Chapter 818

Confidence Intervals for Coefficient Alpha

Introduction

Coefficient alpha, or *Cronbach's alpha*, is a measure of the reliability of a test consisting of *k* parts. The *k* parts usually represent *k* items on a questionnaire or *k* raters. This routine calculates the sample size needed to obtain a specified width of a confidence interval for coefficient alpha at a stated confidence level.

Technical Details

Feldt et al. (1987) has shown that if *CA* is the estimated value of coefficient alpha computed from a sample of size *N* subjects taking a test with *k* items, a 100(1 – *α*)% confidence interval for CA is given by

$$CA_L = 1 - \left[(1 - CA)F_{\frac{\alpha}{2},N-1,(N-1)(K-1)}\right]$$

and

$$CA_U = 1 - \left[(1 - CA)F_{1-\frac{\alpha}{2},N-1,(N-1)(K-1)}\right]$$

Therefore, the width of the confidence interval is $CA_U - CA_L$. Using the above, we can calculate the confidence interval width for any confidence level and sample size.

One-sided limits may be obtained by replacing $\alpha/2$ by α.

Confidence Level

The confidence level, $1 - \alpha$, has the following interpretation. If thousands of samples of *N* items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population correlation is $1 - \alpha$.

Example 1 – Calculating Sample Size

Suppose a study is planned to estimate coefficient alpha with a two-sided 95% confidence interval with a width no wider than 0.1. The researcher would like to examine values of K from 5 to 30 in steps of 5. From past studies, the researcher wants to use a planning estimate of 0.5 for the sample coefficient alpha. The goal is to determine the necessary sample size for each scenario.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Sample Size**
Interval Type	**Two-Sided**
Confidence Level	**0.95**
K (Items per Subject)	**5 10 15 20 25 30**
Width of Confidence Interval	**0.1**
CA (Sample Coefficient Alpha)	**0.5**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Sample Size
Interval Type: Two-Sided

Confidence Level	Number of Subjects N	Items per Subject K	Confidence Interval Width	Sample Coefficient Alpha CA	Confidence Interval Limits	
					Lower	Upper
0.95	963	5	0.1	0.5	0.448	0.548
0.95	855	10	0.1	0.5	0.449	0.549
0.95	825	15	0.1	0.5	0.449	0.549
0.95	810	20	0.1	0.5	0.449	0.549
0.95	802	25	0.1	0.5	0.449	0.549
0.95	796	30	0.1	0.5	0.449	0.549

Confidence Level	The proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the true coefficient alpha.
N	Number of Subjects. The size of the random sample of subjects drawn from the population.
K	The number of items per subject. On a survey, this is the number of questions.
Confidence Interval Width	The width of the confidence interval. It is the distance from the lower limit to the upper limit.
CA	Sample Coefficient Alpha. The planning estimate of the sample coefficient alpha (Cronbach's Alpha).
Confidence Interval Limits	The actual limits that would result from a dataset with these statistics.

Summary Statements

A single-group reliability measurement design, with 5 items per subject, will be used to obtain a two-sided 95% confidence interval for a single coefficient alpha (Cronbach's alpha). The sample coefficient alpha is assumed to be 0.5. To produce a confidence interval with a width of no more than 0.1, 963 subjects will be needed.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	963	1204	241
20%	855	1069	214
20%	825	1032	207
20%	810	1013	203
20%	802	1003	201
20%	796	995	199

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N	The evaluable sample size at which the confidence interval is computed. If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated confidence interval.
N'	The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. After solving for N, N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D	The expected number of dropouts. D = N' - N.

Dropout Summary Statements

Anticipating a 20% dropout rate, 1204 subjects should be enrolled to obtain a final sample size of 963 subjects.

References

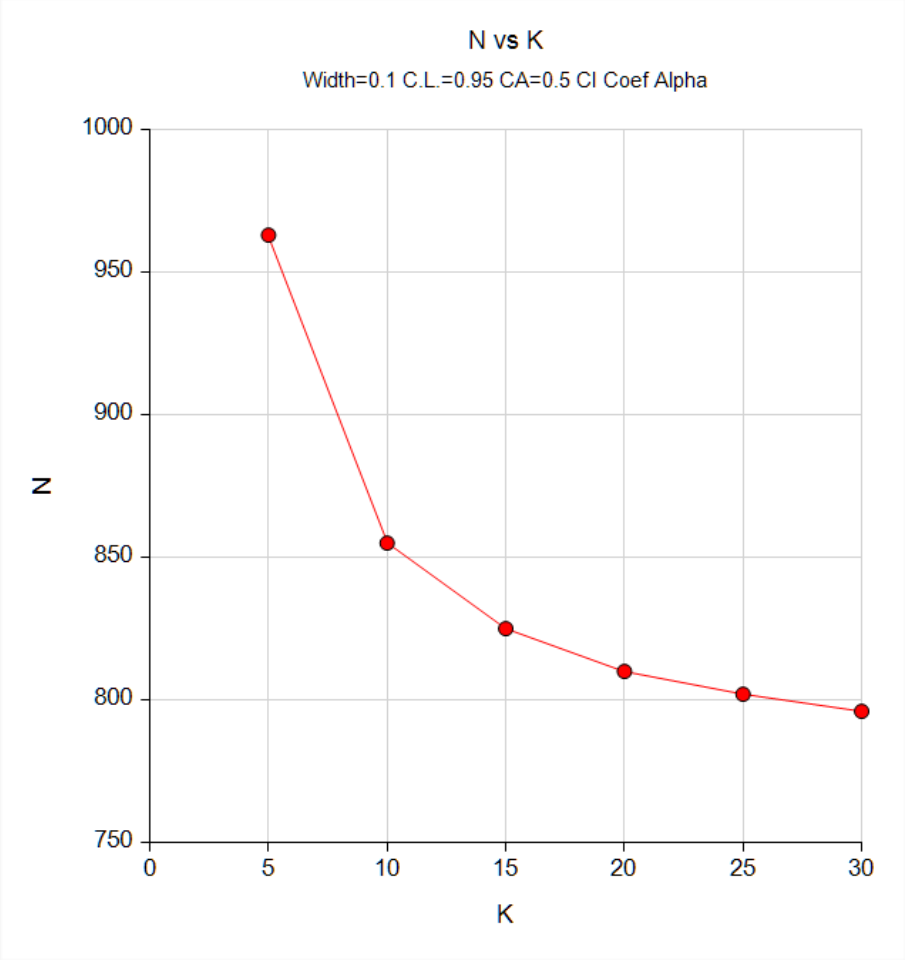
Bonett, D. G. 2002. 'Sample Size Requirements for Testing and Estimating Coefficient Alpha.' Journal of Educational and Behavioral Statistics, Vol 27, No 4, 335-340.

Feldt, L. S., Woodruff, D. J., and Salih, F. A. 1987. 'Statistical Inference for Coefficient Alpha.' Applied Psychological Measurement, Vol 11, No. 1, 93-103.

This report shows the calculated sample size for each of the scenarios.

Plots Section

Plots



This plot shows the sample size versus the value of K.

Example 2 – Validation using Feldt et al. (1987)

Feldt et al. (1987), page 95, give an example calculation for CA = 0.79, N = 41, K=26, and confidence level = 0.90. They find that CAL = 0.704 and CAU = 0.861, for a width of 0.157. We will now validate this routine using this example.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**CI Width or Distance to CA**
Interval Type	**Two-Sided**
Confidence Level	**0.90**
N (Number of Subjects)	**41**
K (Items per Subject)	**26**
CA (Sample Coefficient Alpha)	**0.79**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: CI Width or Distance to Limit
Interval Type: Two-Sided

Confidence Level	Number of Subjects N	Items per Subject K	Confidence Interval Width	Sample Coefficient Alpha CA	Confidence Interval Limits	
					Lower	Upper
0.9	41	26	0.157	0.79	0.705	0.862

PASS matches Feldt's results within rounding.