0: \delta \leq \delta_0$ vs. $H_1: \delta > \delta_0$

Cond. Power	Pred. Power	Target Sample Size N	Current Sample Size nk	Null Diff δ_0	Actual Diff δ_1	Diff of Diffs $\delta_1 - \delta_0$	Std Dev σ_d	Test Stat Zk	Alpha	Futility
0.85265	0.85040	52	26	0.00	0.60	0.60	1.80	2.120	0.025	0.14735

The conditional power of 0.85265 matches the value calculated by hand.

Example 3 – Sample Size Reestimation

Suppose a 2x2 cross-over study has started. It is being conducted to detect a mean change of 1.00 at an alpha of 0.025 using a one-sided t -test and a null difference of 0.0. The target sample size is 52. The standard deviation is expected to be about 1.80.

An interim analysis was run after half the data were collected. This analysis yields a z -test value of 2.12. This test value was the result of transforming the p -value of the t -test with the inverse, standard normal distribution. The sample standard deviation was 4.28.

The data monitoring board would like to have sample size reestimated to obtain a conditional power of 0.8.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may make the appropriate entries as listed below or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size Reestimation
Alternative Hypothesis	H1: $\delta > \delta_0$ (One-Sided)
Conditional Power.....	0.8
Alpha.....	0.025
nk (Sample Size at Look k).....	26
δ_0 (Null Difference).....	0
δ_1 (Actual Difference to Detect)	1
σ_d (Std Dev of Differences).....	4.28
Zk (Current Test Statistic).....	2.12

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for 2x2 Cross-Over T-Test											
Hypotheses: $H_0: \delta \leq \delta_0$ vs. $H_1: \delta > \delta_0$											
Cond. Power	Pred. Power	Target Sample Size N	Current Sample Size nk	Null Diff δ_0	Actual Diff δ_1	Diff of Diffs $\delta_1 - \delta_0$	Std Dev σ_d	Test Stat Zk	Alpha	Futility	
0.80219	0.89316	85	26	0.00	1.00	1.00	4.28	2.120	0.025	0.19781	

Notice that the target sample size has increased from 52 to 85.