Chapter 829

Confidence Intervals for Intraclass Correlation with Assurance Probability (Two-Sided)

Introduction

This routine calculates the sample size needed to obtain a specified half-width of two-sided confidence interval of the intraclass correlation coefficient (ICC). This procedure allows you to set an *assurance probability* that the requested width is achieved. Note that another **PASS** procedure computes the sample size for a one-sided confidence interval for the ICC.

The ICC is the product-moment correlation calculated among observations on the same subject. For example, if you have three raters rating each subject, it is the average correlation among the ratings of the three raters. This procedure is often used in reliability studies.

The ICC analyzed in this procedure comes from a one-way random effects ANOVA model.

Technical Details

Zou (2012) presents formulas used for constructing a two-sided, 100(1 – α)% confidence interval for the ICC. We adopt his notation as we present these formulas.

Suppose that each of *N* subjects (or clusters) is rated by *K* raters. The observations obtained may be from different raters, instruments, or other measurement mechanisms. Such data may be analyzed using a one-way, random-effects, model. The ANOVA model is

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \text{ where } \alpha_i \sim N(0, \sigma_\alpha^2) \text{ and } \varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2).$$

Estimation of ρ

The ICC is defined as

$$\rho = \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma_\varepsilon^2}$$

The ICC is estimated from the mean squares of the ANOVA table as follows

$$r = \frac{MS_B - MS_E}{MS_B + (K-1)MS_E}$$

where MS_B is the between-subject mean square and MS_E is the within-subject mean square.

The large sample variance of r is given by

$$\mathrm{Var}(r) = \frac{2(1-\rho)^2[1+(K-1)\rho]^2}{K(K-1)(N-1)}$$

This can be estimated by replacing ρ with r as follows

$$\widehat{\mathrm{Var}}(r) = \frac{2(1-r)^2[1+(K-1)r]^2}{K(K-1)(N-1)}$$

A large sample, two-sided confidence interval using a Wald statistic is obtained as follows

$$r \mp z_{1-\alpha/2}\sqrt{\widehat{\mathrm{Var}}(r)}$$

This procedure will only provide results for two-sided confidence intervals.

Sample Size Calculation

The sample size is determined so that the probability that the half-width of the requested interval is achieved is above a specified value. This probability is called the *assurance probability* and is referred to as $1-\gamma$.

The assurance probability requirement is written as

$$1-\gamma = \Pr\left(z_{1-\alpha/2}\sqrt{\widehat{\mathrm{Var}}(r)} \le \omega\right)$$

This can be solved for sample size to obtain the following formula

$$N = 1 + \left[\frac{Az_{1-\frac{\alpha}{2}} + \sqrt{\left(Az_{1-\frac{\alpha}{2}}\right)^2 + 4\omega ABz_{1-\gamma}z_{1-\frac{\alpha}{2}}}}{\omega\sqrt{2K(K-1)}}\right]^2$$

where

$$A = (1-\rho)[1+(K-1)\rho]$$

$$B = K - 2 + 2\rho(1-K)$$

This can be rearranged to solve for the assurance probability or ω as well.

Confidence Level

The confidence level, $1 - \alpha$, has the following interpretation. If thousands of samples of n items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population correlation is $1 - \alpha$.

Example 1 – Calculating Sample Size

Suppose a reliability study is planned to find an estimate of the two-sided 95% confidence interval for the ICC. The researcher would like to examine values of *K* from 2 to 10. The goal is to determine the necessary sample size, N, when the assurance probability is 0.9, the half width is 0.1, and the planning value of the ICC is 0.6.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Number of Subjects (N)**
Confidence Level (1 - α)	**0.95**
1 - γ (Assurance Probability)	**0.9**
K (Observations per Subject)	**2 4 6 8 10**
ω (Half-Width of Confidence Interval)	**0.1**
ρ1 (True Value of ρ)	**0.6**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results

Solve For: Number of Subjects (N)
Confidence Interval: Two-Sided
ρ: Intraclass Correlation Coefficient

Confidence Level 1 - α	Number of Subjects N	Observations per Subject K	Half Width of C.I. ω	True Value of ρ ρ1	Target Assurance Probability $1 - \gamma_T$	Actual Assurance Probability $1 - \gamma_A$
0.95	196	2	0.1	0.6	0.9	0.90621
0.95	96	4	0.1	0.6	0.9	0.90443
0.95	77	6	0.1	0.6	0.9	0.90224
0.95	70	8	0.1	0.6	0.9	0.92391
0.95	65	10	0.1	0.6	0.9	0.90879

Report Definitions

Confidence Level is the proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the true ICC.
N is the number of subjects in the random sample drawn from the population.
K is the number of observations obtained for each subject.
ω is the half-width of the symmetric, two-sided confidence interval of ρ.
ρ1 is the true (planning) value of the intraclass correlation.
$1 - \gamma_T$ is the target assurance probability that the half-width of the confidence interval will be less than ω.
$1 - \gamma_A$ is the actual assurance probability that is achieved for this sample size.

References

Zou, G.Y. 2012. 'Sample size formulas for estimating intraclass correlation coefficients with precision and assurance.' Statistics in Medicine, Vol 31, 3972-3981.
Bartko, John J. 1966. 'The intraclass correlation coefficient as a measure of reliability.' Psychological Reports, Vol 19, 3-11.

Summary Statements

A random sample of 196 subjects who are each measured 2 times produces a 95% two-sided confidence interval for the ICC when the true value is 0.6. The half-width of the confidence interval is less than or equal to 0.1. The assurance probability that the actual half-width is no more than the specified value is 0.9. The data will be analyzed using a one-way ANOVA model.

This report shows the calculated sample size for each of the scenarios.

Chart Section

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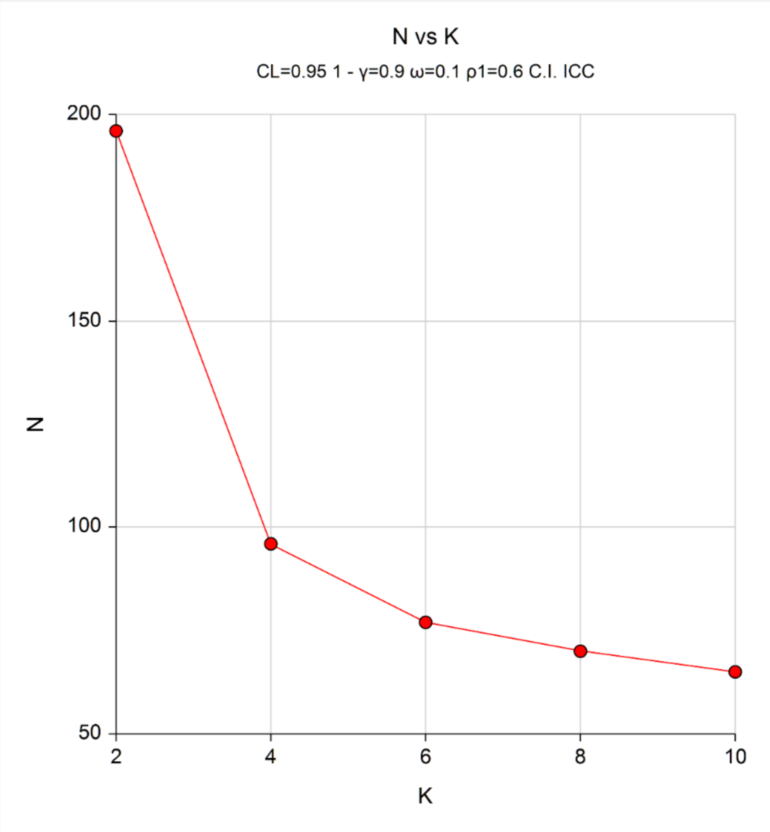

This plot shows the sample size versus the value of K.

Example 2 – Validation using Zou (2012)

Zou (2012), page 3976, in Table 1 gives example calculations of the number of subjects needed of a two-sided confidence interval of ICC when the confidence level is 95%, ρ1 is 0.60, ω is 0.10, $1-\gamma$ is 0.80, and K is 2. The result for N is 183.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Number of Subjects (N)**
Confidence Level (1 - α)	**0.95**
1 - γ (Assurance Probability)	**0.8**
K (Observations per Subject)	**2**
ω (Half-Width of Confidence Interval)	**0.1**
ρ1 (True Value of ρ)	**0.6**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results

Solve For: Number of Subjects (N)
Confidence Interval: Two-Sided
ρ: Intraclass Correlation Coefficient

Confidence Level 1 - α	Number of Subjects N	Observations per Subject K	Half Width of C.I. ω	True Value of ρ ρ1	Target Assurance Probability 1 - γT	Actual Assurance Probability 1 - γA
0.95	183	2	0.1	0.6	0.8	0.80198

PASS matches Zou's results exactly.