Chapter 828

Confidence Intervals for Intraclass Correlation with Assurance Probability (Lower One-Sided)

Introduction

This routine calculates the sample size needed to obtain a specified width of a lower one-sided confidence interval of the intraclass correlation coefficient (ICC). This procedure allows you to set an *assurance probability* that the requested width is achieved. Note that another **PASS** procedure computes the sample size for a two-sided confidence interval for the ICC.

The ICC is the product-moment correlation calculated among observations on the same subject. For example, if you have three raters rating each subject, it is the average correlation among the ratings of the three raters. This procedure is often used in reliability studies.

The ICC analyzed in this procedure comes from a one-way random effects ANOVA model.

Technical Details

Zou (2012) presents formulas used for constructing a lower one-sided, 100(1 – α)% confidence interval for the ICC. We adopt his notation as we present these formulas.

Suppose that each of *N* subjects (or clusters) is rated by *K* raters. The observations obtained may be from different raters, instruments, or other measurement mechanisms. Such data may be analyzed using a one-way, random-effects, model. The ANOVA model is

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \text{ where } \alpha_i \sim N(0, \sigma_\alpha^2) \text{ and } \varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2).$$

Estimation of ρ

The ICC is defined as

$$\rho = \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma_\varepsilon^2}$$

The ICC is estimated from the mean squares of the ANOVA table as follows

$$r = \frac{MS_B - MS_E}{MS_B + (K-1)MS_E}$$

where MS_B is the between-subject mean square and MS_E is the within-subject mean square.

Confidence limits r_L and r_U for ρ are obtained using the formulas

$$r_L = \frac{F_L - 1}{F_L + K - 1}, \qquad r_U = \frac{F_U - 1}{F_U + K - 1}$$

where

$$F_L = \frac{F_O}{F_{1-\alpha/2,V2,V1}}, \; F_U = F_O F_{1-\alpha/2,V1,V2}, \; F_O = \frac{MS_B}{MS_E}, \; V1 = N(K-1), \; V2 = N-1.$$

One-sided bounds may be obtained by replacing $\alpha/2$ by α.

This procedure will only provide results for lower one-sided confidence intervals.

Sample Size Calculation

The procedure focuses on the case where the main concern is that the reliability coefficient, ρ, is not less than a prespecified value, ρ_0. Thus, the primary objective of the study is to determine if the ICC is of acceptable magnitude. Landis and Koch (1977) provide the following guidelines which are often used when select an appropriate value of ρ_0.

ICC Range	Definition
$\rho < 0$	Poor
$0.0 \le \rho < 0.2$	Slight
$0.2 \le \rho < 0.4$	Fair
$0.4 \le \rho < 0.6$	Moderate
$0.6 \le \rho < 0.8$	Substantial
$0.8 \le \rho < 1.0$	Almost Perfect

For example, if one wants a 'substantial' value, they would select $\rho_0 = 0.6$.

The sample size is determined so that the probability that the requested limit is achieved is above a specified value. This probability is called the *assurance probability* and is referred to as $1 - \gamma$.

The assurance probability requirement is written as

$$1 - \gamma = \Pr(\rho_L \ge \rho_0)$$

It turns out that this formulation may also be used to obtain the power of the corresponding hypothesis test.

Donner and Eliasziw (1987) showed that if the one-way model normality assumptions are met, the *assurance probability* could be calculated exactly using the F distribution as follows

$$1 - \gamma = \Pr\left(F \ge C_0 F_{\alpha,v1,v2}\right)$$

where

$$v1 = N - 1$$

$$v2 = N(K-1)$$

$$C_0 = \frac{1 + K\theta_0}{1 + K\theta_1}$$

$$\theta_0 = \frac{\rho_0}{1 - \rho_0}$$

$$\theta_1 = \frac{\rho_1}{1 - \rho_1}$$

$$v1 = N - 1$$

Here, ρ_1 is used to represent the true value of ρ. Care must be taken so that $\rho_0 < \rho_1$.

Zou (2012) presented a close approximation to the assurance probability based on Fisher's transformation which could be solved directly for sample size. The resulting equation is

$$N = 1 + \frac{2K\left(z_\alpha + z_\gamma\right)^2}{\left\{\ln\left[\frac{F(\rho_1)}{F(\rho_0)}\right]\right\}^2 (K-1)}$$

where

$$F(\rho) = \frac{1 + (K-1)\rho}{1-\rho}$$

This can be rearranged to solve for the assurance probability or ρ_0 as well.

In **PASS**, you can choose to use the exact calculation based of the F distribution or the approximate solution based on Fisher's transformation.

Confidence Level

The confidence level, 1 – α, has the following interpretation. If thousands of samples of *n* items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population correlation is 1 – α.

Example 1 – Calculating Sample Size

Suppose a reliability study is planned to find an estimate of the lower one-sided 95% confidence interval for the ICC. The researcher would like to examine values of *K* from 2 to 10. The goal is to determine the necessary sample size, N, when the assurance probability is 0.9, the lower confidence bound of ρ is 0.5, and the lower confidence bound is 0.6.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Number of Subjects (N)**
Calculation Method	**Exact F Distribution**
Confidence Level (1 - α)	**0.95**
1 - γ (Assurance Probability)	**0.9**
K (Observations per Subject)	**2 4 6 8 10**
ρ0 (Lower Confidence Bound of ρ)	**0.5**
ρ1 (True Value of ρ)	**0.6**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results

Solve For: Number of Subjects (N)
Confidence Interval: Lower, One-Sided
Calculation Method: Exact, Based on F Distribution
ρ: Intraclass Correlation Coefficient

Confidence Level 1 - α	Number of Subjects N	Observations per Subject K	Lower Confidence Bound of ρ ρ0	True Value of ρ ρ1	Distance from ρ0 to ρ1 ρ1 - ρ0	Target Assurance Probability 1 - γT	Actual Assurance Probability 1 - γA
0.95	416	2	0.5	0.6	0.1	0.9	0.90045
0.95	202	4	0.5	0.6	0.1	0.9	0.90030
0.95	162	6	0.5	0.6	0.1	0.9	0.90074
0.95	145	8	0.5	0.6	0.1	0.9	0.90049
0.95	136	10	0.5	0.6	0.1	0.9	0.90089

Report Definitions

Confidence Level is the proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the true correlation.
N is the number of subjects in the random sample drawn from the population.
K is the number of observations obtained for each subject.
ρ0 is the lower confidence limit of ρ.
ρ1 is the true value of the intraclass correlation.
ρ1 - ρ0 is the distance from ρ0 to ρ1. It is a measure of the half width of the confidence interval. Since the confidence limits are not symmetric, this is not the exact half width.
1 - γT is the target assurance probability that the confidence interval will include the true value of ρ.
1 - γA is the actual assurance probability that is achieved for this sample size.

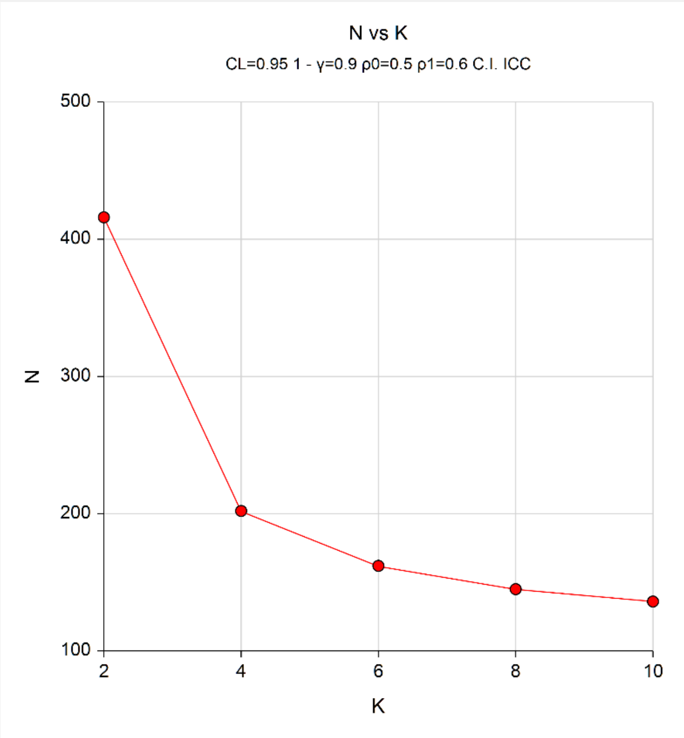
References
Zou, G.Y. 2012. 'Sample size formulas for estimating intraclass correlation coefficients with precision and assurance.' Statistics in Medicine, Vol 31, 3972-3981.
Donner, A, Eliasziw, M. 1987. 'Sample size requirements for reliability studies.' Statistics in Medicine, Vol 6, 441-448.
Bartko, John J. 1966. 'The intraclass correlation coefficient as a measure of reliability.' Psychological Reports, Vol 19, 3-11.

Summary Statements
A random sample of 416 subjects who are each measured 2 times produces a 95% lower confidence bound for the ICC of 0.5 when the true value is 0.6. The assurance probability is 0.9. The data will be analyzed using a one-way ANOVA model.

This report shows the calculated sample size for each of the scenarios.

Chart Section

Chart Section



This plot shows the sample size versus the value of K.

Example 2 – Validation using Zou (2012)

Zou (2012), page 3978, gives example calculations of the number of subjects needed for a one-sided confidence interval of ICC when the confidence level is 95%, ρ1 is 0.60, ρ0 is 0.50, and K is 2. The result for N is 300 using the Fisher's transformation method.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Number of Subjects (N)**
Calculation Method	**Approximate Fisher Transformation**
Confidence Level (1 - α)	**0.95**
1 - γ (Assurance Probability)	**0.8**
K (Observations per Subject)	**2**
ρ0 (Lower Confidence Bound of ρ)	**0.5**
ρ1 (True Value of ρ)	**0.6**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results

Solve For: Number of Subjects (N)
Confidence Interval: Lower, One-Sided
Calculation Method: Approximate, Based of Fisher's Transformation
ρ: Intraclass Correlation Coefficient

Confidence Level 1 - α	Number of Subjects N	Observations per Subject K	Lower Confidence Bound of ρ ρ0	True Value of ρ ρ1	Distance from ρ0 to ρ1 ρ1 - ρ0	Target Assurance Probability 1 - γT	Actual Assurance Probability 1 - γA
0.95	300	2	0.5	0.6	0.1	0.8	0.80022

PASS matches Zou's results exactly. We set the Calculation Method to Exact F Distribution and reran the procedure. This time, N was calculated to be 301. We note that in this case, the results of both methods are very close to each other.