Chapter 296

Confidence Intervals for Cp

Introduction

This routine calculates the sample size needed to obtain a specified width of a Cp confidence interval at a stated confidence level.

Cp is a process capability index used to approximate the *potential* of a process to produce non-defective items. Cp measures the defective rate of a process by comparing its distribution to specification limits. The comparison is based on the process mean and standard deviation. Cp requires the assumption that the measurements are normally distributed and that the process mean is centered between the specification limits.

The formula for the calculation of Cp is

$$Cp = (USL - LSL) / 6\sigma$$

where USL and LSL are the upper and lower specification limits, respectively. Note that in the normal distribution, approximately 99.7% of the data values are within three standard deviations of the mean, so if Cp is a little greater than one, it can be assumed that the data values are all within the specification limits. Thus, a process with a Cp of 2.0 is considered excellent, while one with a Cp of 1.33 is considered adequate.

Technical Details

This procedure is based on the results of Mathews (2010). Since Cp is a simple transformation of the variance, the confidence interval of the variance can be used to obtain the following two-sided, 100(1 – α)% confidence interval for Cp.

$$P\left(\widehat{Cp}\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}} \le Cp \le \widehat{Cp}\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right) = 1-\alpha$$

where $\widehat{Cp}$ is the estimated value of Cp, n is the sample size, and $\chi^2_{df,\varphi}$ is the specific value of the chi-square random variate with *df* degrees of freedom that has probability φ to the left.

One-sided limits may be obtained by replacing $\alpha/2$ by α.

For two-sided intervals, the distance from the sample Cp to each of the limits may be different. So, instead of specifying the distance to the limits (the half width), we specify the width of the entire interval.

Confidence Interval Width

The confidence interval width, confidence level, and sample size are related in the equation

$$Width = \widehat{Cp}\left(\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}} - \sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}}\right)$$

This equation can be used to find *n*, α, or the width.

Confidence Level

The confidence level, 1 – α, has the following interpretation. If thousands of samples of *n* items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population parameter is 1 – α.

Example 1 – Calculating Sample Size

Suppose a study is planned in which the researcher wishes to construct a two-sided 95% confidence interval for Cp such that the width of the interval is no wider than 0.10. The researcher would like to examine Cp values of 1.0, 1.5, 2.0, and 3.0 to determine the effect of the Cp estimate on necessary sample size.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ..... **Sample Size**
Interval Type ..... **Two-Sided**
Confidence Level (1 – Alpha)..... **0.95**
Confidence Interval Width (Two-Sided) **0.10**
Cp **1 1.5 2 3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Sample Size
Interval Type: Two-Sided

Confidence Level	Sample Size N	Confidence Interval Width Target	Confidence Interval Width Actual	Cp	Confidence Interval Limits Lower	Confidence Interval Limits Upper
0.95	769	0.1	0.1	1.0	0.95	1.05
0.95	1730	0.1	0.1	1.5	1.45	1.55
0.95	3074	0.1	0.1	2.0	1.95	2.05
0.95	6916	0.1	0.1	3.0	2.95	3.05

Confidence Level	The proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the true value of Cp.
N	The size of the sample drawn from the population.
Confidence Interval Width	The distance between the lower and upper confidence interval limits.
Target Width	The width that was requested.
Actual Width	The calculated width. This is slightly different from the Target Width because N is an integer.
Cp	Equal to (USL - LSL) / 6σ, where USL and LSL are the upper and lower specification limits and σ is the process standard deviation.
Confidence Interval Limits	The confidence interval of Cp.

Summary Statements

A two-sided 95% confidence interval for Cp is needed. The formula based on the transformation of the variance will be used to calculate the confidence interval. The sample Cp is assumed to be 1. To produce a confidence interval with a width of no more than 0.1, a sample size of 769 will be needed.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	769	962	193
20%	1730	2163	433
20%	3074	3843	769
20%	6916	8645	1729

Dropout Rate The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N The evaluable sample size at which the confidence interval is computed. If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated confidence interval.
N' The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. After solving for N, N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D The expected number of dropouts. D = N' - N.

Dropout Summary Statements

Anticipating a 20% dropout rate, 962 subjects should be enrolled to obtain a final sample size of 769 subjects.

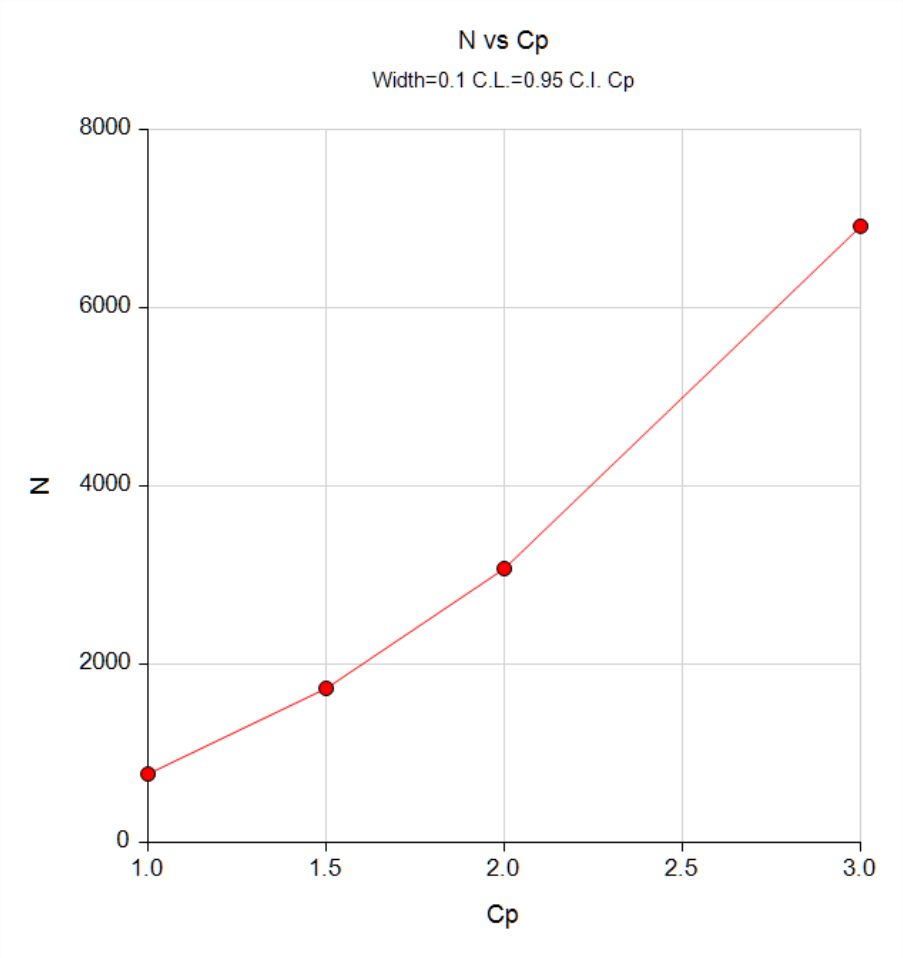
References

Kotz, S. and Johnson, N. 1993. Process Capability Indices. Chapman & Hall.
Mathews, Paul. 2010. Sample Size Calculations: Practical Methods for Engineers and Scientists. Mathews Malnar and Bailey, Inc.

This report shows the calculated sample size for each of the scenarios.

Plots Section

Plots



This plot shows the sample size versus Cp.

Example 2 – Validation using Mathews (2010)

Mathews (2010), page 230, gives an example of a sample size calculation. In this example the value of Cp is 1.0, the confidence level is 90%, and the width is 0.20. The resulting sample size is 136. Note that Mathews uses a normal approximation to the chi-square distribution which may make his results a little different than ours.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ..... **Sample Size**
Interval Type ..... **Two-Sided**
Confidence Level (1 – Alpha)..... **0.90**
Confidence Interval Width (Two-Sided) **0.20**
Cp **1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Interval Type: Two-Sided

Confidence Level	Sample Size N	Confidence Interval Width Target	Confidence Interval Width Actual	Cp	Confidence Interval Limits Lower	Confidence Interval Limits Upper
0.9	137	0.2	0.199	1	0.9	1.099

PASS calculates the sample size to be 137 which is only slightly different than the 136 found by Mathews. Remember that **PASS** uses exact calculation of the chi-square distribution and guarantees that the width requirement is met.